Math 141A Mathematical Logic I

Spring 2022

Instructor:	Assaf Shani	**Time:**	1:30 - 2:45 pm, TTh
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Course Page: Canvas - please monitor the Canvas page regularly for information and updates.

Office Hours: TBA

You are encouraged, and always welcome, to come to office hours. Feel free to contact us by email as well.

Prerequisites: It is required to have taken a rigorous proof-based class in pure mathematics, and be familiar with formal proofs. Specifically, a course at the level of 25ab, 55ab, 101, or 112.

Course Outline: Very roughly speaking, mathematical logic studies mathematical objects by first formalizing them in a precise "mathematical language", and then studying how these objects can be defined (or expressed) in this language.

For example, the "language of fields" allows you to talk about addition $+$, multiplication $\cdot$, and the symbols 0 and 1. The "Field Axioms" can then be formulated as "sentences in this language". Fields, such as the rational numbers, the real numbers, and the complex numbers, are *mathematical models* which satisfy these axioms. Similarly, there is a language for groups, graphs, and other mathematical structures.

We will develop in generality the notion of a "mathematical structure in some language", formalize notions such as *truth* (of statements in a structure), *provability* (of a statement from a set of axioms), and *isomorphism* between structures (that is, when two structures are essentially the same).

One basic question is, given some axioms (assumptions), is there a structure satisfying such axioms? The Completeness theorem, which we will prove, asserts that the answer is always yes, as long as there is no "evident logical contradiction between the axioms". We will also see applications, such as for the study of chromatic numbers of graphs, and the formalization of non-standard analysis.

Other fundamental questions we will investigate is, given some axioms, what do models of these axioms look like? What kind of different properties can these models have? How many models are there? When is there

precisely 1 model, up to isomorphism? (That is, when do the axioms truly describe a unique object.)

Assessment: The grade will be based on the Psets. Possibly with a longer one at the end.

Psets:

- The Psets are a crucial part of the course, and may include important results and definitions.
- *Resubmission*: You may resubmit each Pset once, if necessary, to improve your score. See rules for resubmission below.
- The Psets will be graded strictly. To get credit, your solutions need to show that you understand the material. (This goes hand in hand with the resubmission policy.)
- Late submission will not be accepted.
- It is encouraged to discuss the course material, in particular homework problems, with your classmates.
  However, *your final submitted work must be your own writing in your own words and must not be copied from any source, written or verbal.*
  List your collaborators on each assignment (if any).
- Please make sure your homework submission is legible, clear, and well organized.
  If possible it is recommended to type the assignment (or parts of it) in Latex or a similar program. This is not mandatory, but experience in Latex will also be useful to you later on in your academic studies.
- Solutions to the homeworks will *not* be published.
  It is your responsibility to make sure you understand the material and the Psets. Feel free to ask any questions or seek help at any time.

Resubmission policy:

- You may only resubmit a Pset if a true and honest attempt was made to begin with. *This will be judged by the grader of the Pset.* For example, you may not simply not submit the Pset, and then "resubmit" it late.

  This is the case for each individual question. So if you submitted only some questions of a Pset, you may only resubmit those questions (assuming a true attempt was made).

- *Collaboration is not allowed for resubmission.* After you get your assignment back, if you plan to resubmit, you may no longer collaborate or use any resource. You may not look at the solutions of a classmate or discuss the problems with a classmate, nor look up solutions online. You *are allowed* to discuss the Pset with the teaching staff in office hours.
- Only one resubmission is allowed per Pset. You must resubmit within 2 weeks of receiving the graded Pset.

Grading concerns: Any questions regarding grading must be addressed within two weeks of the material being passed back.

Grading philosophy: When grading, I hope to focus on the key mathematical ideas behind the problem. The best way for you to get the most points is to clearly state what you know and don't know, your ideas and intentions.

Writing things that are irrelevant, completely incorrect, or show a serious lack of understanding of the material, may result in no credit at all. Completely irrelevant arguments may be deemed as not an honest attempt to solve the problem. Also, writing things that may deserve partial credit, but incorrectly presenting them as a full proof or complete solution, may result in no credit at all.

If you only have a partial solution, or some meaningful things to say about the problem, the best thing to do is to say it as it is, explain what you are proving and what you are not, and if possible explain what is missing.

Finally, your solutions are expected to be clear, well written, and mathematically accurate.

Seeking help: Feel free to come to office hours, or contact me by email, regarding anything you would like to discuss. I am here to help! For example, if you have general questions about the class, questions about your standing in the class; questions regarding the material; questions about the assignments.

In this class we will cover several topics, which take time to process and fully understand. Asking questions, getting help, and discussing the material with others, are important parts of the process. It is best to seek help early, and not let things pile up. You are expected to invest a significant amount of time and energy to study the topics of this course, and I am here to help you learn.

Also, please feel free to communicate with me any non-academic issues that may interfere with your ability to fulfil the course work or succeed in the class.

Statement on student wellness: As a student, you may experience a range of challenges that can interfere with learning, such as strained relationships, increased anxiety, substance use, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may diminish your academic performance and/or reduce your ability to participate in daily activities.

If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, I strongly encourage you to seek support. Counseling and Mental Health Services (CAMHS) is here to help: call 617-495-2042 or visit their website at https:// camhs.huhs.harvard.edu/find-help-now. Consider reaching out to a friend, faculty or family member.

Accommodations for students with disabilities: Students needing academic adjustments or accommodations because of a documented disability must present their Faculty Letter from the Accessible Education Office (AEO) and speak with the professor by the end of the second week of the term. Failure to do so may result in the Course Head's inability to respond in a timely manner. All discussions will remain confidential, although Faculty are invited to contact AEO to discuss appropriate implementation.