Math 23a Practice Multiple Choice Questions

1. Which of these functions is **not** uniformly continuous on $(0, 1)$?
   (a) x^2
   (b) $1/x^2$
   (c) $f(x) = 1$ for $x \in (0, 1), f(0) = f(1) = 0$
   (d) $\sin(x)$
   (e) $\frac{\sin(x)}{x}$

2. Let s_n be a sequence of real numbers on a bounded set S, where $\liminf s_n \neq \limsup s_n$. Which of the following is not necessarily true?
   (a) $\lim s_n$ does not exist.
   (b) s_n is not Cauchy.
   (c) $\liminf s_n < \limsup s_n$
   (d) There exists a convergent subsequence.
   (e) s_n has an infinite number of dominant terms.

3. Which of the following is not true about $s_n = \frac{1}{n}$?
   (a) The sequence converges to 0.
   (b) $\lim_{n\to\infty} \displaystyle\sum_{i=1}^{n} s_i = L$, for some finite L.
   (c) $\limsup s_n = 0$.
   (d) The series $\sum(-1)^n s_n$ converges.
   (e) The series $\sum s_n^2$ converges.

4. Let $\sum a_n$ be a conditionally convergent series. Which of the following is not necessarily true?
   (a) The series converges to some finite L.
   (b) The series sum is independent of order of terms.
   (c) $\displaystyle\sum |a_n|$ diverges.
   (d) $\lim(-1)^n a_n = 0$.
   (e) None of the above. They're all necessarily true.

5. Which of the following series converges? **THERE ARE TWO ANSWERS**

   (a) $\sum \frac{x^n}{n!}$, $\forall x$

   (b) $\sum \frac{1}{n+\sin(n)}$

   (c) $\sum (-1)^n n$

   (d) $\sum \sin(n)$

   (e) $\sum \frac{2^n}{\sqrt{n!}}$

6. Which of the following must be true of a continuous function on (a, b)?

   (a) The function achieves its maximum on (a, b).

   (b) The function is bounded.

   (c) For all Cauchy Sequences s_n on the set (a, b), $f(s_n)$ is also Cauchy.

   (d) If $f(a) = 2$, and $f(b) = 5$, then $f(c) = 3$, for some $c \in (a, b)$.

   (e) None of the above.

7. Which of the following is not necessarily true about a uniformly continuous function, f, on $[a, b]$? **THERE ARE THREE ANSWERS**

   (a) The function is bounded.

   (b) The function achieves its maximum on the set (a, b).

   (c) If $f(a) = 4$ and $f(b) = 6$, then $f'(c) = 2$ for some $c \in (a, b)$.

   (d) The derivative f' is bounded.

   (e) If $f'(a) = 3$, and $f'(b) = 4$, then $f'(c) = 3.5$ for some $c \in (a, b)$.

8. Find $\lim_{x \to b} \frac{\sqrt{x} - \sqrt{b}}{x - b}$ for $b > 0$.

   (a) ∞

   (b) $\frac{1}{2\sqrt{b}}$

   (c) 0

   (d) $2\sqrt{b}$

   (e) b

9. Let f be a differentiable function, where all derivatives exist, such that $f(0) = 0$, $f'(0) = 0$, and $|f''(x)| \le M, \forall x$. Which of the following is not necessarily true?

   (a) $f(1) \le \frac{M}{2}$

   (b) 0 is neither a maximum nor a minimum.

   (c) $\forall \epsilon > 0, \exists \delta > 0$ s.t. if $x \in (-\delta, \delta), |f(x)| < \epsilon$

   (d) If $\lim s_n = 0$, then $\lim f(s_n) = 0$.

   (e) None of the above.