# Math 118 Dynamical Systems

Spring 2023

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| **Instructor:** | Assaf Shani | **Time:** | 9:00 - 10:15 am, TTh |
| **Email:** | shani@math.harvard.edu | **Place:** | SC 507 |

**Course Page:** Canvas - please monitor the Canvas page regularly for information and updates.

**Course Assistants:**

- Alex Karbowski - akarbowski@college.harvard.edu
- Calvin Osborne - cosborne@college.harvard.edu

**Office Hours:** TBA

You are encouraged, and always welcome, to come to office hours. Feel free to contact me by email as well.

**Prerequisites:** Familiarity with calculus and linear algebra, Math 21, 22, 25, or 55.
Basic properties of the real numbers, and exposure to formal mathematics, such as the definition of a limit and the $\varepsilon - \delta$ definition of a continuous function. (These may be covered in Math 112, Math 101, Math 55, or others.)
See for example section 1.2 in Devaney's book. See also A.1 in Hasselblatt-Katok book.

**Course Outline:** Our main object of study will be an iterative dynamical system. That is, a space $X$, and a transition function $T\colon X \to X$. The space $X$ will often be the space of real numbers, or the unit circle. The general goal is to study the orbits:

$$x_0,\ \mathrm{T}(x_0),\ \mathrm{T}(\mathrm{T}(x_0)),\ \mathrm{T}(\mathrm{T}(\mathrm{T}(x_0))),\ \ldots$$

where $x_0$ is some initial value.

A central theme in our study will be predictability vs. chaos. That is, when we can predict the long-term behaviour of such orbit, or when the behavior is chaotic and unpredictable.

Our focus will be on the abstract mathematical theory. For example, among the results we will cover will be:

- Finding and analyzing periodic points. Attracting and repelling periodic points.
- Sharkovskii's theorem about periodic points for iterative dynamical systems on the real line.
- Analyzing the dynamical behavior for the functions $T_\lambda(x) = \lambda x(1-x)$, depending on the value of $\lambda$. (In particular, determining when it is predictable and when it is chaotic.)
- Dynamics on the circle: we will try to understand the possible dynamical behavior for arbitrary continuous invertible functions from the circle to itself.
- Symbolic dynamics: "the shift system" on the space of sequences. The shift system, and its subsystems, are important examples of dynamical systems. Furthermore, they provide an essential tool to study the dynamical behavior of other dynamical systems, such as the ones mentioned above.

**Assessment:**

- 80% of the grade will be based on the Psets.
- 20% will be a final project.

**Psets:**

- The Psets are a crucial part of the course, and may include important results and definitions.
- The Psets will be graded strictly. To get credit, your solutions need to show that you understand the material.
- Late submission will not be accepted.
- The 2 Psets with the lowest score will be ignored for the grade calculation.
- It is encouraged to discuss the course material, in particular homework problems, with your classmates.
  However, **your final submitted work must be your own writing in your own words and must not be copied from any source, written or verbal**.
- Please make sure that your homework submission is legible, clear, and well organized.

- Solutions to the homeworks will *not* be published.
  It is your responsibility to make sure you understand the material and the Psets. Feel free to ask any questions or seek help at any time.

**Grading concerns:** Any questions regarding grading must be addressed within *one week* of the material being passed back.

**Grading philosophy:** The best way for you to get the most points is to clearly state what you know and don't know, your ideas and intentions.

Writing things that are irrelevant, completely incorrect, or show a serious lack of understanding of the material, may result in no credit at all. Also, writing things that may deserve partial credit, but incorrectly presenting them as a full proof or complete solution, may result in no credit at all.

If you only have a partial solution, or some meaningful things to say about the problem, the best thing to do is to say it as it is, explain what you are proving and what you are not, and if possible explain what is missing.

Finally, your solutions are expected to be clear, well written, and mathematically accurate.

**Seeking help:** Feel free to come to office hours, or contact me by email, regarding anything you would like to discuss. I am here to help! For example, if you have general questions about the class, questions about your standing in the class; questions regarding the material; questions about the assignments.

In this class we will cover several topics, which take time to process and fully understand. Asking questions, getting help, and discussing the material with others, are important parts of the process. It is best to seek help early, and not let things pile up. You are expected to invest a significant amount of time and energy to study the topics of this course, and I am here to help you learn.

Also, please feel free to communicate with me any non-academic issues that may interfere with your ability to fulfil the course work or succeed in the class.

**Statement on student wellness:** As a student, you may experience a range of challenges that can interfere with learning, such as strained relationships, increased anxiety, substance use, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may diminish your academic performance and/or reduce your ability to participate in daily activities.

If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, I strongly encourage you to seek support. Counseling and Mental Health Services (CAMHS) is here to help:

call 617-495-2042 or visit their website at https:// camhs.huhs.harvard.edu/find-help-now. Consider reaching out to a friend, faculty or family member.

**Accommodations for students with disabilities:** Harvard University values inclusive excellence and providing equal educational opportunities for all students. Our goal is to remove barriers for disabled students related to inaccessible elements of instruction or design in this course. If reasonable accommodations are necessary to provide access, please contact the Disability Access Office (DAO). Accommodations do not alter fundamental requirements of the course and are not retroactive. Students should request accommodations as early as possible, since they may take time to implement. Students should notify DAO at any time during the semester if adjustments to their communicated accommodation plan are needed.