MATHEMATICS E23a, FALL 2016
Linear Algebra and Real Analysis I

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Course Website: `https://canvas.harvard.edu/courses/18236`

Instructor: Paul Bamberg (to be addressed as "Paul," please)
Paul graduated from Harvard in 1963 with a degree in physics and received his doctorate in theoretical physics at Oxford in 1967. He taught in the Harvard physics department from 1967 to 1995 and joined the math department in 2001. From 1982 to 2000 he was one of the principals of the speech recognition company Dragon Systems. If you count Extension School and Summer School, he has probably taught more courses, in mathematics, physics, and computer science, than anyone else in the history of Harvard. He was the first recipient of the White Prize for excellence in teaching introductory physics.

This term, Paul is also teaching Math 152, "Discrete Mathematics," Math 117/E-217, "Probability and Random Processes with Economic Applications," and Math 116, "Real Analysis, Convexity, and Optimization."
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Course Assistants:(all former students in Math 23a or Math E-23a)

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Goals: Math E-23a is a repeat of the 2015 edition of Math 23a, the first half of a moderately rigorous course in linear algebra and multivariable calculus that was designed for students who are serious about mathematics and interested in being able to prove the theorems that they use but who are as much concerned about the application of mathematics in fields like physics and economics as about "pure mathematics" for its own sake. Trying to cover both theory and practice makes for a challenging course with a lot of material, but it is appropriate for the audience!

Students in the course generally are strengthening their background in proof-based mathematics in preparation for graduate wiork in fields like economics, appled mathematics, or statistics

Prerequisites: This course is designed for the student who received a grade of 5 on the Math BC Advanced Placement examination or an A in a college course in single-variable calculus like Math E-16. Probably the most important prerequisite is the attitude that mathematics is fun and exciting.

Our assumption is that the typical Math E-23a student knows only high-school algebra and single-variable calculus and is currently better at formula-crunching than at doing proofs.We do not assume that Math E-23 students have any prior experience in either of these areas beyond solving systems of linear equations in high school algebra.

We will devote four weeks to single-variable real analysis. Real analysis is the study of real-valued functions and their properties, such as continuity, and differentiability, as well as sequences, series, limits, and convergence. This means that if you had a calculus course that touched only lightly on topics like series, limits, and continuity, you will be OK.

Students who register for graduate credit are required to learn and use the scripting language R. This option is also available to everyone else in the course. You need to be only an experienced software user, not a programmer.

Classes

Videos have been made of all the lectures, and these will be posted on the Web site. The course materials for each week list the dates (from 2015) of the relevant videos.

There is also an optional two-hour class on each week's material.

Classes will meet on Friday, Sunday, or Monday following the week in which you are expected to watch the lectures. Members of the class will present the 2-4 required proofs during the first 20-45 minutes. The remainder of the class time will be devoted to problem solving in small groups. If you choose the "participation" option, these will count toward your grade. Lecture videos are crucial background for class!

If you prefer, you can opt out of class participation. In that case you do not attend class, and homework and exams will count for a greater fraction of your grade. You may of course attend office hours to get help with the homework, and you will have access to the solutions to problems that were solved in class and perhaps also to videos of proofs that were presented in class..

Some of the classes will be online, and one will meet at a time that is reasonable for students in Europe. Paul will run a class in Science Center 104 from 2:10-4:00 on Monday afternoons.

The course assistants will hold office hours, some online and some in Cambridge, at which students can get together to discuss homework problems and present proofs to one another. Participation is optional but is strongly recommended.

Yes, this course is a lot of work, but it appears to cover all the math that admissions committees are looking for. Think of it as two courses for the price of one!

There will be no Math E-23b in the spring this year, since Paul is on sabbatical then. However, the topics in Math 23b, multiple intgration and differential forms, are not crcuial for most of the fields in which Extension students are interested. The appropriate followup course in Math E-156, "Mathematical Foundations of Statistical Software,"

Often the first few minutes of a lecture video are devoted to administrative announcements which were relevant only in 2015. You can skip over them.

For each lecture, there is also a scan of Paul's lecture notes on the Web site. Consensus is that you do will do best to take your own notes and use these only as backup.

Some students report that it is efficient to play the easy parts of the lecture at double speed, then to slow down and perhaps even to run twice though difficult proofs or intricate examples.

Exams: There will be two quizzes and one final exam.

Quiz 1:	first weekend in October (module 1, weeks 1-4)
Quiz 2:	first weekend in November (module 2, weeks 5-8)
Final Exam:	in mid-December (module 3, weeks 9-12)

Quizzes can be done at home. You will need to find a informal proctor (family members or roommates are OK) who will sign a statement that you completed the quiz under closed-book conditions, with computers and cell phones turned off. You must complete the quiz at a single sitting during a specified 24-hour period, but there is otherwise no time limit. Two hours should be more than sufficient.

Quizzes will include questions that resemble the ones done in class, and each quiz will include two randomly-chosen proofs from among the numbered proofs in the relevant module. There may be other short proofs similar to ones that were done in lecture and problems that are similar to homework problems.

The final examination will focus on material from the last five weeks of the course. Students who live in New England will take a proctored three-hour exam in Cambridge. Others will have to find an official proctor using the Extension School's Distance Exams office.

If you have an unexpected time confilct for one of the quizzes, contact Paul as soon as you know about it, and special arrangements can be made. Last year, one student took a quiz while flying to Europe for an interview, with the flight attendant serving are her proctor! Time conflicts involving the final exam must be dealt with by the Distance Exams Ofiice.

Textbooks:

Vector Calculus, Linear Algebra, and Differential Forms, Hubbard and Hubbard, fourth edition, Matrix Editions, 2009. You can probably get a used copy at the Harvard Coop or on Amazon, and the authors have found 15 copies that will become available on the Matrix Ediioins website.

We will cover only Chapters 1-3. Almost all the changes in the fifth edition come later in the book.

Ross, Elementary Analysis: The Theory of Calculus, 2nd Edition, 2013.

This will be the primary text for the module on single-variable real analysis. It is available electronically through the Harvard library system (use HOLLIS and search for the author and title). If you like to own bound volumes, used copies can be found on amazon.com for as little as $25, but be sure to get the correct edition!

Lawvere, Conceptual mathematics: a first introduction to categories, 2nd Edition, 2009.

We will only be using the first chapter, and the book is available for free download through the Harvard library system.

Proofs:

Learning proofs can be fun, and we have put a lot of work into designing an enjoyable way to learn high level and challenging mathematics! Each week's course materials includes two proofs. Often these proofs appear in the textbook and will also be covered in lecture. They also may appear as quiz questions.

You, as students, can earn points towards your grade by presenting these proofs to teaching staff and to each other without the aid of your course notes. Here is how the system works:

When we first learn a proof in class, only members of the teaching staff are "qualified listeners." Anyone who presents a satisfactory proof to a qualified listener also becomes qualified and may listen to proofs by other students. This process of presenting proofs to qualified listeners occurs separately for every proof.

You are expected to present each proof before the date of the quiz on which it might appear; so each proof has a deadline date. Distance students may reference the additional document which details how to go about remotely presenting proofs to classmates and teaching staff.

Each proof is worth 1 point. Here is the grading system:

- Presenting a proof to Paul, Joe, one of the course assistants, or a fellow student who has become a qualified listener: 0.95 points before the deadline, 0.8 points after the deadline. You may only present each proof once.

- Listening to a fellow student's proof: 0.1 point. Only one student can receive credit for listening to a proof.

- After points have been tallied at the end of the term, members of the course staff may assign the points that they have earned by listening to proofs outside of section to any students that they feel deserve a bit of extra credit.

Students who do the proofs early and listen to lots of other students' proofs can get more than 100%, but there is a cap of 30 points total.You can almost reach this cap by doing each proof before the deadline and listening twice to each proof.

Either you do a proof right and get full credit, or you give up and try again later. There is no partial credit. It is OK for the listener to give a couple of small hints.

You may consult the official list of proofs that has the statement of each theorem to be proved, but you may not use notes. That will also be the case when proofs appear on quizzes and on the final exam.

It is your responsibility to use the **proof logging software on the course Web site** to keep a record of proofs that you present or listen to. You can also use the proof logging software to announce proof parties and to find listeners for your proofs.

Each quiz will include two questions which are proofs chosen at random from the four weeeks of relevant material. The final exam will have three proofs, all from material after the second quiz. Students generally do well on the proof questions.

Alternatively, you may opt out of the proof logging system. In that case you will need to do one more proof on each exam, and you will be expected, over the course of the term, to upload eight proofs to the course Web site.

Advice for the grade-conscious:

It is easy for a diligent student to get close to 100% for class participation and perhaps even a bit more than 100% for proof logging. However, if you do not attend class faithfully or do not present proofs before the deadline, you would probably have been better off just relying on homework and exams.

Useful software:

- R and RStudio
  This is required only for Extension students who register for graduate credit, but it is an option for everyone. Consider learning R if you
  - are interested in computer science and want practice in using software to do things that are more mathematical than can be dealt with in CS 50 or 51.
  - are thinking of taking a statistics course, which is likely to use R.
  - are hoping to get an interesting summer job or summer internship that uses mathematics or deals with lots of data.
  - want to be able to work with large data files in research projects in any field (life sciences, economics and finance, government, etc.)

  R is free, open-source software. Instructions for download and installation are on the Web site. You will have the chance to use R at the first class, so install it right away, preferably on a laptop computer that you can bring to class.

  On the course Website are a set of R scripts, with accompanying YouTube videos, that explain how to do almost every topic in the course by using R. These scripts are optional for undergraduate-credit students, but they will enhance your understanding both of mathematics and of R.

- LaTeX

  This is the technology that is used to create all the course handouts. Once you learn how to use it, you can create professional-looking mathematics on your own computer.

  The editor that is built into the Canvas course Web site is based on LaTeX.

  One of the course requirements is to upload four proofs to the course Web site in a medium of your choice. One option is to use LaTeX. Alternatively, you can use the Canvas file editor (LaTeX based), or you can make a YouTube video.

  I learned LaTeX without a book or manual by just taking someone else's files, ripping out all the content, and inserting my own, and so can you. You will need to download freeware MiKTeX version 2.9 (see `http://www.miktex.org`), which includes an integrated editor named TeXworks.

  From `http://tug.org/mactex/` you can download a similar package for the Mac OS X.

  When in TeXworks, use the Typeset/pdfLaTeX menu item button to create a .pdf file. To learn how to create fractions, sums, vectors, etc., just find an example in the lecture outlines and copy what I did. All the LaTeX source for lecture outlines, assignments, and practice quizzes is on the Web site, so you can find working models for anything that you need to do.

  The course documents contain examples of diagrams created using TikZ, the built-in graphics editor. It is also easy to include .jpg or .png files in LaTeX. If you want to create diagrams, use Paint or try Inkscape at `http://www.inkscape.org`, an excellent freeware graphics program. Students have found numerous other solutions to the problem of creating graphics, so just experiment.

Use of R:
You can earn "R bonus points" in three ways:

- By being a member of a group that uploads solutions to section problems that require creation of R scripts. These will be available most, but not all, weeks. (about 10 points)
- By submitting R scripts that solve the optional R homework problems (again available most, but not all, weeks). (about 20 points)
- By doing a term project in R. (about 20 points)

To do the "graduate credit" grade calculation, we wiil add in your R bonus points to the numerator of your score. To the denominator, we will add in 95% of your bonus points or 50% of the possible bonus points, whichever is greater. Earning a lot of R points is essential if you are registered for graduate credit. Otherwise,earning more than half the bonus points is certain to raise your percentage score a bit, and it can make a big difference if you have a bad day on a quiz or on the final exam.

Grades: Your course grade will be determined as follows:

- problem sets, 50 points. Your worst score will be converted to a perfect score.
- Optional participation
  - volunteering to present a proof in class, 12 points (1 per week)
  - participation in class, based on attendance, preparation, contributions to problem solving, and posting solutions to the Web site, 10 points.
- With proof logging
  - presenting and listening to proofs, 26 points.
  - two quizzes, 40 points each.
  - final exam, slightly more than 60 points.
- Without proof logging
  - uploading proofs to the Web site, 8 points.
  - two quizzes, 45 points each.
  - final exam, slightly more than 65 points.
- R bonus points, about 50 points in numerator, 25-45 points in denominator.

For graduate students, only a "graduate" percentage score, using the R bonus points, will be calculated. For everyone else, we will also calculate an "undergraduate" percentage score, ignoring the R bonus points, and we will use the higher of the two percentage scores.

The grading scheme is as follows:

Points	Grade
94.0%	A
88.0%	A-
80.0%	B+
75.0%	B
69.0%	B-
63.0%	C+
57.0%	C
51.0%	C-

If you are conscientious about the homework, proofs, and quizzes, you will end up with a grade between B plus and A, depending on your expertise in taking a fairly long and challenging 3-hour final exam, and you will know that you are thoroughly prepared for more advanced courses. For better or worse, you need to be fast as well as knowledgeable to get an A, but an A- is a reasonable goal even if you make occasional careless errors and are not a speed demon. Students who earned a B plus have been successful at getting into PhD programs.

There is no “curve” in this course! You cannot do worse because your classmates do better.

YouTube videos

- The Lecture Preview Videos were made by Kate Penner. They cover the so-called Executive Summaries in the weekly course materials, which go over all the topics, but without proofs or detailed examples.

  If you watch these videos (it takes about an hour per week) you will be very well prepared for the lecture videos, and even the most difficult material will make sense on a first hearing.

- The R script videos were made by Paul. They provide a line-by-line explanation of the R scripts that accompany each week's materials.

  If you are doing the "graduate" option, these scripts are pretty much required viewing, although the scripts are so thoroughly commented that just working through them on your own is perhaps a viable alternative.

  If you are doing just the "undergraduate" option, you can ignore the R scripts completely.

Homework: Homework (typically 8 problems) is assigned weekly. The assignment is included in the same online document as the lecture notes and section problems.

Assignments are due on Wednesdays by 11:59 PM. Submit a .pdf file to the Assignments page on the course Web site. If you write your assignment with pencil and paper, you will need access to a scanner that can create a single .pdf file from all the pages.

Each week's assignment will include a couple of optional problems whose solutions require R scripts. These scripts should be uploaded electronically to the Web site each week. Please include your name as a comment in the script and also in the file name.

The course assistant who grades for your section should return your corrected homework to you electronicall within a week after the due date. If you are not receiving graded homework on schedule, send email to Joe Palin and the problem will be dealt with.

Homework that is submitted after 11:59 PM on the Wednesday when it is due will not be graded. If it arrives before the final exam and looks fairly complete, you will get a grade of 50% for it.

Collaboration and Academic Integrity policy:

You are encouraged to discuss the course with other students and with the course staff, *but you must always write your homework solutions out yourself in your own words.* **You must write the names of those you've collaborated with at the top of your assignment.**

If you collaborate with classmates to solve problems that call for R scripts, create your own file after your study group has figured out how to do it.

Proofs that you submit to the course Web site must be done without consulting files that other students have posted!

If you have the opportunity to see a complete solution to an assigned problem, please refrain from doing so. If you cannot resist the temptation, you must cite the source, even if all that you do is check that your own answer is correct.

You are forbidden to upload solutions to homework problems, whether your own or ones that are posted on the course Web site, to any publicly available location on the Internet.

Anything that you learn from lecture, from the textbook, or from working homework problems can be regarded as "common knowledge" for purposes of this course, and the source need not be cited. Anything learned in prerequisite courses falls into the same category. Do not assume that other courses use some an expansive definition of "common knowledge"!

Week-by-week Schedule:

Month	Date	Topic
Fortnight 1	September 1-11	Fields, vectors and matrices
Week 2	September13-19	Dot and cross products; Euclidean geometry of $\mathbb{R}^n$
Week 3	September 20-26	Row reduction, independence, basis
Week 4	Sept. 27 - Oct. 3	Eigenvectors and eigenvalues
Week 5	October 4-10	Number systems and sequences
	October 8	QUIZ 1 on weeks 1-4
Week 6	October 11-17	Series, convergence tests, power series
Week 7	October 18-24	Limits and continuity of functions
Week 8	October 25-31	Derivatives, inverse functions, Taylor series
Week 9	November 1-7	Topology, sequences in $\mathbb{R}^n$, linear differential equations
	November 5	QUIZ 2 on weeks 5-8
Week 10	November 8-15	Limits and continuity in $\mathbb{R}^n$; partial and directional derivatives
Week 11	November 15-22	Differentiability, Newton's method, inverse functions
Fortnight 12	Nov. 22-Dec. 5	Manifolds, critical points, Lagrange multipliers
	November 24	Thansksgiving
	December ??	FINAL EXAM on weeks 9-12