FINAL PROJECT

Tentative - subject to small changes.

- Deadline: May 5th.
- Collaboration is <u>not</u> allowed for the final project.
- Your submission should be written by you, in your own words, and not be taken from any other source, written or verbal.
- You are allowed to discuss the material with Alex and Calvin, but only the material which we covered in class and in the Psets.
- You are <u>not</u> allowed to discuss with Alex and Calvin (or your fellow students):
  - Particular question which are relevant to your project and were not previously assigned or discussed in class.
  - Guidance and suggestions regarding how to write the final project.
  - Asking to review and give feedback on a written part of the project.
- You may reach out to me to ask for guidance.
- **Each project includes some review and summary of material covered in class and / or read from a book.**
  - Your write-up has to be in your own words, reflect your understanding, and your point of view. Imagine that you are explaining things to someone who is not familiar with it. Try to think about how you want to present things, what ideas you want to emphasize.
  - It must be very clear that you understand what is going on. Here is a quick test: imagine I ask you about a certain sentence or paragraph, why is it there? what does it mean? why is it true? If you are not sure, this is likely reflected in the write-up as well.
  - If you are writing a proof for an exercise or of something you read in a book, it has to be extremely clear that you understand the proof. Again, imagine being asked about a step in the proof. If you are not sure why it is there, or how to justify it, then this gap is very clear to anyone trying to read it.
- Some suggestions:
  - For writing solutions to problems: definitely learn from any feedback, criticism, and guidance provided by Alex and Calvin over the Psets.
  - For writing a proof of something from a book: think of the (many) proofs, especially in [D] and [HK], where you look at it, and it doesn't make sense. You will not get any credit for this kind of write-up. Compare it to the way we presented things in class: we had to justify many steps in the proofs; we tried to understand why these steps are necessary; we analyzed simplified cases; we emphasized where the assumptions of a theorem are used in the proof. (Moreover, a written proof should include more full details than one presented in class.)

1. Projects

Your submission should include:

- A review / summary / story, discussing the relevant material we covered in class, and the new results which are part of the project. **This is the main and most important part of the project.** Your summary should include motivations and particular results. It should show that you understand the results from the course.
- Solution to the designated exercises.
- Full write up of designated theorems.

The presentation is up to you. You may separate the solutions to exercises or proofs of theorems from the summary part, or you can weave them together.

Each project contains a few key emphasized terms, just to give you some ideas. Again it is up to you do decide what you want to say and emphasize, and how you want to say this. There is an expectation that all results from class which are obviously relevant to the subject should be mentioned to some extent.

Note that a particular topic from class will likely be relevant to several (if not all) projects.

Choose one of the following as your final project.

1.1. The family of quadratic maps $F_\lambda(x) = \lambda x(1-x)$ and interval maps.

Basic model related to population dynamics. Bifurcation. Predictable vs chaotic behavior. Restriction on dynamics for continuous interval maps.

Read and write-up proof of [G, Theorem 12.1.1].

Question 1. Exercises 2 and 3 on [G, page 206]. Is this relevant to our study of the quadratic family?

Question 2. Exercises 1 through 4 on [D, page 68]. (We have sketched these ideas in class.)

Read [D, Section 1.17, pages 130-137].

Question 3. Problems 1 through 12 in [D, pages 138-139].

1.2. Maps on the circle.

Topological conjugacy. Degree and rotation number. Emphasis on order-preserving circle homeomorphisms and classification. Denjoy's theorem.

Read and write-up [HK, Section 4.1.4]. (Equidistribution for irrational rotations.) Write up a full proof of Proposition 4.1.7.

Another application of circle rotations:
Read and write-up [HK, Section 4.2.2, pages 111-112]

Question 4 (Denjoy's example)**.** Prove that Denjoy's "counterexample" can be a C^1-diffeomorphism. Exercise 5 in [D, p. 112-113]. (See also [HK, Section 4.4.3])

Question 5. Exercise 4.3.5 in [HK, p.135].

1.3. **On the notion of chaos.** Devaney's three conditions for chaos. Implications and non-implications between them. Circle maps and the quadratic family. Interval maps.

Read [G, Section 17.4], write-up a proof of [G, Theorem 17.4.6]. Note that we defined "transitive" to be part (c) on the theorem. In [G] "transitive" is defined as having some point with a dense orbit (see [G, Definition 6.2.1]). [Remark: (1) The concept of a *complete* and *separable* metric space is important here. See the Baire Category Theorem in [G, Appendix B]. (2) You can also find this result in Section 7.2.1 of [HK]].

Question 6 (G, Exercises 17.4, p. 325)**.** Questions 1, 2, 11.

Read [G, Section 17.5]

Question 7 (G, Exercises 17.5, p. 332)**.** Questions 1, 3, 6.

1.4. **Symbolic dynamics.** The space of sequences and shift map. Relevance to other dynamical systems, e.g., circle maps and quadratic family. Devaney's definition of chaos.

Read and write-up [G, chapter 20], (topological) multiple recurrence theorem and application to Van-der Warden theorem.

Question 8 (G, Exercises 20.1, p. 382)**.** Questions 1,2,3,4.

This is another kind of application for symbolic dynamics. You will need to go over some other concepts from earlier chapters.