Chapter XII

The Theory of Interest, II: Liquidity Preference as a Theory of Spreads

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> To the extent that "money" includes deposit accounts bearing interest, the theory [of liquidity preference] becomes not a theory of the rate of interest but of the gap between different rates of interest, *viz*., the yield on Government securities and the interest on bank deposits. (Dennis Robertson, personal correspondence, to Keynes, February 3, 1935, commenting on a draft of *The General Theory*, in Donald Moggridge (ed), *The Collected Writings of John Maynard Keynes,* vol. 13, *The General Theory and After, Part 1, Preparation,* London: Macmillan St Martins, 1973, p 499.)

> It would be a mistake, which would be as damaging to further analysis of liquidity preference as it would be to classical doctrines, if it were thought that uncertainty and liquidity differentials are the *sine qua non* for the existence of a [positive] rate of interest. Such a view can be compared with a theory of land rent based upon differences in the quality of different kinds of land. I believe that the analogy is not a superficial one. (Paul Samuelson, *Foundations of Economic Analysis*, Cambridge, Mass.: Harvard University Press, 1947, pp 122-123.)

I observed at the outset of the previous chapter that nothing caused the readers of *The General Theory* more grief than liquidity preference. It is thus not surprising that, from the very moment of publication of *The General Theory*, this part of Keynes's overall argument was subjected to intense criticism. Ultimately, liquidity preference provides only a partial explanation of the phenomenon of interest and the level of interest rates. For good reasons (see the epigraph to this chapter) and bad (see the quotation in the previous chapter) Dennis Robertson had strong reservations about liquidity preference, and this chapter at least partly vindicates Robertson's intuition: Keynes's theory tells us why bonds of different quality and maturity commonly offer different yields, but not why the overall level of yields is high or low.

We can see why by asking a simple question: if illiquidity were the sole determinant of interest, what would be the limiting value of the rate of interest as the term to maturity of a bond without default risk gets shorter and shorter? Keynes's answer has to be zero. But this is decisively disproven by the data on the rate charged for overnight loans between banks (the so-called Federal Funds rate, the name deriving from what is actually borrowed and lent: namely, funds on deposit with the Federal Reserve banks). Figure 1 shows the Fed Funds rate over the past 60 years. Only rarely—except for the past eight

Fed Funds Rate, 1954-2014

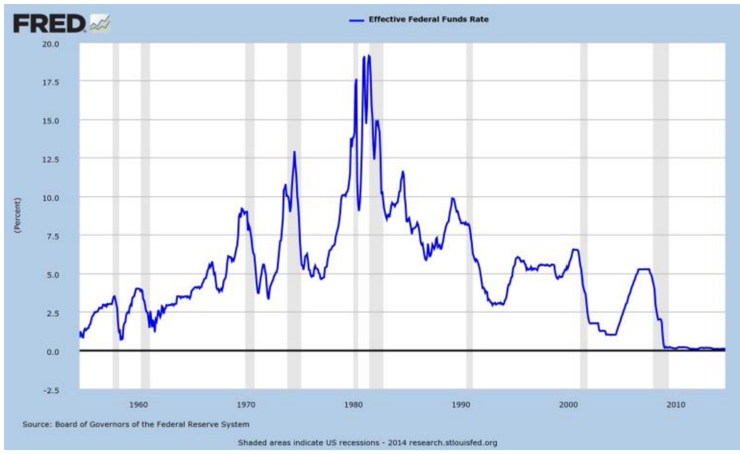

Figure 1

years—has the Fed Funds rate been below 2.5 percent.

The basic problem is that the theory presented in the previous chapter is too simple: liquidity preference “explains” interest in a world with two assets, money and bonds, because the spread determines the interest rate on the bond. Given that the yield of wealth in the form of money is zero, the spread between the yield on bonds and the yield on money *is* the yield on bonds, so in this special case determining the spread is tantamount to determining the level of the (long-term) interest rate.

As a matter of principle, Keynes would no doubt agree with the need to generalize the argument to include short-term interest-bearing assets, but he offers only a fudge in practice:

> We can draw the line between “money” and “debts” at whatever point is most convenient for handling a particular problem. For example, we can treat as money any command over general purchasing power which the owner has not parted with for a period in excess of three months, and as debt what cannot be recovered for a longer period than this; or we can substitute for “three months” one month or three days or three hours or any other period… It is often convenient in practice to include in money time-deposits with banks and, occasionally, even such instruments as (*e.g.*) treasury bills (*The General Theory,* p 167n).

Treating “the line between ‘money’ and ‘debts’” as a matter of convenience actually highlights the limitations of liquidity-preference theory. Once money includes interest-bearing assets, it becomes clear that liquidity preference does not speak to the question of why interest exists or offer an explanation of the overall *level* of interest rates. Liquidity preference becomes a theory of interest rate *differentials* or *spreads*.

John Hicks recognized the difficulty presented by interest-bearing short terms assets but thought that transactions costs resolve the problem. If transactions costs are high enough, then even though liquid short-term assets bearing a positive interest rate are available, a portion of wealth would normally be allocated to cash. At the margin agents would be indifferent between interest-bearing bills and cash, and, as in the cash-bonds model, the spread would be anchored by the zero return to money:

> If people receive payment for the things they sell in the form of money, to convert this money into bills requires a separate transaction, and the trouble of making that transaction may offset the gain in interest. It is only if this obstacle were removed, if safe bills could be acquired without any trouble at all, that people would become willing to convert all their money into bills, so long as any interest whatever was offered… It must be the trouble of making transactions which explains [a positive] short rate of interest. (*Value and Capital*, 2nd edition, pp 164-165)

Ordinary folk, people who in the normal course of events make small transactions, might indeed take transactions costs into account, but these are not generally people who make portfolio choices by weighing the returns to illiquidity against the advantages of liquidity. Almost everybody reading this book has good reasons to save, either to meet predictable needs (college tuition 15 years from now when your 3-year old turns 18, or retirement in 30 years on when you turn 65) or as a precaution against a rainy day on which your job disappears, illness strikes, or some other calamity occurs. However, neither predictable needs nor precaution figures into the theory of liquidity preference. Predictable needs do not require liquidity since portfolios can be tailored to the future date at which resources will be required, and precaution is generally understood to require liquidity irrespective of the rate of interest.

As Chapter XI noted, Keynes identifies the portfolio demand for money with speculation, like the opportunity to buy financial assets on the cheap, or to stave off creditors if a business goes sour. But people who are in a position to speculate in either of these ways will generally be engaging in transactions of a sufficient size that transactions costs won't matter. Hicks correctly saw that relying on transactions costs as an explanation of why people hold cash instead of short-term bills is a stretch for agents engaging in large transactions,

> Relatively large transactions can usually be made with very little more trouble than small transactions, but the total interest offered on a large sum is much larger than on a small sum; thus large capitalists will be tempted to buy bills much more easily than small capitalists. (p 165)

Hicks is driven to the conclusion implicit in Keynes's theory, namely, that zero would be the limiting value of the short-term rate as the term of the bill goes to zero were it not for the presence of small investors for whom transactions costs are significant. But these investors are not significant players in the game of balancing risk against return. So we are left with Hicks's own conclusion

> If… all traders reckon… a particular bill as perfectly safe, then there is no reason why that bill should stand at a discount… (p 165)

Of course, speculators as well as ordinary folk providing for a future with quasi-certainties (children going to college, retirement) and radical uncertainties (job loss, illness) will require cash to meet their obligations, but Hicks is wrong to attribute the phenomenon of interest on short-term bills to the eventual need for cash:

> The imperfect "moneyness" of those bills which are not money is due to their lack of general acceptability; it is this lack of general acceptability which causes the trouble of investing in them, and that causes them to stand at a discount. (pp 165-166)

Hicks here mixes up money as a medium of exchange with money as a store of value. It is the second with which liquidity preference is concerned. In this respect, portfolio money is 180° from transactions money. For transactions, agents need cash, bank deposits, or other forms of legal tender, but this does not necessitate holding cash or deposits as a store of value. If you travel to Mexico, you will need pesos, and if you travel to India you will need rupees, but this does not mean that you will hold pesos or rupees as part of your asset portfolio.

2008 was the last year before the present era of near-zero short-term rates, when bills have become equivalent to cash. In that year checking accounts in the US totaled $881 billion. In the same year, according to the Flow-of-Funds statistics of the Federal Reserve, money-market mutual funds, the paradigmatic vehicle for holding interest-bearing bills, totaled $3,832 billion in assets. As for the assets of the entire non-financial domestic sector, the numbers were $1,348 billion for checking accounts and currency together, vs $2,506 billion for money-market funds (Federal Reserve, *Flow of Funds*).

Liquidity Preference Without Money

There is of course more than one short-term interest-bearing asset, but many of these assets are normally perceived as differing little in default risk. Over the period 1954 to 2014 the Fed Funds rate moved pretty much in tandem with rates on 3-month Treasury bills and 3-month commercial paper, as Figure 2 indicates. For brief periods Treasury bills have sold at a premium (which is to say they yielded

Short Term Interest Rates, 1954-2014

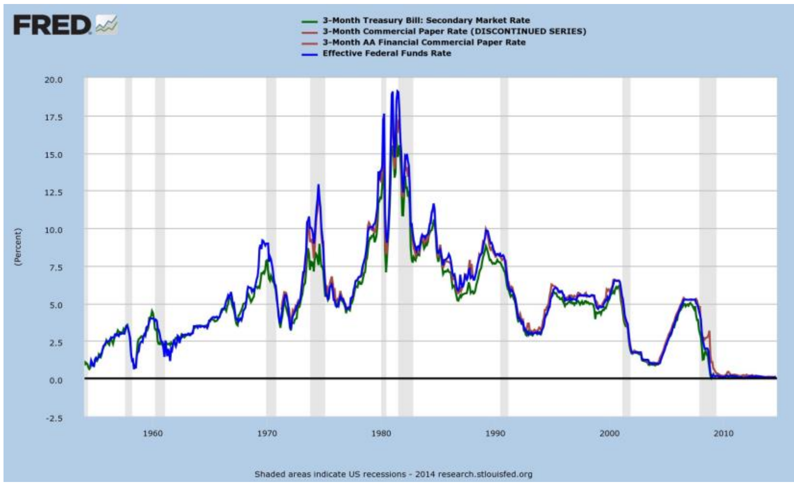

Figure 2

less than the Fed Funds rate and less than commercial paper), but for the most part the market has judged these three securities to be close to perfect substitutes for one another.[1]

What a difference a panic makes. Figure 3 shows the rates on the same assets over the year following

[1] Why aren't short-term bank loans included as substitutes for the short-term paper charted in Figure 2? The answer lies in the form of illiquidity introduced in Chapter XI only to be put to one side, namely, bid-ask illiquidity. Bank loans are generally too idiosyncratic to command the dense markets necessary to eliminate this kind of risk. Securitization is a way of overcoming at least some of the idiosyncrasies of individual bank loans, but it is an imperfect way.

Short Term Interest Rates
September, 2008 to September, 2009

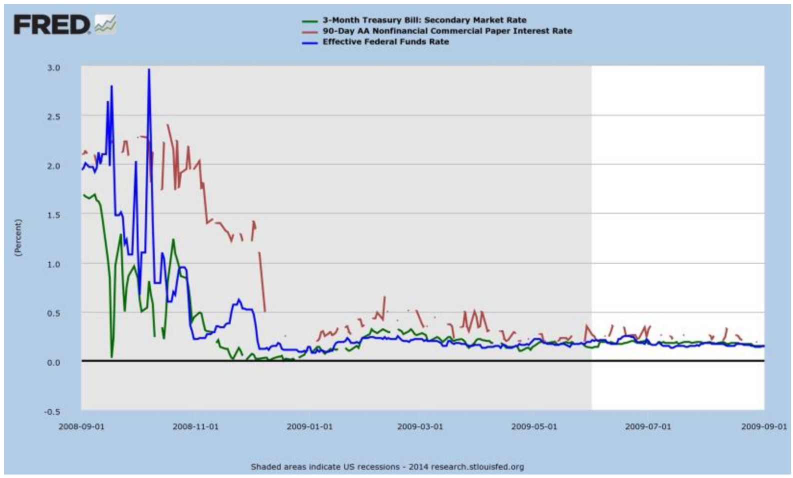

Figure 3

the collapse of Lehman Brothers in September, 2008. Fed Funds, Treasury bills, and commercial paper were no longer perceived to be close substitutes. Indeed, at one point, the Fed Funds rate was more than 2 percentage points above the 3-month Treasury bill rate, and in December, 2008, the market in commercial paper froze up completely. Liquidity preference with a vengeance! Only as the economy bottomed out in early 2009, and it became clear that the Great Recession would not repeat the descent into economic hell of the Great Depression, did commercial paper once again become a plausible substitute for Fed Funds or Treasury bills. After June, 2009, we see the old relationships among these three kinds of short-term assets.

If in normal times liquidity preference plays little role in markets for high-grade short-term commercial paper and short-dated Treasury bills, the spreads between short-term and long-term rates, as well as the spreads between government and private long-term paper, are a different matter. We turn now to analyzing the difference it makes when the liquid alternative to long-term bonds is an interest-bearing short-term asset rather than cash.

Fortunately, we already have in hand an apparatus for modeling these spreads; the logic is the logic of the relationship between bonds and cash in the two asset model. Figure 4 charts the relationship

3 Month Treasury Bill and 10 Year Treasury Note

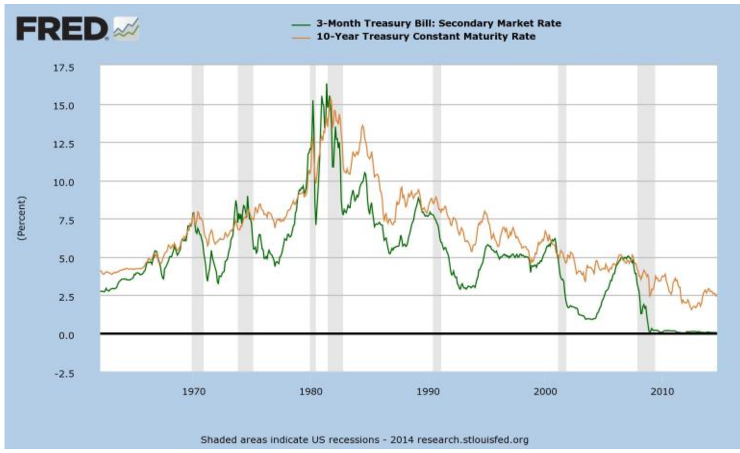

Figure 4

between 3-month and 10-year Treasuries. Figure 4 has three notable features. First, the yield on Treasury bills is generally below the yield on 10 year bonds. Second, the spread is inversely related to the level of yields. Finally, there are occasions—early 2007 is the most recent one—in which the spread is inverted, so that short-term bills yield more than long-term bonds. Both risk aversion and the expectation of reversion to normal can explain a yield premium on longer dated securities, but, as we shall see, risk aversion cannot account for the periods in which short-term bills yield more than long-term bonds.[2] On the other hand, reversion to normal cannot by itself account for the general tendency of bond yields to exceed bill yields.

Of course this is not an either-or situation; we are not obliged to choose between the two hypotheses. If we posit that both risk aversion and reversion to normal are at work, then we can easily account for all three characteristics of Figure 4. The yield on bills is generally below that on bonds because of risk aversion. The spread widens at low levels of interest rates because reversion to normal reinforces risk aversion and the risk premium rises because of greater volatility in bond prices. Exceptional occasions when the term structure is inverted, like late 2000 and 2007, can be explained as times at which reversion to the normal rate and risk aversion are working at cross purposes. Agents are willing to commit to long-term bonds during these periods because they believe on balance that long-term yields will fall; holding these beliefs they are motivated to buy while bonds are perceived to be cheap. In this

[2] At least not as risk aversion has been formulated here, in terms of asset values. Risk aversion can be formulated more generally, for example, in terms of variation in income flows. See, for example, Franco Modigliani and Richard Sutch, "Innovations in Interest Rate Policy," *American Economic Review*, 56: 178-197, 1966. For a more recent treatment, see Campbell and Viceira, 2001, ch 3.

case, expectations of reversion to normal dominate the price-fluctuation risk of holding long-term bonds, and, unusually, agents have to be compensated in the form of higher returns, not for holding illiquid bonds, but for holding short-term paper.

We proceed to derive the relationship between short and long-term yields on the basis of each hypothesis. Then we examine what happens when both hypotheses operate at the same time.

Interest Rate Spreads and Risk Aversion

The simplest way to introduce interest-bearing securities into the picture is to have bills replace money in the agent's endowment. As in Chapter XI, her endowment is $W_0 = \overline{M} + P_B\overline{B}$, but $\overline{M}$ now consists of a stock of Treasury (or commercial) bills rather than a sum of money.[3] The difference from Chapter XI is that bills offer a riskless return of $\rho_s > 0$. The agent's utility function is still $U(E(W), P_BB)$, but for any combination of bills (M) and bonds (B) that satisfies the endowment constraint

$$M + P_BB = \overline{M} + P_B\overline{B}$$

expected wealth is now

$$E(W) = (1 + \rho_s)\,M + \Big(1 + \frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B}\Big)\,P_BB = \overline{M} + P_B\overline{B} + \rho_s M + \Big(\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B}\Big)\Big(\,\overline{M} + P_B\overline{B} - M\Big)$$

and

$$U(E(W), P_BB) = U\Big(\overline{M} + P_B\overline{B} + \rho_s M + \Big(\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B}\Big)\Big(\,\overline{M} + P_B\overline{B} - M\Big),\ \overline{M} + P_B\overline{B} - M\Big)$$

As in the previous chapter, if the long bond is a consol, the case of pure risk aversion is associated with the assumption $\frac{E(\Delta P_B)}{P_B} = 0$, and the condition for an interior solution to the optimization problem is now

$$H(M, P_B\,, \overline{M}\,, \overline{B},\ \rho_s) \ = \ \frac{R}{P_B} \ - \ \rho_s \ + \ \frac{U_2}{U_1} \ = \ 0$$

The picture is in Figure 5.

[3] Winston W. Chang, Daniel Hamberg, and Junichi Hirata, "Liquidity Preference as Behavior toward Risk is a Demand for Short-Term Securities—Not Money," (*American Economic Review*, 73:420-427, 1983), as their title indicates, take the same view as I do with regard to the short-term options available to agents in choosing asset portfolios. Chang *et al* demonstrate that with short-term riskless bills yielding a positive return, optimization precludes holding money. A key difference is that their argument is limited to deriving asset demands as functions of interest rates; they do not investigate the properties of asset-market equilibrium, specifically the property that equilibrium determines the spread between bond and bill rates, not their levels. I am grateful to Professor Korkut Alp Erturk for this reference.

Balancing Return and Risk

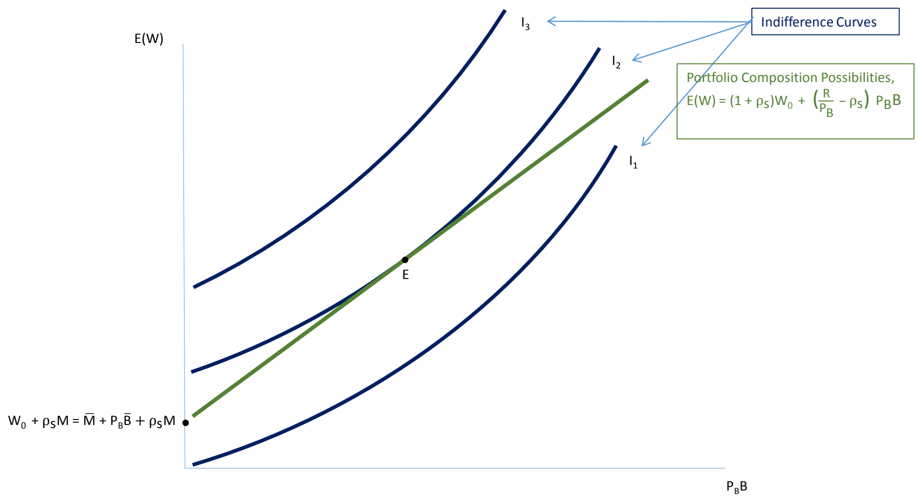

Figure 5

Once again, if we assume homogeneous risk preferences, we must add the auxiliary equations that the demand for bills M* equals the endowment $\bar{M}$

$$M = M^*(P_B\,, \bar{M}\,, \bar{B},\ \rho_s) = \bar{M}$$

and, correspondingly, that the bond price equilibrate financial markets

$$P_B = P_B^*(\bar{M}\,, \bar{B},\ \rho_s) \text{ such that } M^* = \bar{M}$$

In the representative-agent model, the equilibrium bond price and the corresponding interest rate $\rho_c = \frac{R}{P_B}$ are determined endogenously by the optimization process and the requirement that bill and bond demands equal the endowments.

We can extract from this constrained maximization process the relationship between the bill yield ρ_s and the bond yield ρ_c , as in Figure 6. The key result that emerges from the math—see the

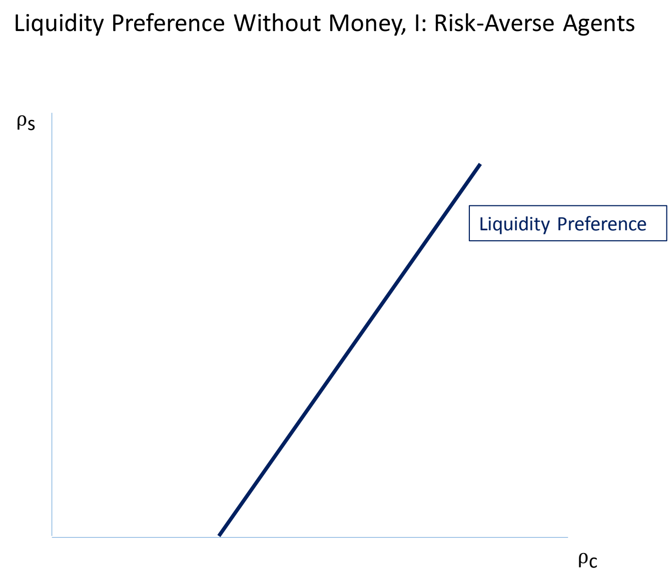

Figure 6

mathematical appendix—is that the equilibrium bond price is a decreasing function of the assumed bill rate, so that the coupon yield increases with the bill rate.

Observe that the construction in Figure 6 implies a liquidity trap, that is, a positive bond yield even as the bill rate approaches its zero lower bound. This result differs from the corresponding result in Chapter XI because the assumptions of the two models differ. In Chapter XI the liquidity-preference schedule answers the question of what happens to the equilibrium bond yield as the endowment of money becomes infinite. Here endowments, both of bonds and bills, are assumed to be given; what varies is the bill yield. And, for given endowments, the equilibrium bond yield goes to a positive limit as $\rho_s \rightarrow 0$.

For present purposes the most important point is that we can infer from the assumption that asset markets determine *only* the spread between the yields on the assets that comprise the market. Robertson was right about this in 1935 and Samuelson was right in 1947.

In a way this should not be surprising. That liquidity preference determines only spreads is the counterpart of a more general limitation of market equilibrium, namely, that with n goods only n – 1 prices emerge, which is to say that only relative prices are determinate. In asset markets there are not enough degrees of freedom to determine the separate yields.

Nor is this result problematic in a world with central banks. For most of the last century monetary policy has consisted of choosing the bill rate with an eye to fixing the bond yield. In other words, the central bank has taken on the task of anchoring the spread at the short end, leaving it to asset markets to determine bond yields and associated hurdle rates of return for new capital expenditure.

Reversion to Normal

But I get ahead of my story. Risk aversion is only one of the arguments for liquidity preference. Does reversion to normal survive any better the substitution of interest-bearing short-term assets for cash? The answer is yes, but the existence of bills as an interest-bearing alternative to cash requires us to flesh out the normal-reversion argument.

In a model with cash and bonds, the short rate is the return on cash; it is fixed at 0 and so cannot revert to anything else. Reversion in the cash-bond case necessarily refers only to the bond yield. In the present model, by contrast, reversion is fundamentally a property of the short-term bill rate, and we derive the trajectory of the breakeven bond yield, the yield that makes an agent indifferent between bonds and bills, from the expected trajectory of short rates.

The starting point is that, in the absence of risk aversion, the willingness of agents to hold both bills and bonds requires the holding yield on bonds (ρ_h) to equal the holding yield on bills (ρ_s). In continuous time $\dot{P}_B$ replaces ΔP_B and the holding-yield equation becomes

$$\rho_h \equiv \frac{R}{P_B} + \frac{E(\dot{P}_B)}{P_B} = \rho_s$$

If this differential equation holds continuously, the expected price at time t is given by its solution

$$E(P_{B,t}) = \int_t^{\infty} Re^{-\int_t^{\tau} \rho_s(x)dx} d\tau$$

where

$$-\int_t^{\tau} \rho_s(x)dx$$

is the discount factor for time τ, namely, the value at time t of \$1 available at a future time τ when the discount rate for each point in time x between t and τ is given by the value of ρ_s at x.

Whether or not it makes financial sense for the agent to hold bonds depends on how the actual price today compares with the expected price, that is, the price based on expected reversion to normal. If the actual price exceeds the expected price, then she is better off putting her financial resources into short-term bills. If the actual price is lower than the expected price, it makes sense to buy the bond. If the two prices are exactly equal, she can expect capital losses to offset the coupon. Stated in terms of yields, today's expected price thus defines an expected coupon yield $\frac{R}{E(P_{B,t})}$ at which the agent will be indifferent between holding bonds and holding bills; she will prefer bonds If the actual coupon yield exceeds the expected yield, bills if the expected yield is below the actual.

In this model reversion to normal of the bill rate drives the expected price and yield of bonds. So how do we characterize the expected path of short-term rates? The simplest story is that the short rate is expected to make up the distance between the current rate ρ_s and the normal rate ρ_s^* at a speed proportional to the distance:

$$\dot{\rho}_s = -\theta(\rho_s - \rho_s^*)$$

If this process is projected into the future, the expected future rate at time τ is given by a weighted average of the current rate and the normal rate, with the weight on the present declining as we move forward in time:

$$\rho_s(\tau) = (1 - e^{-\theta(\tau - t)})\rho_s^* + e^{-\theta(\tau - t)}\rho_s(t)$$

Substituting into the equation for the expected price, we obtain

$$E(P_{B,t}) = \int_t^{\infty} R e^{-\rho_s^*(\tau - t) + \frac{\rho_s(t) - \rho_s^*}{\theta}(e^{-\theta(\tau - t)} - 1)} d\tau$$

and the critical value of the current coupon yield becomes

$$\rho_c^* = \frac{R}{E(P_{B,t})} = \left(\int_t^{\infty} e^{-\rho_s^*(\tau - t) + \frac{\rho_s(t) - \rho_s^*}{\theta}(e^{-\theta(\tau - t)} - 1)} d\tau\right)^{-1}$$

Figure 7 charts the relationship between ρ_s and ρ_c^* on the assumptions $\rho_s^* = 0.04$ and $\theta = 0.1$. If the

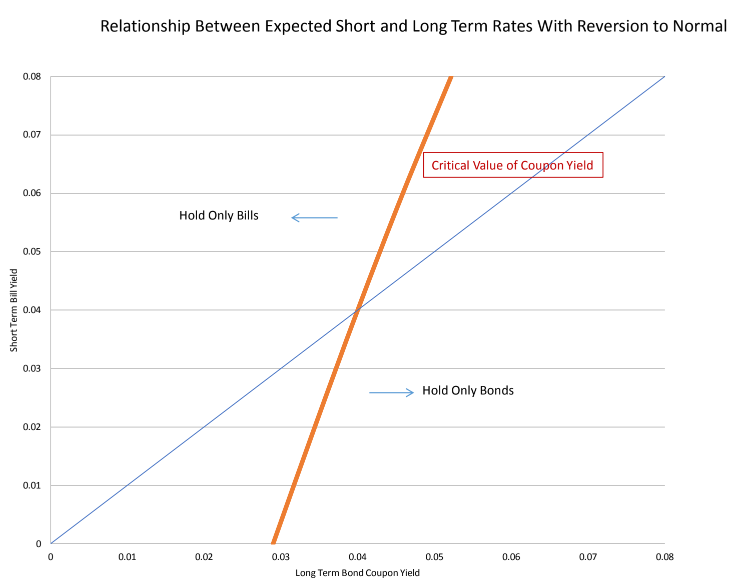

Figure 7

current short-term rate is 0, Figure 7 says that the critical value of the long-term bond yield is 0.029. (The mathemtatical appendix provides a solution to this equation.) If $\rho_c > 0.029$, the agent whose expectations are represented in Figure 7 will commit her portfolio entirely to bonds; if $\rho_c < 0.029$, entirely to bills. Evidently, if all agents are alike, the only long-term yield consistent with agents' holding both bills and bonds is $\rho_c^* = 0.029$. At this coupon yield, all agents believe that capital losses will just cancel out interest earnings and are indifferent between alternative portfolio mixes of bills and bonds.

But if agents have different beliefs about how rapidly ρ_s will revert to normal (or different beliefs about what constitutes normal, or both), then only a subset of agents need be equally comfortable with alternative portfolio mixes. Everybody else will specialize either in bonds or in bills. Figure 8 assumes

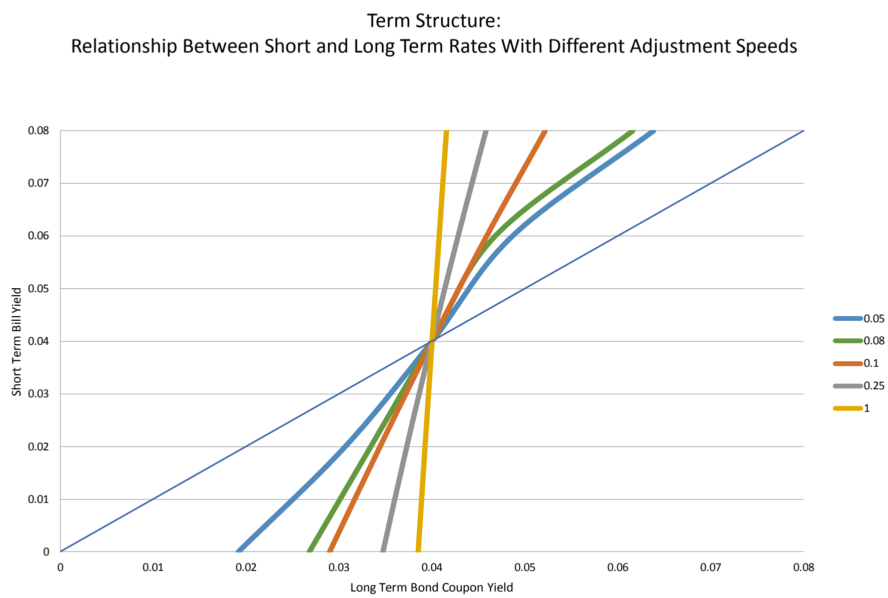

Figure 8

five types of agents differing only in their assumptions about the speed of reversion to normal.

The analysis of equilibrium proceeds as in Chapter XI. If there are n different agents instead of five, the total demand for bills is the sum of the individual demands

$$m(P_B, \rho_s)(\overline{M} + P_B\overline{B})$$

where $m(P_B, \rho_s)$is the number of agents desiring to hold only bills and $\overline{M} + P_B\overline{B}$ is the value of the individual endowment, assumed to be the same for everybody. The picture is in Figure 9.

Number of Agents Desiring to Hold Only Bills, Portfolio Wealth, and Demand for Bills

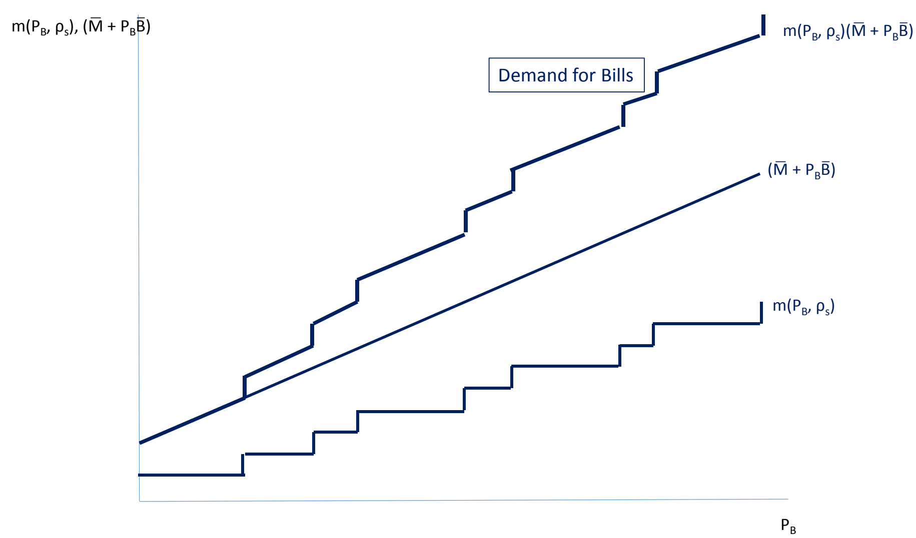

Figure 9

The supply of bills is simply the sum of all the endowments, $n\overline{M}$. Figure 10 puts demand and supply

Demand For and Supply Of Bills With Heterogeneous Agents

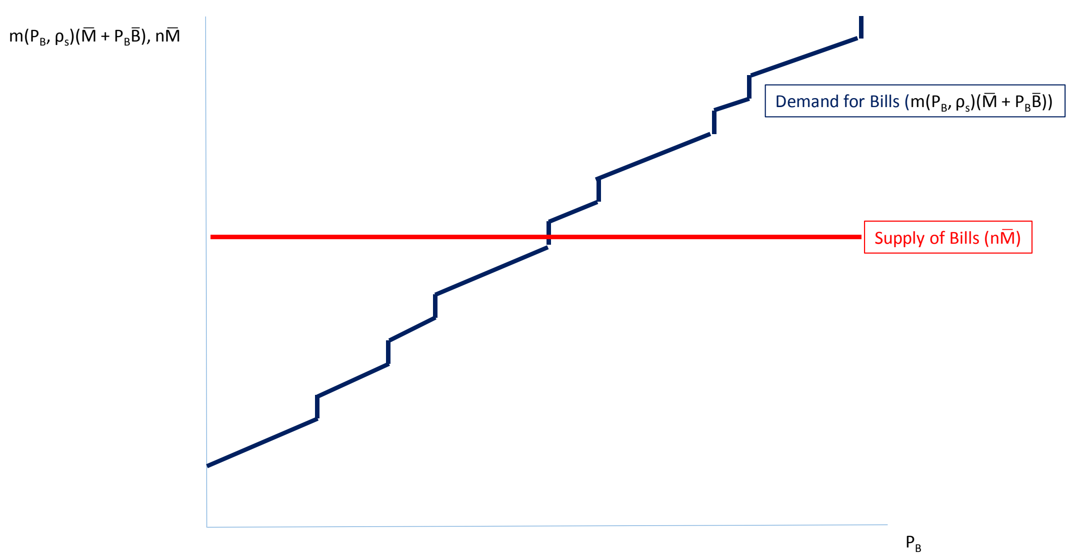

Figure 10

together to obtain the equilibrium bond price and the division of agents between billholders and bondholders. Only one agent holds both bonds and bills. As in the case of risk aversion, the equilibrium bond price presupposes a given bill rate; the equilibrium between demand and supply has only enough degrees of freedom to determine relative prices.

The equilibrium relationship between bill and bond yields, pictured in Figure 11, resembles the case of

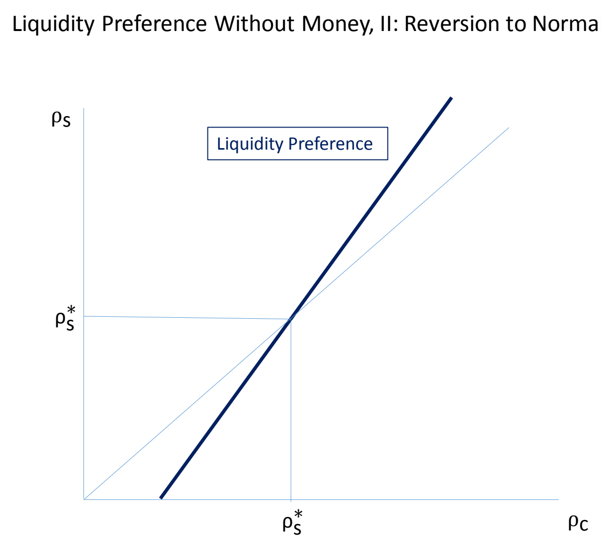

Figure 11

risk aversion in that the equilibrium bond yield is an increasing function of the assumed bill rate. But there is an important difference. When the bill rate is above normal, the relationship between the bond yield and the bill rate is inverted. Instead of the yield premium on bonds associated with risk aversion, short-term bill rates are above the bond yield. This follows from the common feature of the individual breakeven relationships, namely, that whatever is assumed about the speed of adjustment, breakeven bond yields are below the current short rate when the short rate is above normal. So, whereas risk aversion cannot account for the inversion of short and long rates pictured in Figure 4, normal reversion can.

Reversion to normal thus becomes more plausible as an explanation of interest-rate spreads. But reversion to normal is no more adequate by itself than is risk aversion. For one thing, if reversion to normal were the sole force at work, we should expect that, over time, short-term rates would be distributed more or less symmetrically around the normal rate, so that the mean of the difference between the current short-term rate and the normal rate would be 0. This would imply that inversions of the yield premium would be common rather than the rare events they are in Figure 4. Moreover, a major implication of the absence of risk aversion is that *individual* portfolios are specialized to bonds or bills except when agents are on the margin and willing to hold both securities in combination. When all is said and done, the idea of portfolios consisting of only one kind of security is only marginally more palatable than knife edge of homogeneous beliefs, in which demand oscillates wildly between short- and long-dated securities in response to small changes in the spread. There may be wide diversity of opinion, but with risk-neutral agents there is practically no diversification!

Combining Risk Aversion With Reversion to Normal in a Theory of Interest Rate Spreads

Of course, as was the case in Chapter XI, risk aversion and normal reversion are not mutually exclusive theories. Indeed, the two theories are complementary. Risk aversion answers a question to which normal reversion provides no answer, namely, why do agents diversify their holdings? And normal reversion answers a question to which risk aversion provides no answer, namely, why does the term structure sometimes exhibit an inversion of the usual positive spread between long and short coupon yields?

To combine risk aversion and reversion to normal, we go back to the original optimizing problem i relating the holding yield on bonds to the bill rate. In the general case we have

$$E(W) = (1 + \rho_s)\,M + \left(1 + \frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B}\right) P_B B = \bar{M} + P_B\bar{B} + \rho_s M + \left(\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B}\right)\left(\bar{M} + P_B\bar{B} - M\right)$$

and

$$U(E(W), P_B B) = U\left(\bar{M} + P_B\bar{B} + \rho_s M + \left(\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B}\right)\left(\bar{M} + P_B\bar{B} - M\right), \bar{M} + P_B\bar{B} - M\right)$$

So the first-order condition is

$$\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B} - \rho_s + \frac{U_2}{U_1} = 0$$

which can be re-written as

$$\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B} = \rho_s - \frac{U_2}{U_1}$$

or

$$\frac{R}{P_B} - \rho_s = -\frac{E(\Delta P_B)}{P_B} - \frac{U_2}{U_1}$$

We can interpret $-\frac{U_2}{U_1}$ as an illiquidity or risk premium, an amount by which the expected holding yield on bonds must exceed the riskless return available on short-dated assets. This is distinct from the yield premium, $\frac{R}{P_B} - \rho_s$, which takes account of both risk aversion and normal reversion.

When the current short-term rate is below normal, normal reversion simply reinforces risk aversion. The yield premium $\frac{R}{P_B} - \rho_s$ must now reflect the expected capital loss $\frac{E(\Delta P_B)}{P_B}$ as well as the risk premium $-\frac{U_2}{U_1}$. The demand for bonds shifts downward, as in Figure 12.

Balancing Coupon Yield, Capital Loss, and Risk

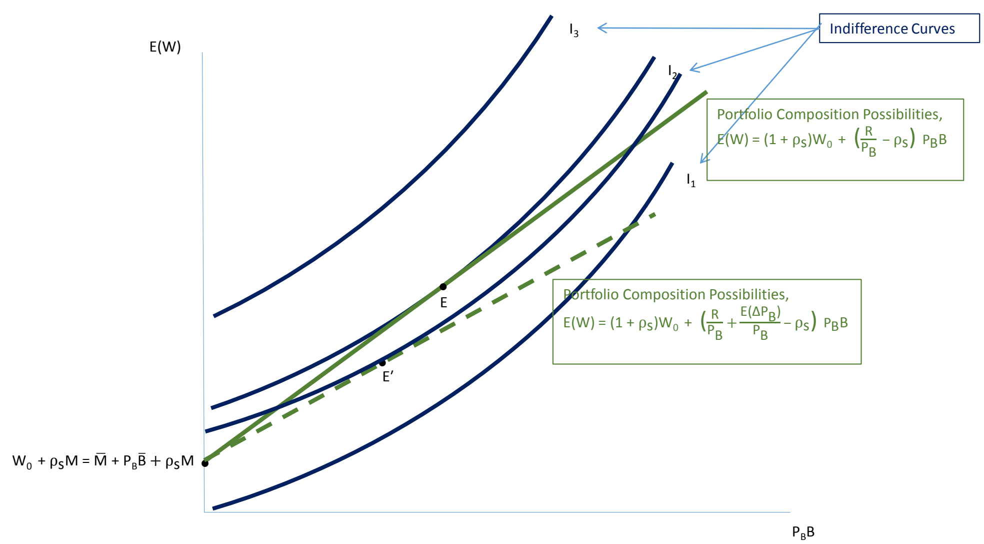

Figure 12

The more interesting case is when the current short rate exceeds the normal rate. In this case normal reversion and risk aversion are working against one another since $-\frac{E(\Delta P_B)}{P_B} < 0$ and $-\frac{U_2}{U_1}$ is positive. If the expected capital gain is great enough, the combined force of normal reversion and risk aversion can be negative, which is to say $-\frac{E(\Delta P_B)}{P_B} - \frac{U_2}{U_1} < 0$. The result is that the yield premium $\frac{R}{P_B} - \rho_s$ becomes negative at the optimum E′, as in Figure 13.

Balancing Coupon Yield, Capital Gains, and Risk

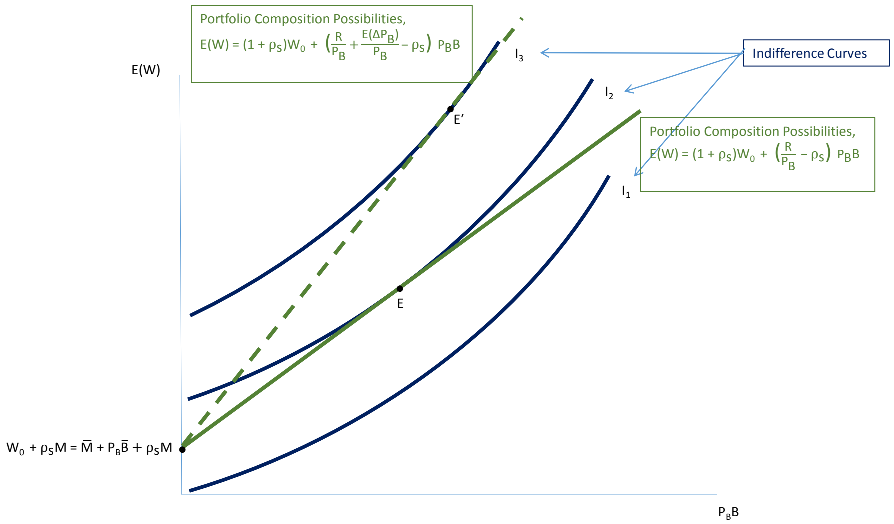

Figure 13

Figure 14 combines these results, showing how normal reversion displaces the liquidity-preference

Liquidity Preference Without Money, III: Combining Risk Aversion and Normal Reversion

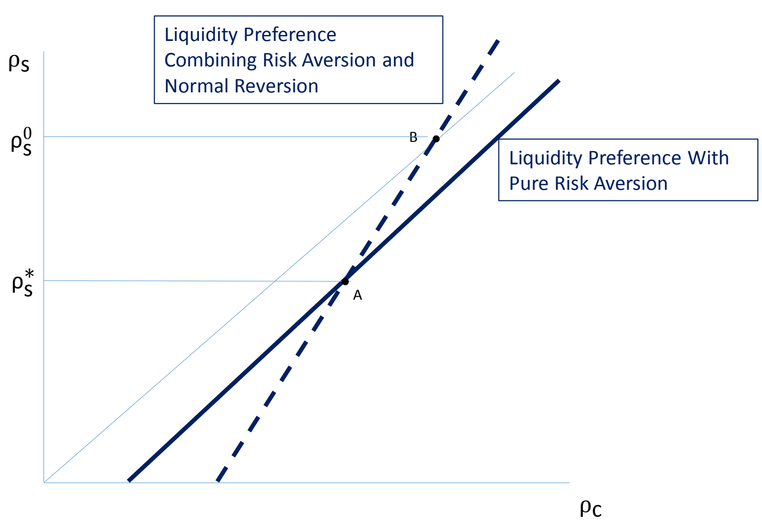

Figure 14

schedule in Figure 6. Observe that at A, corresponding to a bill rate equal to the normal rate, the two schedules intersect. At A an agent who believes that current rates always revert to normal has no difference of opinion with an agent who does not believe at all in reversion to normal; where $\rho_s = \rho_s^*$, both agents share the belief that $\frac{E(\Delta P_B)}{P_B} = 0$. When risk aversion and normal reversion are combined, the short-term rate must be in excess of the normal rate for an inversion of the yield premium to take place. In Figure 14 it is only when the short rate exceeds ρ_s^0 that the prospective return on bonds is sufficient to offset the price risk of holding bonds.

The two extremes of pure risk aversion and pure normal reversion reduce, respectively, to an assumption about portfolio composition possibilities. Pure risk aversion, which is to say risk aversion without normal reversion, can be expressed as $\frac{E(\Delta P_B)}{P_B} = 0$; pure normal reversion reduces to an assumption about the utility function, namely, $-\frac{U_2}{U_1} = 0$. The statistical appendix to this chapter assesses the relative importance of risk aversion and normal reversion in determining the liquidity premium over time. The conclusion is that most of the time normal reversion is irrelevant. But not all the time. At critical junctures—including the period since the financial crisis developed in the fall of 2008—normal reversion not only matters, it is the dominant force driving interest-rate spreads, at least for spreads between Treasury securities.

Default Risk

Up to now we have considered two assets, one of which, bonds, has price risk. But neither entails any default risk. Treasury bills and Treasury bonds are the canonical examples, though as was observed at the beginning of the chapter, high-grade commercial paper as well as Federal Funds debt normally are interchangeable with short-term T-bills. Since our focus is ultimately on how the hurdle rate for private investment decisions is determined, we need to extend the story to take account of the possibility, always present in private undertakings, that the borrower may default.[4] The hurdle rate relevant for private investment is not the yield on Treasuries, but the yield on bonds issued by corporations with a comparable risk of default.

[4] It is not only private issuers of debt who may default but any issuer not in control of the currency in which the debt is denominated. US states and municipalities, not to mention the otherwise sovereign countries that make up the Eurozone, are all subject to default risk. The contrast is with dollar bonds issued by the US Treasury, or for that matter yen bonds issued by the Japanese government, or sterling bonds issued by the UK.

A disclaimer is in order. In the summer of 2011, in order to extract concessions from President Obama, the Republican controlled House of Representatives delayed extension of the debt limit to the last minute, arousing fears of a default. The assumption that default is precluded if debt is issued in a currency controlled by the issuer needs to be qualified to exclude governments divided against themselves.

How do we conceptualize the relationship between yields on Treasuries and yields on corporate bonds? The logic of this comparison is the same as the logic for comparing short and long-term government obligations: agents are assumed to compare the expected holding yields on the two types of bonds, taking account of the impact of default on the expected change in bond price. Risk-averse agents presumably require a premium reflecting the greater price volatility associated with corporate bonds.

Agents choosing alternative combinations of corporate and Treasury bonds can, like agents choosing portfolios of short and long Treasuries, be assumed to maximize a utility function that takes account of both expected wealth at the end of the holding period and the greater risk associated with corporate bonds. Portfolio possibilities are given by

$$P_{CORP}B_{CORP} + P_B B = P_{CORP}\bar{B}_{CORP} + P_B\bar{B}$$

where P_{CORP} and B_{CORP} are prices and quantities of corporate bonds and $\bar{B}_{CORP}$ is the endowment of corporate bonds. As before P_B and B are the price and quantity of Treasuries and $\bar{B}$ is the agent's endowment of Treasuries. Expected wealth is

$$E(W) = \left(1 + \rho_{CORP} + \frac{E(\Delta P_{CORP})}{P_{CORP}}\right) P_{CORP}B_{CORP} + \left(1 + \rho_c + \frac{E(\Delta P_B)}{P_B}\right) P_B B =$$

$$P_{CORP}\bar{B}_{CORP} + P_B\bar{B} + \left(\rho_c + \frac{E(\Delta P_B)}{P_B}\right) P_B B + \left(\rho_{CORP} + \frac{E(\Delta P_{CORP})}{P_{CORP}}\right)\left(P_{CORP}\bar{B}_{CORP} + P_B\bar{B} - P_B B\right)$$

And with the utility function now

$$U(E(W), P_{CORP}B_{CORP}) = U(E(W), P_{CORP}\bar{B}_{CORP} + P_B\bar{B} - P_B B)$$

the first-order condition characterizing agents who hold both types of bonds is

$$\left(\rho_{CORP} + \frac{E(\Delta P_{CORP})}{P_{CORP}}\right) - \left(\rho_c + \frac{E(\Delta P_B)}{P_B}\right) + \frac{U_2}{U_1} = 0$$

As before U_1 reflects the marginal utility of expected wealth, but U_2 now measures the risk of holding corporate bonds relative to the risk of holding Treasuries. Assuming that the two types of bonds are of comparable maturities eliminates differences in price risk that are independent of default risk, so that the difference

$$\frac{E(\Delta P_B)}{P_B} - \frac{E(\Delta P_{CORP})}{P_{CORP}}$$

measures the default risk on the corporate bond. Now the optimization condition is that the yield premium on corporate bonds, $\rho_{CORP} - \rho_c$, is equal to the sum of the default risk, $\frac{E(\Delta P_B)}{P_B} - \frac{E(\Delta P_{CORP})}{P_{CORP}}$, and the risk premium, $-\frac{U_2}{U_1}$. In the absence of risk aversion, the yield premium is exactly equal to the default risk, but with risk aversion the yield premium must be higher than the default risk to compensate for the added risk of default.

Figure 15 summarizes the results of adding default risk to liquidity preference. The spread between

Liquidity Preference Without Money, IV:
Optimizing Between Corporate and Treasury Bonds

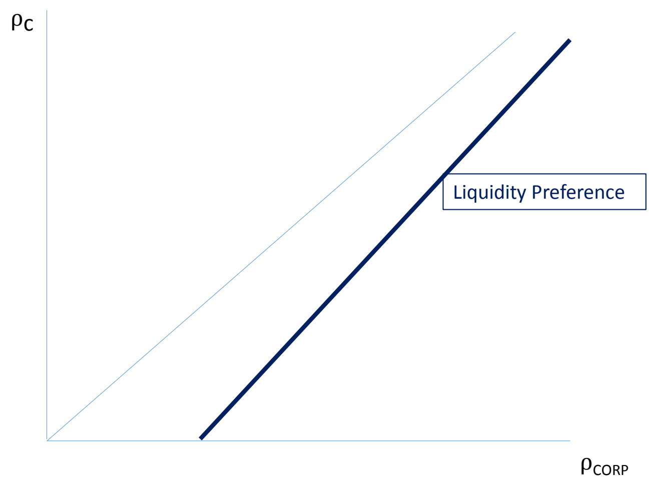

Figure 15

Treasury and corporate bonds is pictured as decreasing with the Treasury yield. This is intended to reflect the increase in both perceived and actual default risk as times of slack aggregate demand, when the Federal Reserve typically reduces Treasury yields but corporate default risk, and hence the spread $\rho_{CORP} - \rho_c$ increases. This is particularly salient at times of financial panic. From 1990 to 2007, the difference between the yield on corporate bonds and the yield between Treasuries of comparable maturity suggest an implicit default risk on the lowest investment-grade corporate bonds (Moody's Baa rating) of the order of 1.5 percent per year.[5] By contrast, in the year following the collapse of Lehman Brothers, the implicit default risk rose on average to 4 percent, peaking just above 5.5 percent in December, 2008.

Figure 16 shows how the relationship between short and long Treasuries is modified by the addition of

[5] This difference varied between 1.2 and 2.8 percent, averaging just under 1.7 percent. I am here measuring default risk by the difference between yields to maturity on Moody's index of long-term Baa-rated corporate bonds and 30-year US Treasury bonds. According to Moody's Analytics, their long-term corporate bond yield index is "based on seasoned bonds with remaining maturities of at least 20 years… [with] maturities as close as possible to 30 years" (http://credittrends.moodys.com/chartroom.asp?r=3, accessed 11/15/2013). When I last checked (personal communication, 11/15/2013), the average maturity of the index was 28 years.

Liquidity Preference Without Money, V: Adding in Default Risk

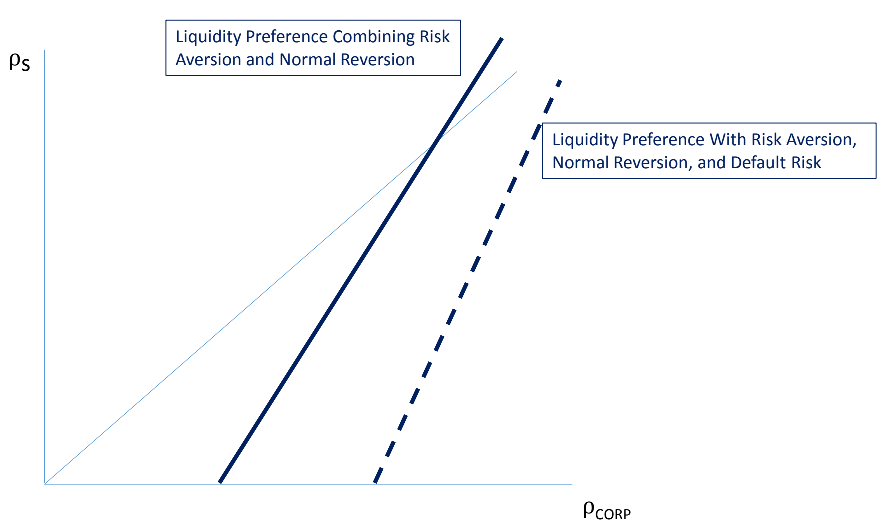

Figure 16

default risk. It is still theoretically possible to have an inverted term structure—possible for the corporate bond to have a lower yield to maturity than a short-term Treasury bill—though it takes a higher short-term rate to offset the higher price risk when default is part of the picture. In fact, even though the term structure of Treasuries exhibited inversion six times in the period covered by Figure 4, the short-term bill yield never rose above the corporate-bond yield, as Figure 17 shows.

3 Month Bills, 10 Year Treasury Bonds, and Baa Corporate Bonds

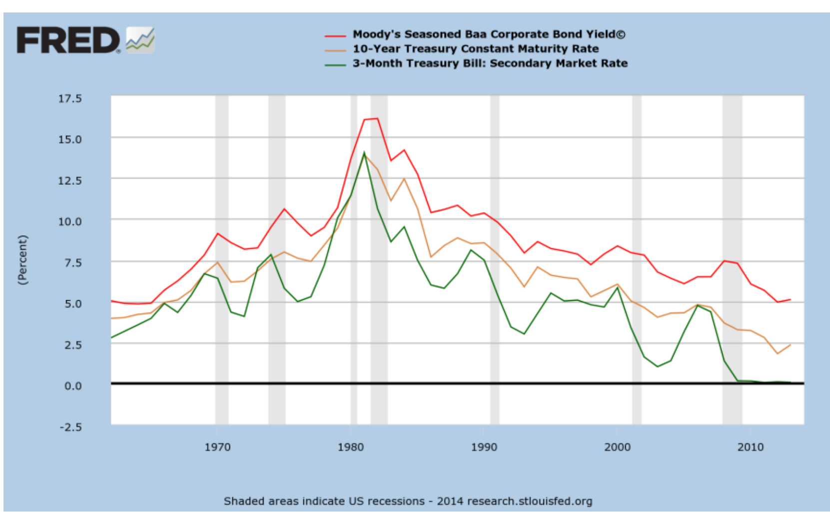

Figure 17

If we couple the construction of the in Figure 16 with the investment-demand and saving schedules, we can determine the hurdle rate of interest, the level of aggregate demand, and the demand for transactions money, as in Figure 18. Liquidity preference may provide only a theory of spreads, but with

Determination of Aggregate Demand

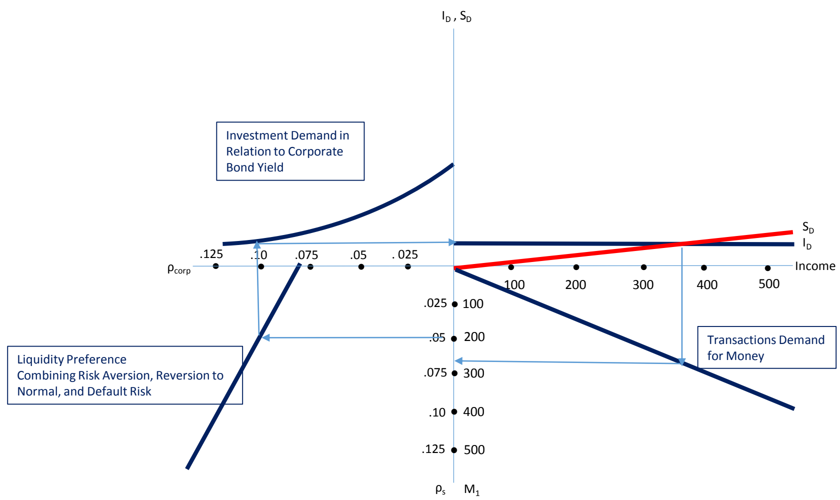

Figure 18

a central bank's hand on the steering wheel, a theory of spreads is all that is needed to determine aggregate demand: the central bank fixes the short-term bill rate, and the bond markets take care of the rest.

How does the central bank do it? Under present US conditions, with the Federal Reserve paying interest on reserves as well as charging interest for loans, the process is quite transparent. The Fed simply announces the operative short-term rate, in the US the target Federal Funds rate. In principle the Fed Funds rate is a "corridor," with the rate paid on reserves as the floor and the rate charged at the discount window as the ceiling. But given the large volume of excess reserves—just under $2 trillion in December, 2016, down from a peak of almost $2.7 trillion in 2014—the floor has determined the operative short-term rate since the inception of the present regime in the midst of the financial meltdown in October, 2008.

Before the present regime was instituted, the Fed paid no interest on reserves and relied on its control of reserves to implement its choice of policy rate. From 1994, when the Fed began to announce a target for the Federal Funds rate, until 2008, it was able to vary the policy rate without much actual change in the overall reserve position of the banks, relying on the effect of announcing changes to accomplish the desired result (Benjamin Friedman, 2000; Benjamin Friedman and Kenneth Kuttner 2011). Not surprising: like any monopolist, the Fed could set whatever price it chooses for its "product" (namely, bank reserves) and therefore for close substitutes—provided it was prepared to supply the quantity demanded at that price.[6]

The model developed in this chapter suggests a very different monetary regime from the two regimes discussed in Chapter XI. There we contrasted the regime implicit in the first-pass model with the regime implicit in the second-pass model. In the first-pass model, the central bank sets the hurdle interest rate and deploys open-market operations to adjust the mix of money and bonds in the hands of the public so that the desired hurdle rate is compatible with asset-market equilibrium. The quantity of transactions money is fixed separately in accordance with the level of output and the goods-price level. In the second-pass model, the amount of money available for agents' portfolios is what is left over after transactions demands are met. The equilibrium bond yield has to make money demand equal to this supply. The result in Chapter XI was that risk aversion generated a perverse LM schedule because the equilibrium bond yield varies directly rather than inversely with the amount of portfolio money.

The difference between the monetary regimes explains why the relationship between portfolio money and the equilibrium bond yield is positive in Figure XI-11, XI-12, and XI-13 but the relationship between bill rates and bond yields is positive in Figure 18 even if risk aversion alone drives liquidity preference. Here the operative assumption is divorce between portfolio money and transactions money. Agents are assumed to hold given amounts of bills ($\bar{M}$) and bonds ($\bar{B}$). Unlike the two models of the previous chapter, these amounts do not vary. The central bank is assumed to choose the bill rate, corresponding to which is the spread determined by the liquidity-preference schedule in the third quadrant of Figure 18. The other end of the spread is the bond yield at which desired holdings of bills (M) and bonds (B) are equal to the given endowments, so that money and bond markets are in equilibrium. This equilibrium bond yield provides the hurdle rate of interest, which determines level of investment demand in the second quadrant of Figure 18 and the level of aggregate demand in the first quadrant. The central bank, along with the banking system, is assumed to provide the requisite amount of transactions money, as determined by transactions demand in the fourth quadrant. For the economy to be on its aggregate-demand schedule, transactions money (and a corresponding amount of commercial loans and commercial paper)—the M_1 of the LM schema—must of course dovetail with the hurdle rate of interest, but there is no feedback from the quantity of transactions money to asset markets. If prices of goods were to double, M_1 would necessarily double too, but this would have no effect on agents' endowments of bills and bonds, and no effect on the equilibrium price of bonds or the corresponding hurdle rate.

The Federal Reserve's first reaction to the financial crisis and the ensuing recession was to reduce the bill rate to its lower bound of zero. But this produced only a liquidity trap: the spread between bill rate and the corporate bond yield—see Figure 17—was a whopping 750 basis points (100 basis points = 1

[6] With massive excess reserves, the Fed is no longer in the position of a monopolist, but with a floor and a ceiling on the rates at which it borrows and lends, the only impediment to controlling the Fed Funds rate is self inflicted, namely, its acceptance of deposits from government sponsored enterprises like the mortgage giants Fannie Mae and Freddie Mac. The Fed does not pay interest on these deposits, which gives the GSE's an incentive to undercut the interest rate paid to banks.

percentage point) throughout 2009. Given the endowments of bonds and bills in the hands of the public, this was all the Fed could do.

But the endowments of bills and bonds are not immutable. In terms of Figure 18, "quantitative easing," so-called, was an action to change the endowments and by this means to shift the liquidity-preference schedule. In the mathematical appendix to this chapter, it is shown that for changes in the endowments that are wealth preserving, that is, for which

$$M + P_B B = \overline{M} + P_B \overline{B} = \text{const.}$$

the sign of the derivative $\frac{dP^*}{d\overline{B}}$ is ambiguous. An increase in $\overline{M}$ and a corresponding decrease in $\overline{B}$ can shift the liquidity-schedule inwards, as in Figure 19, but it is possible for the new equilibrium to

Determination of Aggregate Demand at the Zero Lower Bound

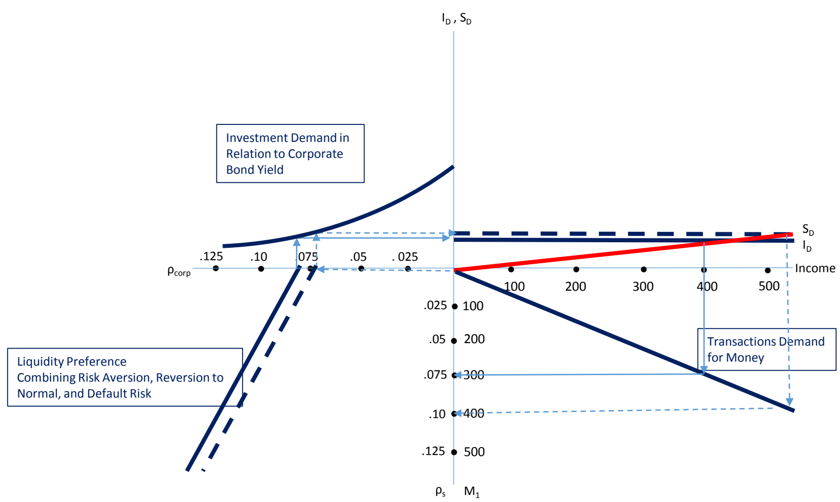

Figure 19

correspond to a lower bond price, a higher bond yield, and a correspondingly lower rate of investment demand. Such an equilibrium can be discounted however since it implies the dynamic system is unstable.[7] The reason for the ambiguity is the same reason why the response of money demand to the

[7] In *The General Theory*, Keynes offered little by way of hands-on suggestions for implementing his ideas. But he made this exception:

price of bonds is ambiguous: the change in the bond price creates a wealth effect along with the substitution effect, and this wealth effect may require a lower bond price to equilibrate bill and bond demands with their respective supplies when there are fewer bonds and more bills in the hands of the public.

A second novelty in Fed policy as recovery slowed was the commitment to maintain the short-term rate virtually at zero. Reversion to normal has been framed as a process in which the current value of the short-term interest rate is fixed by the central bank, and its expected evolution follows a path of gradual adjustment to normal. Under this assumption the central bank may set $\rho_s(0)$ as low as it chooses, even at zero, as was effectively the case from the fall of 2008 to the end of 2015, but the expectations embodied in θ determine the current value of the bond coupon yield. This is what makes a liquidity trap possible.

However, θ is not etched in stone; like $\rho_s(0)$, θ is a variable under partial control of the central bank. A commitment to maintain ρ_s below the normal rate in effect reduces θ, and the longer the duration of its commitment, the lower the current coupon yield. The effect is, as in Figure 19, to shift the liquidity-preference schedule inwards, with the difference that the shift is more pronounced the further the economy is from the normal rate.

The limit to the central bank's control over θ is the credibility of its commitment. In the limit, a credible commitment to $\rho_s = 0$ for the entire term of a Treasury bond drives θ to 0 over this whole period. The result is that the Treasury bond increases in value today so that the gradual fall in the price of the bond exactly offsets the coupon, and the yield to maturity is zero. Observe that in contrast with a commitment to maintain $\rho_s = 0$ for a definite amount of time, a commitment that expires when a trigger is pulled (for example, the unemployment rate reaching 5 percent) leaves the path of θ uncertain because of the uncertainty as to when the trigger event will happen.

Real and Nominal Rates: Is Central Bank Freedom Limited by Necessity?

So far the only limit to a central bank's power over the short-term rate is a zero lower bound. This is actually a bit anachronistic. If cash, with its zero nominal holding yield, is an option, a zero lower bound for bills makes perfect sense, but as long as there are costs to holding cash (storage, insurance, and the like), negative short rates become theoretically—and more important, practically—possible. The continuing recession in Europe led the European Central Bank to experiment with negative short-term rates in the form of charging banks interest on reserves. Indeed, for a large part of 2016, negative rates

> Perhaps a complex offer by the central bank to buy and sell at stated prices gilt-edged bonds of all maturities, in place of the single bank rate for short-term bills, is the most important practical improvement which can be made in the technique of monetary management. (p 206)

He not only anticipates the limits of a "bills only" policy, he offers the remedy which apart from a short episode ("Operation Twist") in the 1960s was not implemented until the Fed, under Ben Bernanke, introduced quantitative easing.

were not confined to the short end of the spectrum: ten-year government bond yields in Germany, Switzerland, and Japan also were negative.[8]

This does not mean a central bank can set short-term rates as far into negative territory as it might like, or that there are no limits on how far bond yields can fall. The costs of holding cash, even if not zero, limit the discretion of central banks as well as the limits of the market to drive bond yields down. Nothing short of a cashless economy or Silvio Gesell's stamped money (Keynes, 1936, ch 23) would remove the limit posed by the zero nominal yield of cash. Carrying costs do shift the lower bound to the short rate into negative territory, but no further than the charge represented by these carrying costs. At the lower end the freedom of the central bank to set the short rate is still quite limited.

More interesting is the question of the freedom of the central bank to control the *real* rate of interest, which differs from the nominal rate by the amount of inflation. The mainstream view is that at least in the long run, when frictions and imperfections are overcome, the real rate of interest (what Knut Wicksell [1907] calls the "natural rate") is determined by forces of productivity and thrift. In the older view put forward by Irving Fisher in 1896 and by Wicksell as well, a central bank can temporarily set the short-term rate of interest at a level incompatible with the natural rate, but economic forces will eventually make the central bank adjust to the natural rate. If, for instance, the central bank sets the short-term rate at such a low level that the bond yield—Wicksell's "money rate of interest"—is below the natural rate, economic activity will be stimulated. But with the economy normally at full employment, there would be no outlet for the stimulus other than to raise prices. Higher prices, however, would require more transactions money, and price inflation would eventually bump up against a fixed money supply (Wicksell, 1907, pp 116-117). Only by bringing the short rate, and thus the money rate of interest, into line with the natural rate can the inflationary pressure be relieved, and transactions demand brought into line with the supply of money.[9]

A more contemporary mainstream story accepts the Fisher-Wicksell view that the real rate is determined by productivity and thrift and is in the long run independent of central bank policy. The difference is that the central bank is free to choose the short-term rate and the corresponding long rate, but the central bank's choice of interest rate is purely nominal, with only a transitory effect on the natural rate, that is, on the real rate. In consequence the central bank is choosing a rate of inflation, which is the difference between the nominal rate chosen by the central bank (Wicksell's money rate) and the real rate (the natural rate) determined in the real economy by desired investment and saving at full employment. In the equation linking real and nominal rates,

---

[8] As of this writing (early 2017), Swiss ten-year government bond yields are still in negative territory. The yields in question are yields to maturity, not coupon yields. See Chapter XI for an explanation of the difference, which is important for finite-maturity bonds but plays no role in the world of consols in which we have been operating.

[9] Liquidity preference offers an argument that leads to the same end: assuming a fixed supply of money that serves either to facilitate transactions or as a store of wealth in agents' portfolios, an increased demand for transactions money directly reduces the amount of money available for portfolios, and this reduction can be accommodated only by an increase in the bond yield.

$$\rho_{NOMINAL} - \rho_{REAL} = \phi$$

where ϕ represents the rate of inflation, causality is read from left to right, from the real rate and the nominal rate to the rate of inflation.

The theory of liquidity preference developed in this chapter tells a different story. With the short-term rate fixed, the yield on a long-term bond is determined by the combination of risk aversion, reversion to normal, and default risk, as in Figure 18. With only bonds and bills available, portfolio choices are purely nominal, but nominal rates determine real rates. A rearrangement of the equation linking nominal and real rates, also read from left to right,

$$\rho_{NOMINAL} - \phi = \rho_{REAL}$$

implies a very different causality.

Inflation becomes endogenously determined in asset markets if agents can purchase physical commodities as a hedge against price changes. Assume that gold stands in for a representative basket of commodities, so the rate of change in the price of gold is a proxy for inflation. In a two-asset world, gold and short-term bills, the equilibrium yield from holding gold is the yield at which agents are just willing to hold gold and bills in the proportions afforded by their relative supplies. Consider Figure 20.

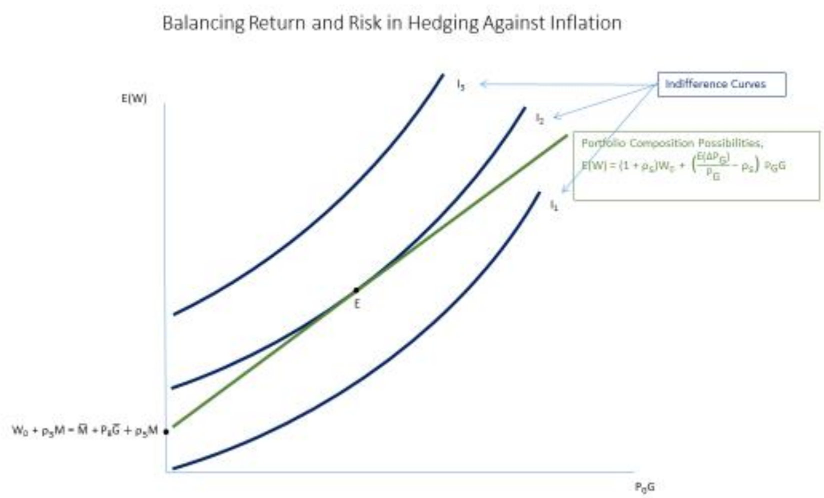

Figure 20

If the price of gold is P_G , the optimization process is strictly analogous to the process represented in Figure 5. The price change ΔP_G is the money return on holding gold and the expected return $\frac{E(\Delta P_G)}{P_G}$ has the role that the return $\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B}$ plays in bills vs bonds decisions.

I assume that $E(\Delta P_G)$ is independent of P_G . This is tantamount to assuming that the probability distribution of the future price of gold is independent of the present price, so that the expected rate of inflation decreases with the present price of gold. With this assumption, the positive relationship between the bill rate and the *bond yield* depicted in Figure 6 translates into a positive relationship between the bill rate and the *expected rate at of inflation* in Figure 21. The crucial role of risk aversion is

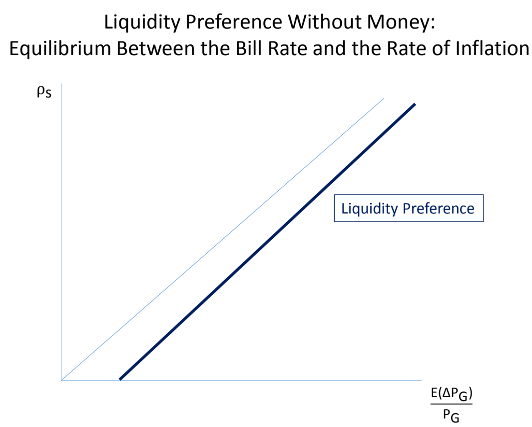

Figure 21

that, at an equilibrium between the demand for bills and the supply (and inconsequence the demand for gold and its supply), the rate of inflation must be higher than the bill rate to compensate for the variability of the inflation rate.

If we include bonds along with gold and bills, the condition for the demands for all three assets to equal the given supplies is that the asset with the higher price variability command the higher illiquidity premium. Thus if the price of gold is less variable than the price of the bond, the spread between the rate of inflation and the bill rate will be smaller than the spread between the bond yield and the bill rate. And vice versa.

The nominal bond yield determines a nominal hurdle rate, and the rate of inflation transforms this nominal rate into a real hurdle rate. The picture is in Figure 22. The central bank is assumed to set the

Determination of Aggregate Demand in an Inflationary Environment

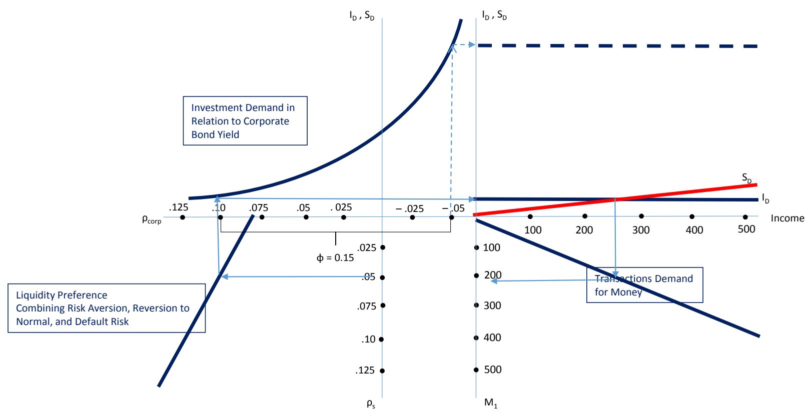

Figure 22

nominal short rate equal to 0.05, and the endogenously determined spread makes the nominal bond yield equal to 0.10. The equilibrium rate of inflation is 0.15, so the real hurdle rate is –0.05. In the first quadrant investment demand is boosted from the solid line to the dashed line. As Figure 22 is drawn, most of the additional investment has a negative real rate of return, being profitable only because the nominal return is in excess of the nominal hurdle rate. That it is possible for the central bank to impose a regime of negative real rates of interest and drive the marginal productivity of capital into negative territory does not make this a prudent policy.

What Happens If There is No Central Bank?

The short answer is that without a central bank *the level of aggregate demand is indeterminate.* Asset-market equilibrium determines the spread between the bill rate and the (corporate) bond yield, but in the absence of a central bank there is no mechanism for anchoring the spread. With the bill rate indeterminate, the bond yield, the hurdle rate for private investment, is also indeterminate, and so are the resulting levels of investment demand and aggregate demand. In the second-pass model of Chapter III, the problem with capitalism left to its own devices was that for any specification of investment demand, the propensity to save, liquidity preference, transactions demand, the money wage rate, and the overall supply of money, there is a determinate amount of aggregate demand, but this level of aggregate demand may fall short of what is necessary to match the full-employment supply of output. The problem is actually much deeper: once liquidity preference is recognized to provide only the spread between various rates of interest, the level of aggregate demand is itself indeterminate. In effect we have an IS schedule but no LM schedule. So we can't pin down a point on the IS schedule to associate with each potential level of the price of goods.

The difference with Chapter III is in the monetary regime. Instead of money that can be used either for transactions or as a store of wealth, but not both, money in the present model is limited to transactions. Paradoxically we are back to a world in which the quantity equation holds, with transactions demand given by $M_1 = \alpha PY$, with α the inverse of the output velocity of money. But not the quantity *theory*. Causality does not run from M_1 to P or Y, but from PY to M_1. As J Laurence Laughlin (1911) observed in debate with Irving Fisher on the operation of the quantity equation, equilibrium between transactions-money demand and transactions-money supply is achieved because of the ability of the banking system to create transactions money endogenously to satisfy transactions demand.[10] But without a determinate level of interest rates, the levels of P and Y are themselves indeterminate.

Conclusions

This chapter addressed the major shortcoming of liquidity preference as a theory of interest: the alternative to holding bonds is not to hold cash or bank deposits, but short-term bills that normally offer an interest payment to their owners. The cash-bonds model, unlike the rigid money-wage model, turns out not to be a part of the scaffolding that can be jettisoned once the building is in place!

When all is said and done, Keynes's critics were right to question liquidity preference as a theory of interest. Liquidity preference is instead a theory of interest rate *differences*. In a world of money and bonds liquidity preference nevertheless provides a coherent and complete theory of the rate of interest, but only because a theory of differences between yields on the short-term asset (money) and the long-term asset (bonds) is necessarily a theory of the rate of interest on the long-term asset. The nominal return on money is by definition zero, so the difference between the two rates is simply the nominal return on bonds.

This result does not generalize to a more realistic world in which wealth holders choose among an array of assets of varying terms, yields, and default risks, an array which does not include money. In this world the writ of liquidity preference runs no further than the spreads between these various yields. In a world in which a central bank steers the economy by imposing a short-term interest rate, this limitation is not, in principle, a problem because all that is needed is a theory of spreads.

As in the money-bonds world of the previous chapter, liquidity preference is a big tent in which risk aversion and reversion to normal as well as default risk can influence the structure of interest rates. And as in the simpler model, none of these motives for holding liquid assets is sufficient by itself to account for observed patterns of interest-rate structures. Risk aversion leads to the prediction that interest rate differentials do not depend on the existing short rate, but that bond yields will always be above short-term rates. As we shall see in the statistical appendix, the data reveal some dependence of the spread on the level of the bill rate, and we have already observed in Figure 4 instances of an

[10] In a capitalism left to its own devices, it is plausible to assume that the bills held in portfolios as liquid assets are commercial paper since there is presumably no government and no government bills or bonds. But the quantity of bills with which agents' portfolios are endowed would only by chance be sufficient to cover the transactions needs of businesses.

inverted term structure in which yields to maturity on ten-year Treasury notes are below short-term rates.

Reversion to normal accounts for the widening of the difference between long and short yields at low interest rates, but by itself would predict that inversions of the term structure should be as frequent as the usual term structure, in which yields rise with bond term. Neither aversion to the risks of bond-price fluctuations nor reversion of interest rates to normal accounts for the persistent gap between yields on government securities and yields on corporate bonds of comparable maturity. Here liquidity preference theory has to appeal to another kind of risk, namely, the risk of default.

The various motives for holding more liquid assets are not mutually exclusive. We can imagine agents who embody both risk aversion and a belief in reversion to normal. Or we can imagine that some agents are risk averse without believing rates will revert to normal and others are risk neutral while believing in reversion to normal. Either way, we will get a liquidity-preference relationship between short rates and long rates, and more to the point, between riskless short rates and the hurdle rate that governs investment decisions. Even if the short-term rate is equal to its zero lower bound, the hurdle rate will be positive. And higher short-term rates (normally) correspond to higher long-term yields, which is the essence of liquidity preference.

A critical difference remains between the orthodox theory of interest and liquidity preference. In the mainstream view, real rates of interest are determined by productivity and thrift, by investment demand and the supply of saving. A central bank can temporarily depart from the resulting "natural" rate of interest, but market forces will force a return to the natural rate in the long run. At best the central bank, through its control of the nominal rate of interest, determines the rate of inflation.

Liquidity preference argues that causality runs in the opposite direction. The starting point is a nominal rate of interest determined by the central bank and an endogenously determined rate of inflation, and the result is the real rate of interest.

There remains the theoretical question of importance to the negative critique that Keynes offered in *The General Theory.* What would determine the interest rate, or more accurately, the spectrum of interest rates, in a perfectly competitive economy left to its own devices, an economy without a government and more particularly without a central bank to anchor the spread between the returns available on bills and bonds with differing maturities and differing default risks? The answer is even more disturbing than Keynes imagined: the problem is not a discrepancy between two determinate levels of output, the level of output determined by the data of Keynes's theory—aggregate demand and goods supply determined by profit maximization—and the level of output at full employment. Rather the first of these two levels of output, the one depending on aggregate demand, is indeterminate once the spread loses its mooring in a bill rate fixed by the central bank.

Mathematical Appendix to Chapter XII

The Shape of the Indifference Map

The restrictive assumptions on the indifference map of the utility function $U(E(W), P_B B)$ are (1) the slope *increases* as we move upwards, that is, holding the value of bonds constant; and (2) the indifference-curve slope increases if the agent increases her holdings of bonds while maintaining a given level of expected wealth. These two conditions together imply (3) the slope *increases* as we move along any indifference curve, that is, holding utility constant. Define the slope of the indifference curve going through the point $\langle y, x\rangle = \langle E(W), P_B B\rangle$ as

$$h(y, x) = \frac{dy}{dx} = h(E(W), P_B B) = -\frac{U_2}{U_1}$$

An increase in the indifference-curve slope holding $P_B B$ constant means that the derivative of h with respect to expected wealth is positive, which is to assume

$$h_1 = -\frac{U_{21}U_1 - U_{11}U_2}{U_1^2} > 0 \qquad (1)$$

By the same token an increase in the slope $-\frac{U_2}{U_1}$ holding E(W) constant implies

$$h_2 = -\frac{U_{22}U_1 - U_{12}U_2}{U_1^2} > 0 \qquad (2)$$

Together, these two results imply that the slope increases as we move upward along any indifference curve. The change in the slope along an indifference curve is given by

$$h_1\frac{dy}{dx} + h_2 = \left(-\frac{U_{21}U_1 - U_{11}U_2}{U_1^2}\right)\left(-\frac{U_2}{U_1}\right) - \frac{U_{22}U_1 - U_{12}U_2}{U_1^2} > 0 \qquad (3)$$

The Implications of Optimizing Behavior

The agent is assumed to maximize a utility function characterized by (1) and (2) and hence by (3):

$$U(E(W), P_B B) = U\Big((1 + \rho_s)\, M + \Big(1 + \frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B}\Big)P_B B,\ P_B B\Big)$$

subject to the portfolio constraint

$$M + P_B B = \overline{M} + P_B\overline{B}$$

in which M represents bills and B bonds, with $\overline{M}$ and $\overline{B}$ endowments. Substituting from the portfolio constraint into the utility function, we can reduce the optimization problem to a choice of M:

$$U(E(W), P_B B) = U\Big(\overline{M} + P_B\overline{B} + \rho_s M + \Big(\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B}\Big)\Big(\overline{M} + P_B\overline{B} - M\Big),\ \overline{M} + P_B\overline{B} - M\Big)$$

for which we can write the first-order condition for an interior solution as

$$H(M, P_B, \overline{M}, \overline{B}, \rho_s) \equiv \frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B} - \rho_s + \frac{U_2}{U_1} = 0 \qquad (4)$$

Treating P_B as a parameter we can write the solution to the first-order condition as

$$M = M^*(P_B, \overline{M}, \overline{B}, \rho_s)$$

Taking the total derivative of the function

$$H(M^*(P_B, \overline{M}, \overline{B}, \rho_s), P_B, \overline{M}, \overline{B}, \rho_s) = 0$$

with respect to P_B tells us how the demand for bills (M) is related to the price of bonds. We have

$$H_1 \frac{\partial M^*}{\partial P_B} + H_2 = 0$$

so that

$$\frac{\partial M^*}{\partial P_B} = -\frac{H_2}{H_1}$$

Differentiating (4) gives

$$H_1 = \left(-\frac{U_{22}U_1 - U_{12}U_2}{U_1^2}\right)\left(\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B} - \rho_s\right) - \frac{U_{22}U_1 - U_{12}U_2}{U_1^2}$$

$$H_2 = \frac{-\frac{R + E(\Delta P_B)}{(P_B)^2}U_1^2 + (U_{21}U_1 - U_{11}U_2 + U_{22}U_1 - U_{12}U_2)\overline{B} + (U_{21}U_1 - U_{11}U_2)\frac{R + E(\Delta P_B)}{P_B}(\overline{B} - B)}{U_1^2}$$

Given the restrictions (1) and (2), H_1 is positive. But the sign of H_2 is ambiguous unless $\overline{B} \geq B$, which is to say, unless the optimal quantity of bonds is no greater than the endowment. So in principle the demand for bills can increase or decrease with the bond yield $\frac{R}{P_B}$. The reason was noted in the body of this chapter: there is a substitution effect as well as a wealth effect. A higher bond price (lower bond yield) reduces the excess of the holding yield on bonds over the bill rate—the substitution effect—but at the same time increases the wealth of the agent.

A Representative-Agent Model

In a representative-agent model, in which $B = \overline{B}$ (and $M = \overline{M}$), the ambiguity in the sign of $\frac{\partial M^*}{\partial P_B}$ is resolved. With $B = \overline{B}$ we have

$$\operatorname{sgn}\frac{\partial M^*}{\partial P_B} = -\frac{\operatorname{sgn} H_2}{\operatorname{sgn} H_1} = -\frac{-}{+} = +$$

so the demand for bills unambiguously decreases as the bond yield increases.

We can also determine what happens when the endowment of bills or bonds increases. If $M = \overline{M}$, then $M^* = \overline{M}$ and P_B is no longer a free parameter. Instead $P_B = P_B^*(\overline{M}, \overline{B}, \rho_s)$ such that

$$H(M^*(P_B^*(\overline{M}, \overline{B}, \rho_s)\,,\, \overline{M}\,,\, \overline{B},\ \rho_s),\, P_B^*(\overline{M}, \overline{B}, \rho_s)\,,\, \overline{M}\,,\, \overline{B},\ \rho_s) = 0$$

If we differentiate this equation with respect to $\overline{M}$, we obtain

$$H_1\left(\frac{\partial M^*}{\partial P_B}\frac{\partial P_B^*}{\partial \overline{M}} + \frac{\partial M^*}{\partial \overline{M}} \right) + H_2\frac{\partial P_B^*}{\partial \overline{M}} + H_3 = 0$$

Since

$$M = M^*(P_B^*(\overline{M}, \overline{B}, \rho_s) = \overline{M}$$

we have

$$\frac{\partial M^*}{\partial P_B}\frac{\partial P_B^*}{\partial \overline{M}} + \frac{\partial M^*}{\partial \overline{M}} = 1$$

So

$$\frac{\partial P_B^*}{\partial \overline{M}} = -\frac{H_1 + H_3}{H_2}$$

With

$$H_3 = \left(\frac{U_{22}U_1 - U_{12}U_2}{U_1^2}\right) \left(\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B} + 1\right) + \frac{U_{22}U_1 - U_{12}U_2}{U_1^2}$$

the sum $H_1 + H_3$ simplifies to

$$H_1 + H_3 = \left(\frac{U_{22}U_1 - U_{12}U_2}{U_1^2}\right) \left(1 + \rho_s\right)$$

and we have

$$\operatorname{sgn} \frac{\partial P_B^*}{\partial \overline{M}} = -\frac{-}{-} = -$$

which is to say that the equilibrium bond *yield* is an increasing function of the endowment of bills.

Equilibrium Spreads in a Representative-Agent Model

For a given endowment of bills, we calculate the relationship between the bond price and the bill rate by taking the total derivative of $H(M^*(P_B^*(\overline{M}, \overline{B}, \rho_s)\,,\, \overline{M}\,,\, \overline{B},\ \rho_s),\, P_B^*(\overline{M}, \overline{B}, \rho_s)\,,\, \overline{M}\,,\, \overline{B},\ \rho_s)$ with respect to ρ_s and equating it to 0. This gives

$$H_2\frac{\partial P_B^*}{\partial\, \rho_s} + H_5 = 0$$

The derivative H_5 is

$$H_5 = -1 + \frac{U_{21}U_1 - U_{11}U_2}{U_1^2} M$$

so that

$$\text{sgn}\,\frac{\partial P_B^*}{\partial \rho_s} = -\frac{-}{-} = -$$

and the equilibrium bond yield is a positive function of the bill rate.

Quantitative Easing

In the body of this chapter, quantitative easing is represented as a wealth-preserving exchange of bonds and bills between the public and the monetary authority. That is, $M^*(P_B^*, \overline{M}, \overline{B}, \rho_s) = \overline{M}$ and $B = \overline{B}$, so

$$M^*(P_B^*, \overline{M}, \overline{B}, \rho_s) + P_B^*(\overline{M}, \overline{B}, \rho_s)B = \overline{M} + P_B^*(\overline{M}, \overline{B}, \rho_s)\overline{B}$$

When the central bank buys bonds from the public, the corresponding change in the public's stock of bills is given by

$$\frac{d\overline{M}}{d\overline{B}} = -P_B^* - \frac{dP_B^*}{d\overline{B}}\overline{B}$$

To solve for the subsequent price adjustment, we set the total derivative of H with respect to $\overline{B}$ equal to 0:

$$H_1\frac{dM^*}{d\overline{B}} + H_2\frac{dP_B^*}{d\overline{B}} + H_3\frac{d\overline{M}}{d\overline{B}} + H_4 = 0$$

Since $M^* = \overline{M}$ for all $\overline{M}$, we have $\frac{dM^*}{d\overline{B}} = \frac{d\overline{M}}{d\overline{B}}$, and we can write $\frac{dP_B^*}{d\overline{B}}$ as

$$\frac{dP_B^*}{d\overline{B}} = \frac{P_B(H_1 + H_3) - H_4}{H_2 - \overline{B}(H_1 + H_3)}$$

The new element is

$$H_4 = \frac{P_B\left(\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B} + 1\right)(U_{21}U_1 - U_{11}U_2) + P_B(U_{21}U_1 - U_{11}U_2)}{U_1^2}$$

So $P_B H_3 = H_4$ and

$$P_B(H_1 + H_3) - H_4 = P_B H_1 = \frac{-P_B\left(\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B} - \rho_s\right)(U_{21}U_1 - U_{11}U_2) - P_B(U_{21}U_1 - U_{11}U_2)}{U_1^2}$$

We now have

$$\frac{dP_B^*}{d\overline{B}} = \frac{P_B H_1}{H_2 - \overline{B}(H_1 + H_3)} = \frac{-P_B\left(\frac{R}{P_B} + \frac{E(\Delta P_B)}{P_B} - \rho_s\right)(U_{21}U_1 - U_{11}U_2) - P_B(U_{21}U_1 - U_{11}U_2)}{-\frac{R + E(\Delta P_B)}{(P_B)^2}U_1^2 + \overline{B}[(U_{22}U_1 - U_{12}U_2) - \rho_s(U_{21}U_1 - U_{11}U_2)]}$$

Since the last term of the denominator is positive, the sign of the denominator is ambiguous

$$\text{sgn}\frac{d\mathrm{P}_{\mathrm{B}}^{*}}{d\bar{\mathrm{B}}} = \frac{+}{\pm} = \pm$$

As the stock of bonds in the hands of the public falls, the equilibrium price of bonds may fall, in which case bond yields would rise.

However, in a full dynamic model in which

$$\frac{\dot{\mathrm{P}}_{\mathrm{B}}}{\mathrm{P}_{\mathrm{B}}} = \omega\left(\mathrm{B} - \bar{\mathrm{B}}\right)$$

and

$$\dot{\mathrm{P}}_{\mathrm{B}} = -\,\omega\left(\mathrm{M}^{*}(\mathrm{P}_{\mathrm{B}}, \bar{\mathrm{M}}, \bar{\mathrm{B}}, \rho_{s}) - \bar{\mathrm{M}}\right)$$

$\dot{\mathrm{P}}_{\mathrm{B}}$ must increase when the supply of bonds in the hands of the public falls, so the equilibrium associated with a positive $\frac{dP_{B}^{*}}{d\bar{\mathrm{B}}}$ is unstable. We have

$$\frac{\partial\dot{\mathrm{P}}_{\mathrm{B}}}{\partial\bar{\mathrm{B}}} = -\,\omega\left(\frac{\partial\mathrm{M}^{*}}{\partial\bar{\mathrm{M}}}\frac{\partial\bar{\mathrm{M}}}{\partial\bar{\mathrm{B}}} + \frac{\partial\mathrm{M}^{*}}{\partial\bar{\mathrm{B}}} - \frac{\partial\bar{\mathrm{M}}}{\partial\bar{\mathrm{B}}}\right) = -\,\omega\left(-\frac{\mathrm{H}_{3}}{\mathrm{H}_{1}}(-\mathrm{P}_{\mathrm{B}}) - \frac{\mathrm{H}_{4}}{\mathrm{H}_{1}} - (-\mathrm{P}_{\mathrm{B}})\right) = -\,\phi\left(\mathrm{P}_{\mathrm{B}}\right)$$

Along with the equilibrium condition, this gives us the picture in Figure 1. A decrease in $\bar{\mathrm{B}}$ coupled with

Disequilibrium Price Change as a Function of the Quantity of Bonds in the Public's Hands

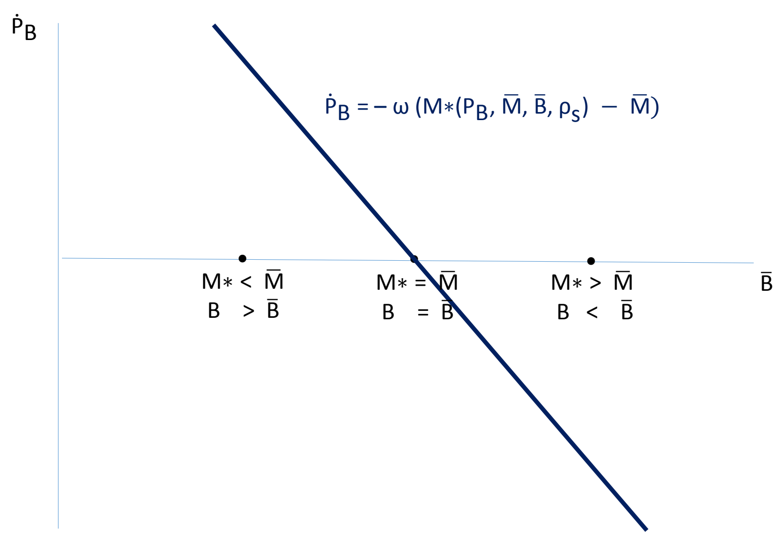

Figure 1

the corresponding increase in $\bar{\mathrm{M}}$ dictated by the constraint

$$\overline{M} + P_B\overline{B} = \text{const.}$$

drives up the price of bonds. This leads to a new equilibrium price only if the new P_B^* exceeds the original price. This is the case only if

$$\text{sgn}\frac{dP_B^*}{d\overline{B}} = -$$

Normal Reversion: Calculating the Consol Yield as a Function of ρ_s , ρ_s^*, $\rho_s(0)$, and θ

In the body of this chapter I derived an integral expression for a consol yield ρ_c when the evolution of the bill rate is governed by the equation

$$\dot{\rho}_s = -\theta(\rho_s - \rho_s^*)$$

Namely,

$$\rho_c(0) \equiv \frac{R}{P_B} = \left(\int_0^\infty e^{-\rho_s^* t + \frac{\rho_s(0) - \rho_s^*}{\theta}(e^{-\theta t} - 1)}\, dt\right)^{-1}$$

To solve the expression on the right-hand side numerically, the text makes use of a formula derived in Marglin (1970):

$$\rho_c(0) \equiv \frac{R}{P_B} = \left\{\frac{1}{\rho_s^*}\left(1 + \sum_{n=1}^{\infty}\left[\left(\frac{\rho_s(0) - \rho_s^*}{\theta}\right)^{-n}\prod_{j=1}^{\infty}\left(\frac{\rho_s^*}{\theta} + j\right)^{-1}\right]\right)\right\}^{-1}$$

Statistical Appendix to Chapter XII

What Do the Data Say?

Twenty years ago John Campbell (1994) offered a comprehensive assessment of the empirical state of play of interest-rate determination, based largely on joint work with Robert Shiller. Campbell argued that the data contradict a key provision of pure normal reversion (the "pure expectations hypothesis" in Campbell's terminology).[1] The hybrid hypothesis that both normal reversion and risk aversion are at work (Campbell's "expectations hypothesis") doesn't fare any better. Campbell regresses changes in bond yields on spreads between yields on bonds of various maturities and the short rate and finds

> In these regressions, the spread is scaled so that if the expectations hypothesis holds, the slope coefficient should be one. In fact, all but one of the slope coefficients are negative; all are significantly less than one, and some are significantly less than zero. When the long-short yield spread is high, the long yield tends to fall, amplifying the yield differential between long and short bonds, rather than rising to offset the yield spread as required by the expectations hypothesis. (1994, pp 138-139)

Nobody to my knowledge has contradicted Campbell's assessment. For good reason. Campbell is right that the data contradict the expectations hypothesis, in both its pure and hybrid forms, at least as the post World War II literature has framed the argument. For Campbell

> The pure expectations hypothesis of the term structure is the theory that interest rates are expected to move… to equalize expected returns [holding yields] on short- and long-term investment strategies. The expectations hypothesis is the slightly weaker proposition that the difference between the expected returns on short- and long-term investment strategies is

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[1] Campbell's negative conclusions themselves were hardly novel. A quarter century before Campbell, Ed Kane wrote

> It is generally agreed that, *ceteris paribus*, the fertility of a field is roughly proportional to the quantity of manure that has been dumped upon it in the recent past. By this standard, the term structure of interest rates has become ... an extraordinarily fertile field indeed. ("The Term Structure of Interest Rates: An Attempt to Reconcile Teaching with Practice," *Journal of Finance*, vol 25, 1970, pp 361-74.)

In 1989 Kenneth Froot offered pretty much the same assessment, although his language was considerably less colorful:

> If the attractiveness of an economic hypothesis is measured by the number of papers which statistically reject it, the expectations theory of the term structure is a knockout. (Kenneth Froot, "New Hope for the Expectations Hypothesis of the Term Structure of Interest Rates," *The Journal of Finance*, vol. 44 no. 2, 1989, pp 283-305, p 283)

Nonetheless, the rigor and comprehensiveness of Campbell's analysis make his summary the obvious starting point for assessing the data.

> constant, although it need not be zero as required by the pure hypothesis. (Campbell, 1994, p 137).

The problem is that Campbell and others neither offer nor test a theory of the term structure, but rather a term-structure theory of changes in interest rates.

Inverting Causality: From a Theory of Spreads to a Theory of Long-Bond Yields

The difference is more than semantic. For our purposes the important point is that in the expectations hypothesis that emerges from *The General Theory* causality runs in the opposite direction from causality in the theory deployed and tested by Campbell and the legions who have worked the expectations street over the last many decades. In one respect only was Keynes on the same page as the post-war term-structure theorists. Keynes too was concerned with the behavior of the long-term rate of interest, and he thought he had discovered the key to this behavior in liquidity preference. In short, Keynes thought he had arrived at *the* theory of interest—a theory of interest-rate levels as well as a theory of spreads—because he believed that the spread between long and short-rates could be anchored in a zero rate of interest for short-term riskless debt. For Keynes, a zero rate of interest was the natural rate for an ultra-short term security. For this reason he thought there was no loss of generality in identifying the short-term security with cash. In a model with only two securities, cash and long bonds, the spread and the long-bond yield are the same thing. The problem is that this model does not have the generality that Keynes imputed to it.

In the more general case, in which short-term assets as well as long-term bonds pay interest, the Keynesian expectations hypothesis is that expectations about the future course of interest rates, along with risk aversion, determine the yield premium, today's spread between the yield on Treasury bonds and Treasury bills. Default risk must be added in if the analysis is extended to corporate obligations. Campbell, in line with virtually the entire post-World War II literature, turns liquidity-preference theory upside down, treating the expectations hypothesis as a hypothesis about how spreads affect the course of future interest rates.

This might matter relatively little were it not for a second innovation, namely, the insistence, beginning in the 1970s, on marrying the inverted expectations hypothesis to rational expectations, an idea that would have been summarily rejected by Keynes. In its married life the expectations hypothesis becomes a hypothesis not only about how present spreads relate to future rates, but also the hypothesis that agents correctly predict the future (up to a random error). Framed this way, looking for confirmation of the expectations hypothesis is a fool's errand. It is hard to see how anybody could believe that any data—except perhaps Nostradamus's, certainly not interest-rate spreads—could accurately forecast future rates.[2]

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[2] It was hardly a shotgun wedding. Indeed the marriage of expectations to *rational* expectations is easier to understand than the shift from a theory of spreads to a hypothesis about forecasting. One of the very first applications of rational expectations was to the relationship of long- and short-term rates (Thomas Sargent, "Rational Expectations and the Term Structure of Interest Rates," *Journal of Money, Credit, and Banking*, vol. 4, No.

It would be interesting to pinpoint when and how the fateful leap was made from a theory of spreads to a hypothesis about the future course of interest rates. It is clear that in *The General Theory* causality ran from beliefs about the future to the spread between long and short rates. But confusion about what Keynes had actually achieved—a theory of spreads—and what he thought he had accomplished—a theory of the interest rate—may have contributed to the inversion of causality that has characterized the postwar literature.

Pre-publication correspondence shows that Dennis Robertson, at least, was aware of the confusion, but Robertson's observation, quoted in the epigraph to this chapter, was ignored by Keynes. Robertson's subsequent criticism, though recognizing a basic incompleteness in Keynes's liquidity-preference theory, struggled, and ultimately failed, to make the key distinction between a theory of *interest* and a theory of *interest spreads* that he had made so cogently in private correspondence.

John Hicks, widely credited as the intellectual father of the expectations hypothesis, blends risk aversion and normal reversion, just as subsequent versions do, but Hicks's main purpose seems to have been to use the combination of the two arguments to make the point that, in view of risk aversion, a high bond yield was a necessary but not sufficient condition for a negative premium on the long bond: "The short rate can only lie above the long rate if the short rate is regarded as *abnormally* high" (*Value and Capital*, Oxford: Clarendon Press 1946 [1939], p 152, emphasis added). Otherwise, it is clear that causality for Hicks, as for Keynes, runs from beliefs about the future course of interest rates to the current spread, not vice-versa (1946 [1939], Chapter 11). But it is also clear that Hicks failed to understand the distortion introduced by Keynes in identifying short-term assets with money, for in Chapter 13 of *Value and Capital* he argues that it is only because of transactions costs that short-term bills offer positive returns.

In the postwar period, Paul Samuelson was crystal clear about the difference between a theory of interest and a theory of spreads. Following on Robertson's 1935 letter to Keynes, the epigraph to Chapter XII quoted Samuelson:

> It would be a mistake, which would be as damaging to further analysis of liquidity preference as it would be to classical doctrines, if it were thought that uncertainty and liquidity differentials are the *sine qua non* for the existence of a [positive] rate of interest. Such a view can be compared with a theory of land rent based upon differences in the quality of different kinds of land. I believe that the analogy is not a superficial one. (*Foundations of Economic Analysis*, Cambridge, Mass.: Harvard University Press, 1947, pp 122-123.)

But Samuelson was much less clear about how a theory of spreads relates to the determination of the overall level of interest rates. Judging from successive editions of his textbook, he never got beyond a model in which all capital is fungible and there are complete markets in all capital assets.[3] Consequently the spread is not only anchored by the marginal productivity of capital, but the anchor operates at every moment of time, so marginal productivity is continuously equal to the hurdle rate. Investment is driven by saving, which in turn responds to agents' wealth preferences. In the 11th edition, the last authored

1, 1972), 74-97. The fit between theorizing interest-rate forecasts and rational expectations must have appeared irresistible, love at first sight.

[3] I have consulted the 2nd, 3rd, and 11th editions of *Economics,* published respectively in 1948, 1955, and 1980.

solely by Samuelson, he writes that net investment ceases when the interest rate is “low enough to choke off all desire to save,… low enough to make the community’s average propensity to consume equal to 100 percent of income.” (*Economics*, 11th edition, New York: McGraw-Hill, 1980, p 562)

Samuelson was hardly alone in believing in a theory in which the rate of interest is determinate, but at least he saw the crucial Keynesian point that this determination has to take place in asset markets, in markets for stocks of capital assets, rather than in markets limited to current flows of additional capital. As was noted in Chapter II, subsequent views been much less astute in theorizing an alternative to liquidity preference, endorsing the pre-Keynesian view of interest-rate determination by the demand for, and supply of, current saving, at least in the long run.

From Consols to Long Bonds, From Coupon Yields to Yields to Maturity

Before we turn to empirical tests of various aspects of liquidity-preference theory, the theory has to be modified to take account of an important difference between the models we have laid out in Chapters XI and XII and the real world, namely, the virtual absence of consols from bond markets. In the US, consols have never been a regular part of Treasury debt, though apparently some of the debt issued in connection with financing the Panama Canal, long since retired, took this form.[4] In the UK, the original home of the consol, this particular debt instrument has gone the way of the dodo even though there are a few surviving relics. (At present only about £2.6 billion of a national debt of £1.5 trillion are consols. https://www.gov.uk/government/news/chancellor-to-repay-the-nations-first-world-war-debt; https://www.gov.uk/government/statistics/public-sector-finances-bulletin)

There is instead a spectrum of bonds and bills of various finite maturities, running, in the US, from one day to 30 years. It is for this reason that we speak of a yield *curve*, representing the relationship between the yield to maturity and the bond’s term. Figure 1 represents the yield curve for Treasury

[4] Some private railroad bonds issued in the 19th and 20th centuries had maturity dates so far in the future that they might as well have been perpetuities. The West Shore Railroad, whose track was leased for 475 years by the New York Central in the 19th century (with an option to renew for another 500 years), issued bonds that were coterminous with the expiration of the initial lease in 2361. The Canadian Pacific Railway also issued perpetual debentures in the 19th century. According to the *Toronto Globe and Mail*, C$31 million were still outstanding in 2011 (http://www.theglobeandmail.com/globe-investor/why-cps-old-time-bondholders-have-a-big-say-in-the-future/article4105862/ accessed 12/09/2014).

Preferred stock is a conditional consol, for which the specified coupon payment can be omitted (or postponed) under certain circumstances.

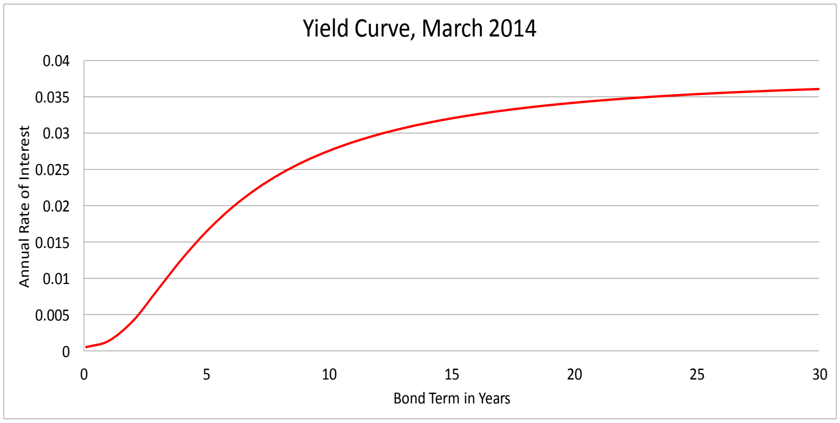

Figure 1

obligations in March, 2014, when the short term rate was virtually zero and the longest bond, maturing in March, 2044, offered a yield to maturity of 3.6 percent.

By definition, the yield to maturity is the interest rate which just makes the present value of the bond's lifetime returns equal to the current price of the bond. In continuous time

$$P(m,t) = \int_t^{t+m} Re^{-\rho_m(m,t)\cdot(\tau-t)}\,d\tau + e^{-\rho_m(m,t)\cdot m} = -\frac{R}{\rho_m}\left(e^{-\rho_m(m,t)\cdot m} - 1\right) + e^{-\rho_m(m,t)\cdot m}$$

where t is the calendar date at which the bond price is evaluated and m is the term to maturity. For a consol, we can define the yield to maturity as the value of ρ_m as $m \to \infty$; the equation reduces to

$$P(m,t) = \frac{R}{\rho_m}$$

so that

$$\rho_m = \rho_c = \frac{R}{P}$$

That is, the limiting yield to maturity coincides with the coupon yield $\frac{R}{P}$.[5] (For notational convenience, the subscript B in the expression P_B is omitted in this appendix. The bond price is denoted by P or by P(m, t) when the extra detail is necessary. The price of goods, which was previously denoted by P, does not enter the analysis here.) By contrast, in the case of finite maturities, the two yields will generally

[5] In a world of consols, there is, strictly speaking, no yield to maturity because a consol never matures. In any case we have no need for this concept because we can work with the coupon yield, $\frac{R}{P}$. In a world of finite-maturity bonds, the yield to maturity plays a separate and distinct role.

differ. Only when the bond trades at par, that is, at its redemption value, P = 1, will the two conceptually distinct yields coincide in value.

Whatever the term to maturity, the holding yield continues to be the sum of the *coupon yield* and the expected (percentage) change in the bond's price,

$$\rho_h(m,t) \equiv \frac{R}{P(m,t)} + \frac{E(\dot{P})}{P(m,t)}$$

And market equilibrium, as characterized by equality between the *expected* holding yield on long bonds and the sum of the short rate plus an illiquidity premium, $-\frac{U_2}{U_1}$, continues to hold for finite-maturity bonds. Except that $-\frac{U_2}{U_1}$ now depends on the term to maturity since the longer the life of the bond, the greater the expected variability of its price and the greater the sensitivity of utility to the value of bonds in the portfolio. Denoting the illiquidity premium $-\frac{U_2}{U_1}$ by $\alpha(m)$, we have the equilibrium condition for agents to hold both bonds and bills:

$$\frac{R}{P} + \frac{E(\dot{P})}{P} = \rho_s(t) + \alpha(m)$$

The spread between the coupon yield and the short-term rate continues to be equal to the difference between the illiquidity premium and the change in price:

$$\frac{R}{P} - \rho_s(t) = \alpha(m) - \frac{E(\dot{P})}{P}$$

In the case of consols this result made it easy to deal with the two polar cases where only risk aversion or only normal reversion is at play. The first assumption—no risk aversion—implies $\alpha = 0$, whereas the second assumption—no reversion to normal—implies $\frac{E(\dot{P})}{P} = 0$. The first of these two assumption carries over to a world of finite bond maturities: the absence of risk aversion implies $\alpha(m) = 0$ whatever the term of the bond. But the second assumption does not imply $\frac{E(\dot{P})}{P} = 0$ when m is finite.

Unlike consols, every finite maturity bond is characterized by a terminal condition, namely, a condition that the price must approach the redemption value of the bond as we approach the redemption date.[6] Although this terminal condition is independent of whether risk aversion or normal reversion is what drives bond prices, its effect plays out very differently in the two cases.

[6] The analogous condition in the case of consols is a so-called transversality condition, an externally imposed limiting value for the price of the consol. Without such a transversality condition, there is nothing to prevent an infinitely long price bubble from equating the holding yield on a consol to any value of the short-term interest rate, with or without a risk premium. For example, suppose the short-term interest rate is fixed at 5 percent. If a consol with a $5 coupon were priced at $200, giving it a coupon yield of 2.5 percent, it can offer a holding yield of 5 percent if it increases in value to $205. In subsequent years, the capital gain would have to be progressively greater than $5 for the holding yield to remain at 5 percent, but there is nothing to prevent this scenario other than the *assumption* that bubbles eventually pop. A transversality condition in effect rules out infinite bubbles.

In the case of pure risk aversion (no reversion to normal), expected bond price and yield will have the general shape of Figure 2. Figure 2 assumes that the short-term rate is 0 today, and will continue to be

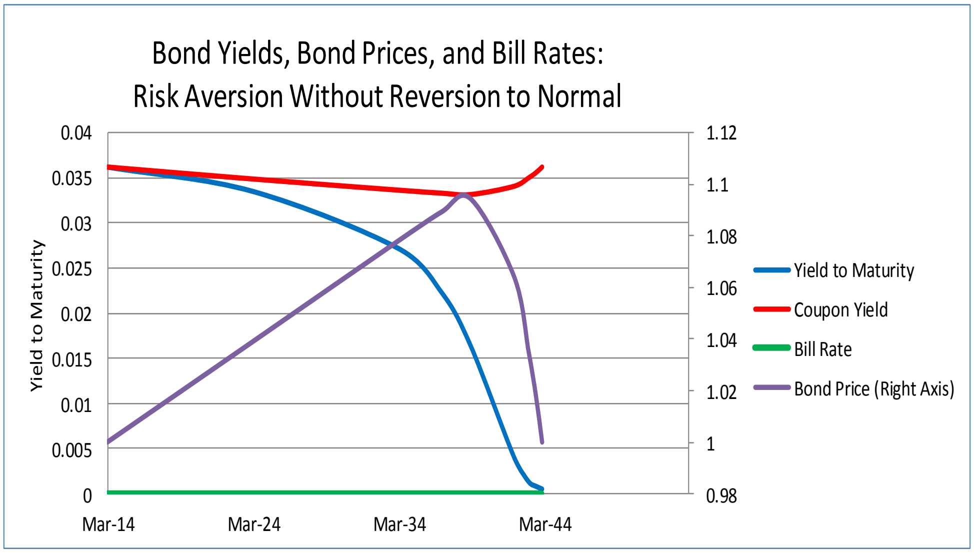

Figure 2

equal to 0 as far out as the eye can see (or rather the mind can imagine). It represents the trajectory of a 30-year bond with a par value of \$1 and a coupon of \$0.036, reflecting the actual 30-year yield to maturity on a bond issued at par in March, 2014.

How do we account for the shapes of the various schedules in Figure 2? Observe first that the yield-to-maturity schedule is the mirror image of the yield curve in Figure 1: the long bond starts life as a 30-year bond but over time morphs into bonds of successively shorter maturities. At every point in the bond's life, the condition of market equilibrium is that the holding yield on the long bond equal the short rate plus a risk premium $\alpha(m)$, where m is the time remaining until the bond matures:

$$\frac{R}{P} + \frac{E(\dot{P})}{P} = \rho_s + \alpha(m)$$

In 2043 the 30-year bond issued in 2014 will be equivalent to a 1-year bill in price risk, and, therefore, equivalent in its return to a short-term bill issued in 2043 (apart from tax treatment and bid-ask liquidity considerations). This is to say that in 2043 the 30-year bond issued in 2014 requires a very small premium to offset its price volatility. With the 1-year bill rate expected to remain at the near-zero level obtaining in 2014, and with no risk premium on the 2014 vintage bond ($\alpha \cong 0$), in 2043-44 the holding yield on this bond is now expected to be near 0. But this can only happen if the price is expected to fall by about as much as the coupon, namely, by about \$0.036. This tells us that the price in 2043 must be in the vicinity of \$1.036 since we assume with certainty that the bond will be redeemed for \$1 in 2044.

What comes down must first have gone up. Long bonds, by assumption, start life at par, so the price of the 30-year bond issued in 2014 must rise early in its life to be able to come down at the end of its life. If, as expected, the bond price begins to rise upon issuance, both the yield to maturity and the coupon yield initially fall since the two yields start out life together.

Evidently it is no longer the case that $\frac{E(\dot{P})}{P} = 0$. Rather, the expected value of bond-price changes with the remaining maturity of the bond. When risk aversion rules, price changes are expected to be positive towards the beginning of the bond's life and negative at the end.

This contrasts sharply with the expected price behavior of the same 30-year bond in a world of reversion to normal, without risk aversion. Figure 3 shows the expected course of the yield to maturity, the

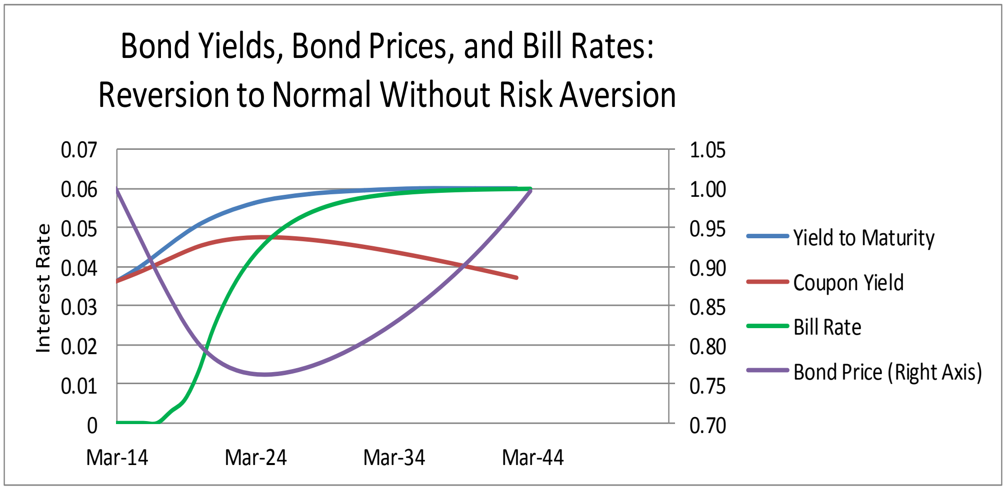

Figure 3

coupon yield, and the price of the same 30-year bond issued at par with a coupon rate of 3.6 percent, along with the expected course of the short-term bill rate, under the assumption that the bill rate will revert to a normal rate of 6 percent at the rate θ = 0.25, after an initial three-year period at 0 and two more years of slower adjustment. The differences between Figures 2 and 3 evidently hinge on the different assumptions about the trajectory of the bill rate as well as on the relationship between the holding yield and the short rate. Because the short-term rate is expected to rise, the yield to maturity on the long bond must also rise as we approach the redemption date. The bond price must initially fall since market equilibrium continues to require

$$\frac{R}{P} + \frac{E(\dot{P})}{P} = \rho_s + \alpha(m)$$

But in the absence of risk aversion, α = 0 for all m, not just at the short end of the term structure, so when the bond is issued at par, the expected bond price must fall in order to equalize returns on bills and bonds. As the price falls, the coupon yield rises; the coupon yield is the ratio of a fixed coupon to a varying price. Once the short-term rate catches up to the coupon yield, the downward price trajectory is reversed, and the price once again reaches par when the bond is redeemed.[7]

[7] It is of course possible that the expected price change is zero over time, but this requires a fortuitous combination of the rates of change of ρ_s and $\alpha(m)$. For $\frac{E(\dot{P})}{P}$ to equal 0, we must have

$$\frac{R}{P} = \rho_s + \alpha(m) = \text{const}$$

Observe that the yield curve, a static picture at one point in time, itself tells us nothing, or rather very little, about the roles of risk aversion and normal reversion in the determination of the spread between long and short rates. The only difference between the two theories that might reveal itself in the yield curve is the incompatibility between pure risk aversion and a downward sloping (inverted) yield curve. As has been observed, Hicks argued that a downward sloping yield curve presupposes that "the short rate is regarded as abnormally high" (*Value and Capital*, 1946 [1939], p 152).[8]

In any case, an upward sloping yield curve is consistent both with risk aversion and with reversion to normal. To illustrate this consistency I have constructed a hypothetical yield curve on the basis of an assumed illiquidity-premium function α(m) with no reversion to normal, and constructed the *same* yield curve on the basis of reversion to normal absent risk aversion. As can be seen in Figure 4, this hypothetical yield curve does a good job of approximating the actual schedule for maturities above three years

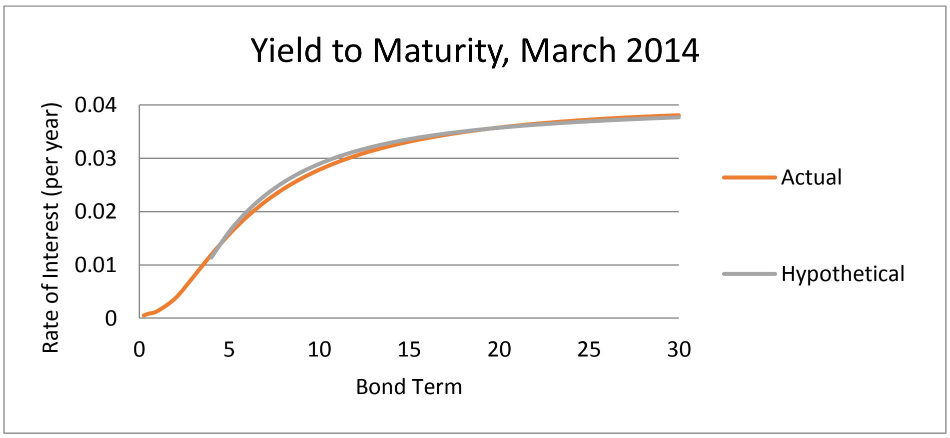

Figure 4

If, however, we project the yield curve 10 years forward, the two hypotheses lead to very different results. Pure risk aversion implies that the yield curve does not change over time, pure normal reversion implies that the yield curve flattens out. So, under the hypothesis of pure risk aversion, we would expect a 30-year bond issued in 2024 to have the same yield to maturity as one issued today. Under normal reversion, the 30-year bond issued in 2024 is expected to yield the average of short-term rates forecast *today* for 10 to 40 years hence. Figure 5 pictures today's forecasts for the two hypothetical

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which is to say that ρ_s must increase over time at exactly the same rate that α(m) is decreasing.

[8] I say "might reveal itself" because I have been unable to prove Hicks's conjecture for coupon bonds, though it is clearly true for zero-coupon bonds. Zero-coupon bonds are artificially constructed bonds which, like actual Treasury bills, have no coupon, the return being the difference between the price today and the redemption price. As we shall see, theoretical as well as empirical analysis is usually formulated in terms of zero-coupon bonds.

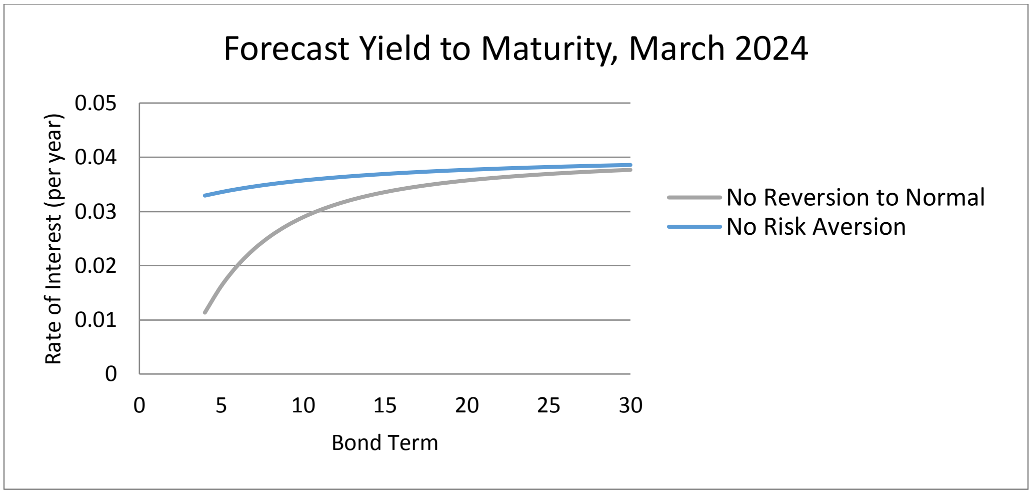


Figure 5

yield curves, one corresponding to pure risk aversion, the other to pure normal reversion.

Sorting Out Risk Aversion and Normal Reversion in a World of Zero-Coupon Bonds

Coupon bonds are the real-world norm, but in exploring the implications of the two theories, it is useful to assume that all bonds provide returns not by periodic payments of interest, but by virtue of a difference between the price paid at the time of purchase and the redemption value when the bond matures (assumed to be $1).[9] Such bonds, so-called zero-coupon bonds, are unavailable even in theory in a world of consols (except as a limiting case), because nobody would hold a bond that offers no periodic payment and will never be redeemed.

Zero-coupon bonds have become the focus of both the theoretical and empirical literature on term structure because, absent consols, it is simpler to analyze a bond with only one payment than a bond with periodic payments and a final payment of a different amount. Real-world, m-year coupon bonds can be understood as composite securities put together from m zero-coupon bonds, each corresponding to a single payment of interest (assumed to take place once per year), with the last payment including the repayment of principal.[10]

[9] The two yield curves in Figure 4 are calculated for zero-coupon bonds , which is why the 30-year bond yields about 3.8 percent in Figure 4, as against 3.6 percent in Figure 3. Actual Treasury zero-coupon bonds are limited to issues with an initial maturity of 1 year or less; the terminological distinction between bills and bonds is precisely whether the obligations do (bonds) or do not (bills) carry a coupon.

[10] By separating the interest payments from the repayment of interest, bond dealers have created interest-only and principal-repayment (zero-coupon) bonds in derivatives markets. These derivatives are called STRIPs (for *S*eparate *T*rading of *R*egistered *I*nterest and *P*rincipal of *S*ecurities).

One obvious difference is that ordinary coupon bonds generally begin their lives at par (and are often called "par bonds"), whereas the prices of zero-coupon bonds increase over time regardless of whether risk aversion or normal reversion is the driving force. Or, rather, bond prices will be expected to increase. Actual bond prices may fall, but nobody will hold a zero-coupon bond that is expected to fall in price as long as holding cash is costless. By contrast, yields to maturity, which differ from changes in bond prices when yields change over time, can be expected to rise or fall, depending on whether risk aversion or reversion to normal is calling the tune.

As is the case for ordinary bonds that begin life at par, the trajectories of zero-coupon bonds of the same maturity and the same initial yield to maturity—one reflecting normal reversion and the other risk aversion—must start out together and end at the same point. The difference between the price trajectories in Figures 2 and 3 translates into the difference in Figure 6. Both bonds start from

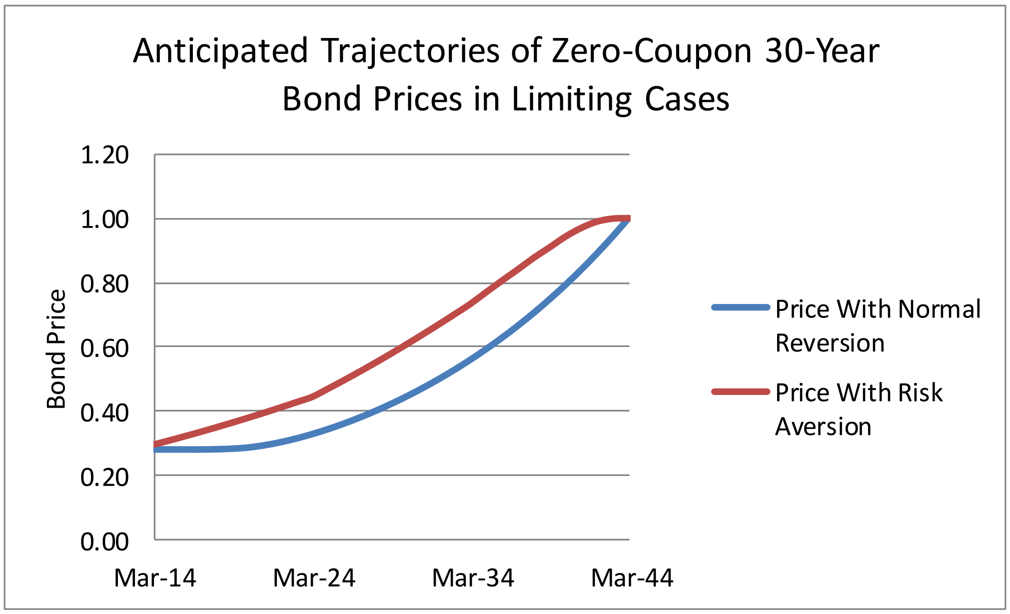


Figure 6

a value of just over \$0.30 when first issued, corresponding to a yield to maturity of 0.038. But the slope of the schedule depicting normal reversion is flat at the outset, and the schedule depicting risk aversion is flat when the bond matures.

To see why, start from the relationship between the price of a zero-coupon bond and the yield to maturity. By definition, the yield to maturity is the interest rate which makes the return on purchasing the bond today equal to the value of the bond at maturity:

$$P(m,t)e^{\rho_m(m,t)\cdot m} \equiv 1$$

Equivalently, the price today is equal to the present value of the redemption price, discounted at the yield to maturity.

$$P(m,t) \equiv e^{-\rho_m(m,t)\cdot m}$$

The holding yield on the bond is just the (percentage) rate of price appreciation,[11]

$$\frac{\hat{\dot{P}}}{P} \equiv \rho_m - \frac{\widehat{d\rho_m}}{dt} m$$

where $\frac{\widehat{d\rho_m}}{dt}$ is the total derivative of ρ_m , taking account of the interdependence between the remaining time to maturity m and the calendar time t, m = T – t, with T the redemption date:

$$\frac{\widehat{d\rho_m}}{dt} m \equiv \frac{\widehat{\partial\rho_m}}{\partial t} m - \frac{\partial\rho_m}{\partial m} m$$

The holding-yield condition is that the expected price increase be equal to the sum of the short rate and the illiquidity premium

$$\frac{\hat{\dot{P}}}{P} \equiv \rho_m - \frac{\widehat{d\rho_m}}{dt} m = \rho_s(t) + \alpha(m)$$

The first order differential equation

$$\frac{\widehat{d\rho_m}}{dt} - \frac{\rho_m}{m} = -\frac{\rho_s(t) + \alpha(m)}{m}$$

has the solution

$$\rho_m(m,t) = m^{-1}\left(-\int \hat{\rho}_s(t)\,dt - \int \alpha(m)\,dt\right) + m^{-1}c = m^{-1}\left(-\int \hat{\rho}_s(t)\,dt + \int \alpha(m)\,dm\right) + m^{-1}c$$

To solve for the constant term, c, we take limits

$$\lim_{m\to 0} \rho_m(m,t) = \rho_m(0,T) = \lim_{m\to 0} \frac{\widehat{\rho}_s(t)(-m) + \alpha(m)m}{m} + \frac{c}{m}$$

Since $\rho_m(0, T) = \lim_{m\to o} \hat{\rho}_s(t) = \rho_s(T)$, we have $c = \lim_{m\to o} \alpha(m)\, m = 0$. Hence

$$\rho_m(m,t)\, m = -\int \hat{\rho}_s(t)\,dt + \int \alpha(m)\,dm$$

In words: the holding-yield condition implies that discounting at the yield to maturity is equivalent to discounting at the average expected short-term rate plus the corresponding illiquidity premium. The relationship between bond price, yield to maturity, and short-term rates is

$$P(m,t) \equiv e^{-\rho_m(m,t)\cdot m} = e^{-\int_t^{t+m} \hat{\rho}_s(t)\,d\tau + \int_0^m \alpha(\tau)\,d\tau} = e^{-\int_t^{t+m}[\alpha(m+t-\tau) + \hat{\rho}_s(\tau)]d\tau}$$

where $\alpha(m+t-\tau)$ is the illiquidity premium at time τ, $m+t-\tau$ reflecting the time remaining until the bond matures, and $\hat{\rho}_s(\tau)$ represents the estimate of the short-term interest rate at τ. The holding-yield condition can be derived from this equation by taking total derivatives on both sides, but there is new information in the limits of integration. Take logarithms on both sides

[11] "Hats" in general denote forecasts, as distinct from actual, unhatted, values.

$$\ln P(m,t) \equiv -\rho_m \cdot m = -\int_t^{t+m} \left[\alpha(m+t-\tau) + \hat{\rho}_s(\tau)\right] d\tau$$

and then take *partial* derivatives with respect to both calendar time t and maturity m. We obtain two new conditions that characterize the instantaneous relationship between the yield to maturity, forecasts of short-term rates, and the illiquidity premium.

First we differentiate the discount factor $\rho_m \cdot m$ with respect to t, holding m constant. The resulting partial derivative $\widehat{\frac{\partial \rho_m}{\partial t}}$ is the estimate of the rate of change over time of the yield to maturity on an m-period bond, holding maturity constant, that is, the rate of change of the constant-maturity yield:

$$\widehat{\frac{\partial \rho_m}{\partial t}}\, m = \alpha(0) + \hat{\rho}_s(t+m) - \alpha(m) - \rho_s(t) + \int_t^{t+m} \frac{\partial \alpha}{\partial t} d\tau$$

The last term can be seen to be equal to $\alpha(m) - \alpha(0)$ by making the substitution $\omega = \tau - t$ and noting that

$$\int_t^{t+m} \frac{\partial \alpha}{\partial t} d\tau = -\int_0^m -\frac{d\alpha}{d\omega} d\omega = \alpha(m) - \alpha(0)$$

Thus the equation reduces to the *consistent forecasting condition*

$$\widehat{\frac{\partial \rho_m}{\partial t}}\, m = \hat{\rho}_s(t+m) - \rho_s(t)$$

which says that the anticipated change in the constant-maturity discount factor, $\rho_m \cdot m$, must equal the difference between the expected gain at the end of the bond's life, $\hat{\rho}_s(t+m)$, and the loss at the beginning, $\rho_s(t)$.[12]

A second condition is obtained by differentiating $\rho_m \cdot m$ with respect to maturity, holding time constant. The derivative $\frac{\partial \rho_m}{\partial m}$ is the rate of change of the yield to maturity along the yield curve. (Since the yield curve is known to the agent, this is not an estimate; hence no hat.) This gives the *forward-rate condition*

$$\rho_m(m,t) + \frac{\partial \rho_m}{\partial m}\, m = \hat{\rho}_s(t+m) + \alpha(m)$$

The left-hand side is the forward rate implicit in the yield curve. The forward rate is the yield that an agent can obtain in the future if she enters into a swap in which she makes offsetting sales and purchases of bonds that mature at time $t + m$ and time $t + m + \varepsilon$. If time is divided into discrete periods, we can imagine an agent making a short sale of one bond maturing $m - 1$ periods hence and with the proceeds buying bonds maturing m periods hence. At the present time she has no cash outlay; the cash outlay takes place $m - 1$ periods from now, when she must redeem the bond she sold short today. A

[12] Observe that the consistent-forecasting condition obtained by differentiating the definite integral is different from the corresponding derivative of the indefinite integral. The upper limit of integration introduces the term $\hat{\rho}_s(t+m)$. A similar change characterizes the forward-rate condition (below).

market equilibrium in which agents hold both long and short term securities requires the forward rate to be equal to the expected short rate plus the illiquidity premium.[13]

Why does the forward rate equal the sum of the yield to maturity, $\rho_m(m,t)$, and the maturity-weighted change along the yield curve, $\frac{\partial \rho_m}{\partial m} m$? An example might help. Suppose time is divided into discrete periods of one year each. Assume that for bonds with terms of zero to four years, the yield to maturity is 1 percent; that is, $\rho_m(m,t) = 0.01$, and that the yield to maturity on five-year bonds is 2 percent, $\rho_m(5,t) = 0.02$. The agent of the previous paragraph—let's call her Naomi—sells a four-year bond short and buys five-year bonds. With continuous compounding, the bond she sells, with a redemption value of $1, nets her $0.961. This allows her to buy 1.062 five-year bonds, since the five-years are worth $.905 each.

The cost of the five-year bond is offset for four years by the short sale of the four-year bond. During this period Naomi has no money at risk and earns nothing. At the end of the fourth year, however, Naomi has to return the bond she borrowed to initiate the process, and to do so she has to lay out $1 to purchase a bond to make the lender whole. At the end of year five, she receives $1.062 when it comes time to redeem the five-year bonds. The net return of $0.062 can be broken down into a 1 percent yield (the yield on four-year bonds), plus an additional 1 percent per year over five years, or 6 percent in all

$$\rho_m(4,t) + \left(\frac{\Delta \rho_m}{\Delta m}\right)_{m=5} \times 5 = 0.01 \quad + \quad (0.01 \times 5)$$

which is precisely the formula on the left-hand side of the forward-rate condition.

Equilibrium requires that the forward rate equal the expected short rate at time t + m *plus* an illiquidity premium $\alpha(m)$. Why is an illiquidity premium necessary to equilibrate expectations about future short rates and opportunities for future gain available with certainty today (assuming no default)? If the short rate expected in year 5 is say, 2 percent, how could Naomi's offsetting transactions yield 6 percent? After all, she lays out $1 at the end of Year 4 and reaps $1.06 at the end of year 5, a short-term return on a short-term investment. The answer is that, even though no cash is required of Naomi until year 5, she is committed to the transaction from the get-go and thus runs the price risks that would accompany premature (literally) unwinding of her positions. This commitment, rather than the actual laying out of cash, is the reason for an illiquidity premium in the first place.

The forward rate only provides upper bounds for $\hat{\rho}_s(t+m)$ and $\alpha(m)$ because it tells us only what the sum $\hat{\rho}_s(t+m) + \alpha(m)$ must be when Naomi is indifferent between committing to the (partially)

[13] Observe that the forward-rate condition contains no new information since it can be derived from the holding-yield condition and the forecasting-consistency condition. The forward rate is conceptually different from the holding yield even though both reflect short-period returns from long bonds. The holding yield reflects the return to a commitment today with respect to the agent's portfolio over the next ε years, whereas the forward rate reflects a commitment m years hence to hold a bond maturing a further ε years in the future. If the forecast $\hat{\rho}_s(t+m)$ turns out to be accurate, today's forward rate for a point in time m years hence is equal to the holding yield on an $(m+\varepsilon)$-year bond at that future time.

offsetting purchase and sale today and holding short-term bonds in the future.[14] A given value of the forward rate is compatible with any combination of the expected short-term rate and the illiquidity premium that sums to this value. That is, any given forward rate is compatible with a high value of $\hat{\rho}_s(t+m)$ and a low value of $\alpha(m)$, or vice versa. Of course, the total absence of risk aversion implies that the forward rate on long bonds must be exactly equal to the expected future short rate for the forward market to be in equilibrium. At the other extreme, the absence of expectations of reversion to normal means that the expected short rate is equal to today's short rate. In the example, a forward rate of 6 percent for year five is compatible with an expected short rate of 6 percent coupled with an illiquidity premium of 0, or an illiquidity premium of 0.06 coupled with an expected short rate of 0. Or with in-between values that sum to 0.06.

If we combine consistent forecasting with the forward-rate condition, we obtain the holding-yield condition

$$\frac{\hat{P}}{P} \equiv \rho_m(m,t) + \frac{\partial \rho_m}{\partial m} m - \frac{\widehat{\partial \rho_m}}{\partial t} m = \rho_s(t) + \alpha(m)$$

Rearranging terms, we obtain a relationship between the expected change in the yield to maturity (taking account of the change in term to maturity) and the spread between long-bond yield and short-term rate:

$$\frac{\widehat{\partial \rho_m}}{\partial t} m - \frac{\partial \rho_m}{\partial m} m = \rho_m(m,t) - \rho_s(t) - \alpha(m)$$

Analysis of this equation allows us to understand the very different ways that expectations have been understood to interact with interest rates. As I have argued in this chapter and the last, Keynes's liquidity-preference theory properly understood is a theory of how interest-rate spreads are determined by expectations about the future course of interest rates and risk aversion. So if causality is read as going from right to left, we have

$$\rho_m(m,t) - \rho_s(t) = \frac{\widehat{\partial \rho_m}}{\partial t} m - \frac{\partial \rho_m}{\partial m} m + \alpha(m)$$

with the yield premium the dependent variable and the terms on the right-hand side the independent variables. The key to teasing out the separate impact of risk aversion from the separate impact of normal reversion is the behavior of $\frac{\widehat{\partial \rho_m}}{\partial t}$ and $\frac{\partial \rho_m}{\partial m}$, particularly how these terms relate to the current short-term rate of interest. If we combine the forecasting-consistency condition

$$\frac{\widehat{\partial \rho_m}}{\partial t} m = \hat{\rho}_s(t+m) - \rho_s(t)$$

[14] When the forward rate is equal to zero, the yield curve itself tells us something about market expectations with respect to short-term rates. In this case equilibrium requires $\rho_s(t+m) + \alpha(m)$ to be equal to zero. With both $\rho_s(t+m)$ and $\alpha(m)$ constrained to be non-negative, this in turn implies $\rho_s(t+m) = \alpha(m) = 0$.

and the forecasting equation

$$\hat{\rho}_s(\tau) = (1 - e^{-\theta(t-\tau)})\rho_s^* + e^{-\theta(t-\tau)}\rho_s(t)$$

the solution to the differential equation

$$\rho_m(m,t) + \frac{\partial\rho_m}{\partial m} m = \frac{\widehat{\partial\rho_m}}{\partial t} m + \rho_s(t) + \alpha(m)$$

becomes

$$\rho_m(m,t) = m^{-1}\int_0^m \alpha(\tau)d\tau + \left(1 - \frac{1-e^{-\theta m}}{\theta m}\right)\rho_s^* + \frac{1-e^{-\theta m}}{\theta m}\rho_s(t) + m^{-1} c$$

We solve for the constant term c by invoking the boundary conditions, $\rho_m(0,t) = \rho_s(t)$ and $\alpha(0) = 0$.

Thus c = 0, and the spread is given by

$$\rho_m(m,t) - \rho_s(t) = m^{-1}\int_0^m \alpha(\tau)d\tau + \left(1 - \frac{1-e^{-\theta m}}{\theta m}\right)\rho_s^* + \left(\frac{1-e^{-\theta m}}{\theta m} - 1\right)\rho_s(t)$$

The two limiting cases of no risk aversion and no normal reversion are characterized by

$$\rho_m(m,t) - \rho_s(t) = \left(1 - \frac{1-e^{-\theta m}}{\theta m}\right)\rho_s^* + \left(\frac{1-e^{-\theta m}}{\theta m} - 1\right)\rho_s(t) \qquad \text{no risk aversion}$$

$$\rho_m(m,t) - \rho_s(t) = m^{-1}\int_0^m \alpha(\tau)d\tau \qquad \text{no normal reversion}$$

One consequence of assuming away normal reversion is to limit the yield curve to the general shape of Figure 1, with a positive yield premium, one that increases with the term to maturity. If we add the assumption of an upper limit to $\alpha(m)$ as $m \to \infty$, we obtain an asymptotic limit to the yield to maturity. The absence of normal reversion thus rules out inverted yield curves, in which the spread between long and short bonds falls with the maturity of the bond. That from time to time we actually observe inverted yield curves, as for example in early 2007, contradicts the possibility that the yield curve is

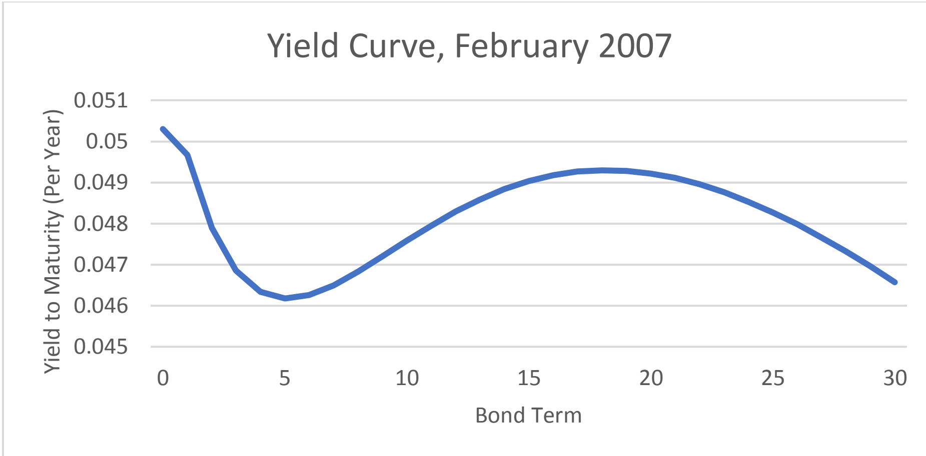


Figure 7

determined by risk aversion alone.

Testing Whether Both Risk Aversion and Normal Reversion Matter

Testable differences between risk aversion and normal reversion follow from the implications of the two hypotheses for regressions of the yield premium on the short-term rate

$$\rho_m(m,t) - \rho_s(t) = a_0 + a_1\,\rho_s(t) + \varepsilon$$

In the absence of risk aversion, this regression should give

$$a_0 = \left(1 - \frac{1 - e^{-\theta m}}{\theta m}\right)\rho_s^* > 0$$

$$-1 < a_1 = \left(\frac{1 - e^{-\theta m}}{\theta m} - 1\right) < 0$$

And in the absence of normal reversion the resulting coefficients should be

$$a_0 = m^{-1}\int_0^m \alpha(\tau)d\tau > 0$$

$$a_1 = 0$$

In between the two limiting cases, we would expect

$$a_0 = m^{-1}\int_0^m \alpha(\tau)d\tau + \left(1 - \frac{1 - e^{-\theta m}}{\theta m}\right)\rho_s^* > 0$$

$$-1 < a_1 = \left(\frac{1 - e^{-\theta m}}{\theta m} - 1\right) < 0$$

with both the constant term and the coefficient on the short rate larger in absolute value the greater is m.

A regression of the yield premium on the short rate is evidently complicated by the possibility of errors in the measurement of the short rate. The short rate relevant for the present exercise is one specific observation within a given month, or perhaps an average of several observations, but the measured short rate is an average of *all* observations during the month. Since the short rate appears on both sides of the equation with different signs, the measurement error will bias the OLS regression coefficient towards zero.

This problem can be addressed by instrumenting the independent variable, and an obvious instrument is at hand, namely, the rate of inflation. The relevance of this instrument is hardly in question, but its exogeneity depends on what is assumed about the relationship between inflation and long-bond yields. For the moment I assume that inflation directly affects only the current short rate and that the direct effect feeds through to long-bond yields via expectations about future short rates, which are still assumed to be driven by the equation

$$\hat{\rho}_s(\tau) = (1 - e^{-\theta(t-\tau)})\rho_s^* + e^{-\theta(t-\tau)}\rho_s(t)$$

The key point is that the *normal* rate of interest is assumed to be unaffected by the current inflation rate. On this assumption, the rate of inflation will not be correlated with the error term, so that the exogeneity condition is satisfied.[15]

Inflation (INF) is measured by the urban CPI, also on a monthly basis. I use year on year changes in the index to eliminate noise in the monthly data, but even so separate regressions over periods of rising inflation and periods of falling inflation (before and after 1980) make it clear that the coefficient on inflation in the first stage is markedly higher in the second period. This makes sense if the short rate actually depends on a distributed lag of past and present inflation since lagged inflation will be lower than current inflation in the first period and higher in the second. Without introducing an explicit lag function, this effect can be approximated by adding a dummy variable in the first stage (DUMINF), which takes the value 0 until June 1980 and is equal to the inflation rate thereafter.

The results of the first-stage regression of the short rate (TB3MS = 3 month Treasury Bill Rate, Secondary Market Rate) and the second-stage regression of the difference between the long rate and the short rate (YLDPREM = Difference between the yield to maturity of a zero-coupon 10-year T-note and the short rate) are

[15] Later in this appendix we examine in more detail the nexus between the rate of inflation and interest-rate forecasting.

$$TB3MS = .000949 + .697\ INF + .405\ DUMINF$$
$$\quad\quad (.0002) \quad (.033) \quad (.050)$$
$$R^2 = .57$$

$$YLDPREM = .00284 - .313\ TB3MS$$
$$\quad\quad (.0009) \quad (.020)$$
$$R^2 = .38$$

Robust Standard Errors in Parentheses

Observations: 497 F(2, 494) = 223.7

Variable	Obs	Mean	Std. Dev.	Min	Max
------------	-------	-------------	-------------	----------	----------
YLDPREM	497	0.0015	0.0012	-0.0028	0.0036
TB3MS	497	0.0044	0.0027	0.0000083	0.0136
INF	497	0.0036	0.0025	-0.0017	0.0123

Interest and inflation are per month, so that 0.001 corresponds to 12 percent per year.

A couple of pictures will help us understand these results. Figure 8 is a scatter of the yield premium vs

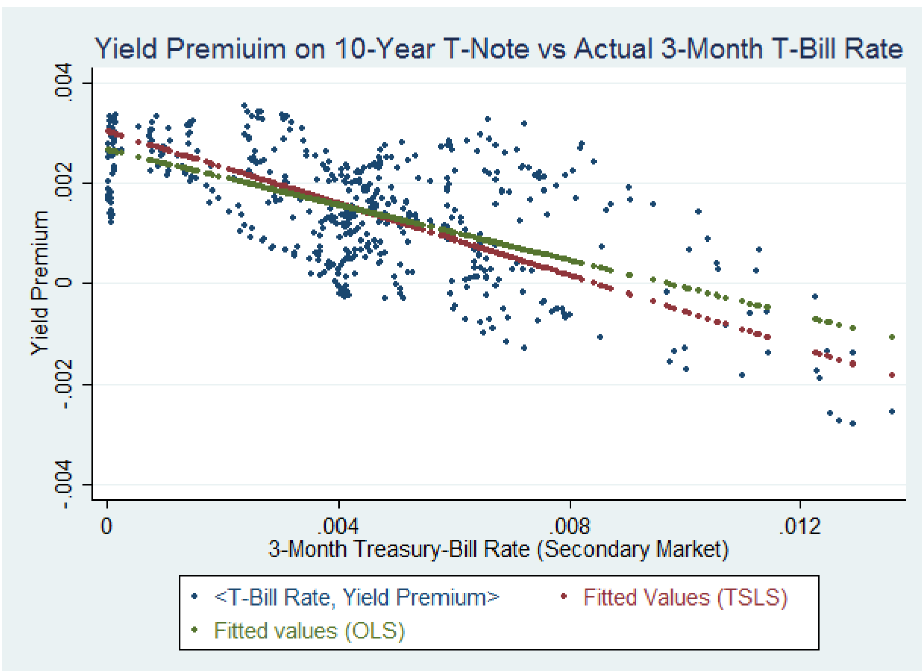


Figure 8

the short rate, along with the fitted values of the two-stage regression (and the OLS values for comparison). The next picture, Figure 9, shows the fitted equation with a plot of the yield premium

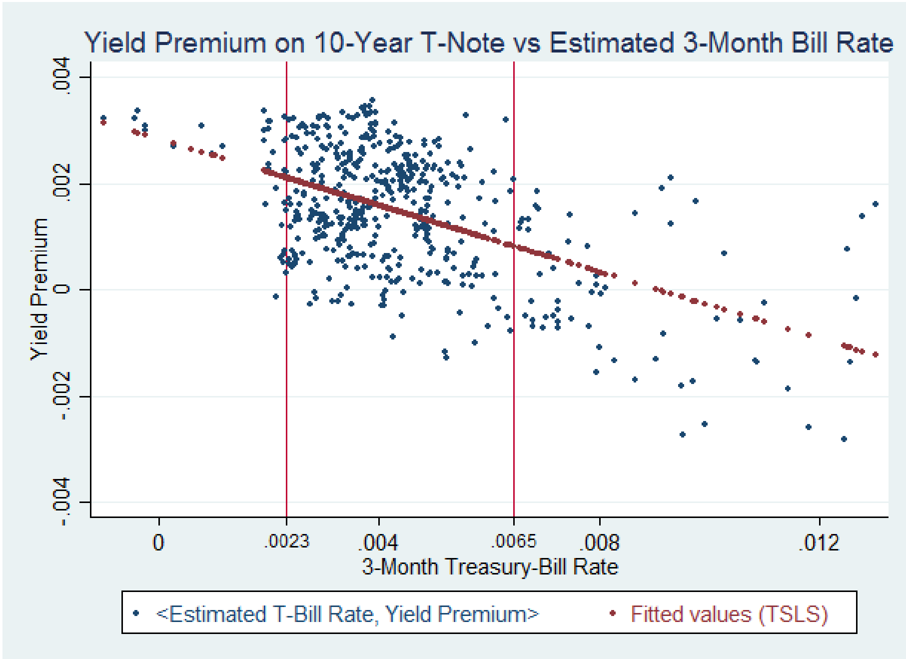


Figure 9

against the estimate of the current short-term rate calculated in the first stage. The vertical red lines demarcate an interval of one standard deviation on each side of the mean of the fitted short-term rate. Figure 9 makes clear that the strength of the regression derives from the outliers, when estimated 3-month bill rates are more than one standard deviation away from the mean, less than .0023 and greater than .0065. For if we repeat the regression limiting the sample to observations within the one standard deviation interval, the second-stage coefficient a_1 is closer to zero and has a much smaller t-value. The coefficient goes from –0.31 to –0.19 and its t-value falls from 15.6 to 4.2.

$$\underset{(.0003)}{TB3MS = .0006} + \underset{(.060)}{.795\ INF} + \underset{(.068)}{.401\ DUMINF}$$

$$R^2 = .28$$

$$YLDPREM = \underset{(.0002)}{.00237} - \underset{(.045)}{.185\ TB3MS}$$

$$R^2 = .15$$

Robust Standard Errors in Parentheses

Observations: 399 $\quad$ F(2, 396) = 91.2

These results support the plausible notion that both normal reversion and risk aversion play a role in determining the spread between the long-bond yield and the short-term rate. The negative coefficient on TB3MS supports a role for normal reversion, while the fact that this result is driven mostly by the outliers suggests that normal reversion is much less relevant most of the time.

We can exploit the data for other maturities to test the idea that both normal reversion and risk aversion are at play. Normal reversion predicts that both the constant term (a_0) and the coefficient (a_1) will be larger in absolute value as the bond maturity lengthens. And risk aversion predicts that the constant term becomes larger with m. But the ratio $\frac{a_0}{a_1}$ tells one story when risk aversion is absent and another when it is present. Or rather, the story told by this ratio may be dispositive of whether risk aversion is present in the data. In the general case we have

$$\frac{a_0}{a_1} = -\frac{m^{-1}\int_0^m \alpha(\tau)d\tau \;\; + \left(1-\frac{1-e^{-\theta m}}{\theta m}\right)\rho_s^*}{\left(1-\frac{1-e^{-\theta m}}{\theta m}\right)} = -\frac{\theta\int_0^m \alpha(\tau)d\tau}{\left(\theta m-\left(1-e^{-\theta m}\right)\right)} - \rho_s^*$$

The right hand side will change with m unless $\theta = 0$, which is to say no normal reversion, or unless

$$\alpha(m)\left(\theta m - \left(1-e^{-\theta m}\right)\right) = \left(1-e^{-\theta m}\right)\theta\int_0^m \alpha(\tau)d\tau$$

This can happen for one of two reasons, either a fortuitous coincidence of parameter values that makes the left- and right-hand sides equal, or if $\alpha(m) = 0$ for all m. On the other hand if $\frac{a_0}{a_1}$ changes with bond maturity, then it must be because the first term changes with m, which in turn requires $\alpha(m) > 0$ for *some* m. In other words, a tendency for $\frac{a_0}{a_1}$ to change with m is inconsistent with the assumption that only reversion to normal matters for the determination of interest-rate spreads.

In the event, Table 1 suggests that the illiquidity premium changes markedly over intermediate bond

Table 1. Regressions of Yield Premium Against Short-Term Bond Rate							
		Sample Period					
	1971-2012				1985-2012		
Maturity (Years)	Coeff (a_1)	Const (a_0)	a_0/a_1	θ	Coeff (a_1)	Const (a_0)	a_0/a_1
1	-0.01052	0.00049	-0.04658	0.0018	0.04838	0.00023	0.00480
2	-0.07502	0.00097	-0.01296	0.0066	0.01068	0.00058	0.05399
3	-0.12564	0.00135	-0.01076	0.0076	-0.03432	0.00091	-0.02661
4	-0.16738	0.00167	-0.00998	0.0079	-0.07789	0.00123	-0.01573
5	-0.20247	0.00194	-0.00959	0.0078	-0.11854	0.00151	-0.01275
6	-0.23223	0.00218	-0.00937	0.0077	-0.15575	0.00177	-0.01138
7	-0.25755	0.00238	-0.00924	0.0075	-0.18931	0.00201	-0.01061
8	-0.27899	0.00256	-0.00916	0.0072	-0.21920	0.00222	-0.01012
9	-0.29726	0.00271	-0.00910	0.0070	-0.24552	0.00240	-0.00978
10	-0.31267	0.00284	-0.00907	0.0067	-0.26849	0.00256	-0.00955
11					-0.28837	0.00270	-0.00938
12					-0.30548	0.00282	-0.00925
13					-0.32012	0.00293	-0.00914
14					-0.33259	0.00301	-0.00906
15					-0.34317	0.00309	-0.00900
16					-0.35213	0.00315	-0.00894
17					-0.35970	0.00320	-0.00889
18					-0.36609	0.00324	-0.00884
19					-0.37148	0.00327	-0.00880
20					-0.37604	0.00329	-0.00875
21					-0.38043	0.00331	-0.00870
22					-0.38377	0.00332	-0.00866
23					-0.38663	0.00333	-0.00861
24					-0.38909	0.00333	-0.00857
25					-0.39124	0.00333	-0.00852
26					-0.39311	0.00333	-0.00847
27					-0.39477	0.00332	-0.00842
28					-0.39626	0.00332	-0.00837
29					-0.39761	0.00331	-0.00831
30					-0.39884	0.00329	-0.00826

maturities and then levels out. For the sample period 1971-2012, $\frac{a_0}{a_1}$ falls over the range of 1-4 years. Observe the relative stability of the implicit estimate of θ, except for the shortest maturity. For maturities between 2 and 10 years, θ varies between 0.0066 and 0.0079, indicating a relatively slow adjustment of the expected short rate to the normal rate.

However, the range of bond maturities for the sample period 1971-2012 is limited to 10 years. It is possible to extend the range, but to run the regression over the full spectrum of bond maturities, 1 to 30 years, requires us to begin with data from the 1980s. Despite the omission of data in which yields were rising—the 1970s—the coefficients in the 7-10 year range don't change very much. By contrast, at the short end of the spectrum a_1—contrary to the predictions of both risk aversion and normal reversion—is significant at the 99 percent level but has the wrong sign. For the shorter sample period, the ratio $\frac{a_0}{a_1}$ changes over a range of maturities up to 7 or 8 years. The limiting value of $\frac{a_0}{a_1}$ as m increases is the sum of the upper limit to α(m) and the normal rate ρ_s^*. Unfortunately the data do not really permit an estimate of this limiting value, but the data are reasonably clear in rejecting the hypothesis that only normal reversion matters.

Is the Illiquidity Premium Constant?

So far it has been assumed that the liquidity premium depends only on the (remaining) term until the bond matures. Both theory and data suggest otherwise. Refet Gürkaynak and Jonathan Wright survey a vast literature on variable illiquidity premia ("Macroeconomics and the Term Structure," *Journal of Economic Literature*, 50:331-367 [2012]). The theory developed in the Chapter XI and this chapter suggests specific reasons why the illiquidity premium ought to vary. One is variation in the relative supplies of bonds and bills. Suppose the bill rate is given. Then the common-sense view is that the more bonds in the mix, the lower the price of bonds and the higher the yield. Conversely, the more bills, the lower the bill price and the higher the bill yield. Thus the yield premium ought to vary directly with the proportion of bonds in the mix of Treasury obligations. A convenient measure of the bond:bill ratio is the average maturity (AVGMAT) of Treasury obligations.

As we have seen in the mathematical appendix to this chapter, this result does not necessarily hold in a comparative-statics context. But if we limit ourselves to stable equilibria, common sense is vindicated by the math.

A second reason why the illiquidity premium varies, namely, the rate of unemployment, is less obvious. As a measure of business conditions, unemployment might be a reason for default rates to vary, but default is not an issue for Treasury obligations. However, as a proxy for the degree of uncertainty about the economic future, even without default risk it is possible for the unemployment rate to influence the degree of perceived liquidity risk through an effect on the volatility of bond prices. Liquidity preference as aversion to risk suggests that the more volatile are bond prices, the greater will be the illiquidity premium.

We can test this relationship by asking whether or not the volatility of bond-price changes is systematically related to the unemployment rate. According to Figure 10, there is a clear relationship.

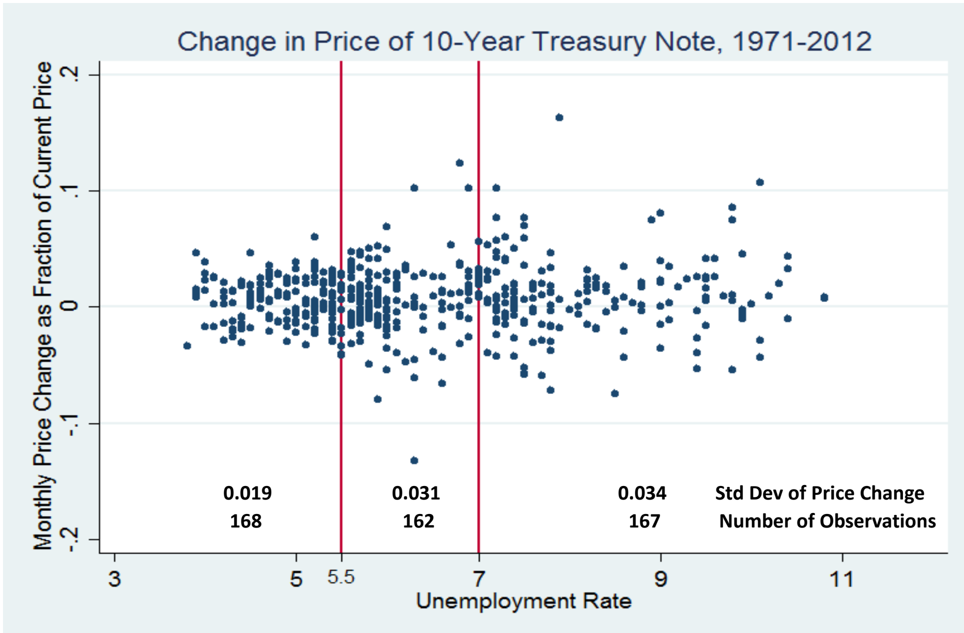


Figure 10

The vertical axis measures monthly price changes of zero-coupon bonds, estimated by the formula

$$\frac{\dot{P}}{P} = \rho_m(m,t) + \frac{\partial \rho_m}{\partial m} m - \frac{\partial \rho_m}{\partial t} m$$

In which the price change $\frac{\dot{P}}{P}$ and the shift in the yield curve $\frac{\partial \rho_m}{\partial t}$ are actual rather than expected price changes. At low levels of unemployment the standard deviation of bond-price changes is 2/3 its value at higher levels, so the unemployment rate makes sense as a proxy, if not a direct cause, of bond-price volatility.

If we linearize an illiquidity-premium function that depends on the unemployment rate (UNRATE, measured as a percentage of the civilian labor force), the average maturity of Treasury obligations (AVGMAT, measured in months), and bond maturity ($\alpha^*(m)$), we have the illiquidity premium as a constant term $\alpha^*(m)$ that is modified by the impact of the unemployment rate and the mix of bonds and bills[16]

$$\alpha(m, \text{UNRATE}, \text{AVGMAT}) = \alpha^*(m) + b_1\text{UNRATE}(t) + b_2\text{AVGMAT}(t)$$

The differential equation relating the holding yield on the bond to the bill rate becomes

[16] UNRATE is the deviation from the mean unemployment rate (6.43 percent) and AVGMAT is the deviation from the mean average maturity (56.2 months) over the 497 observations.

$$\frac{\widehat{d\rho_m}}{dt} - \frac{\rho_m}{m} = -\frac{\rho_s(t) + \alpha^*(m) + b_1 UNRATE(t) + b_2 AVGMAT(t)}{m}$$

And the solution becomes

$$\rho_m(m,t) = m^{-1}\left(\int \hat{\rho}_s(t+m)\, dm + \int \alpha^*(m)\, dm\right) + b_1 UNRATE(t) + b_2 AVGMAT(t) + m^{-1} c$$

As before, examination of the limiting case as $m \to 0$, gives $c = 0$. With short rates forecast by the equation

$$\hat{\rho}_s(\tau) = (1 - e^{-\theta(t-\tau)})\rho_s^* + e^{-\theta(t-\tau)}\rho_s(t)$$

the yield premium is now related to the short-term bill rate by the equation

$$\rho_m(m,t) - \rho_s(t) = m^{-1}\int_0^m \alpha^*(\tau)d\tau + \left(1 - \frac{1 - e^{-\theta m}}{\theta m}\right)\rho_s^* + \left(\frac{1 - e^{-\theta m}}{\theta m} - 1\right)\rho_s(t) + b_1 UNRATE(t) + b_2 AVGMAT(t)$$

Once again we estimate the equation

$$\rho_m(m,t) - \rho_s(t) = a_0 + a_1 \rho_s(t) + b_1 UNRATE(t) + b_2 AVGMAT(t) + \varepsilon$$

by two-stage least squares, using the same instruments as before. The results are

$$\underset{(.0002)}{TB3MS = .000974} + \underset{(.044)}{.661\ INF} + \underset{(.071)}{.453\ DUMINF} - \underset{(.00006)}{.00176\ UNRATE} - \underset{(.000009)}{.0000137\ AVGMAT}$$

$R^2 = .58$

$$YLDPREM = \underset{(.00009)}{.000274} - \underset{(.019)}{.291\ TB3MS} + \underset{(.00002)}{.000375\ UNRATE} + \underset{(.000002)}{.0000228\ AVGMAT}$$

$R^2 = .60$

Robust Standard Errors in Parentheses

$F(4, 492) = 115.34$

Variable	Obs	Mean	Std. Dev.	Min	Max
------------	-------	-------------	-------------	----------	----------
YLDPREM	497	0.0015	0.0012	-0.0028	0.0036
TB3MS	497	0.0044	0.0027	0.0000083	0.0136
INF	497	0.0036	0.0025	-0.0017	0.0123
UNRATE	497	6.43	1.58	3.8	10.8
AVGMAT	497	56.22	12.41	29	74

The coefficient on the short-term rate hardly changes. But in place of the constant term

$$a_0 = m^{-1}\int_0^m \alpha(\tau)d\tau + \left(1 - \frac{1 - e^{-\theta m}}{\theta m}\right)\rho_s^* = m^{-1}\int_0^m \alpha(\tau)d\tau - a_1\,\rho_s^* = 0.00284$$

we have

$$m^{-1}\int_0^m \alpha^*(\tau)d\tau + \left(1 - \frac{1 - e^{-\theta m}}{\theta m}\right)\rho_s^* + b_1\text{UNRATE(t)} + b_2\text{AVGMAT(t)}$$

$$= m^{-1}\int_0^m \alpha^*(\tau)d\tau - a_1\,\rho_s^* + b_1\text{UNRATE(t)} + b_2\text{AVGMAT(t)}$$

$$= .00274 + .000375\ \text{UNRATE} + .0000228\ \text{AVGMAT}$$

which fluctuates considerably around its mean, as Figure 11 shows. Evidently we cannot sort out

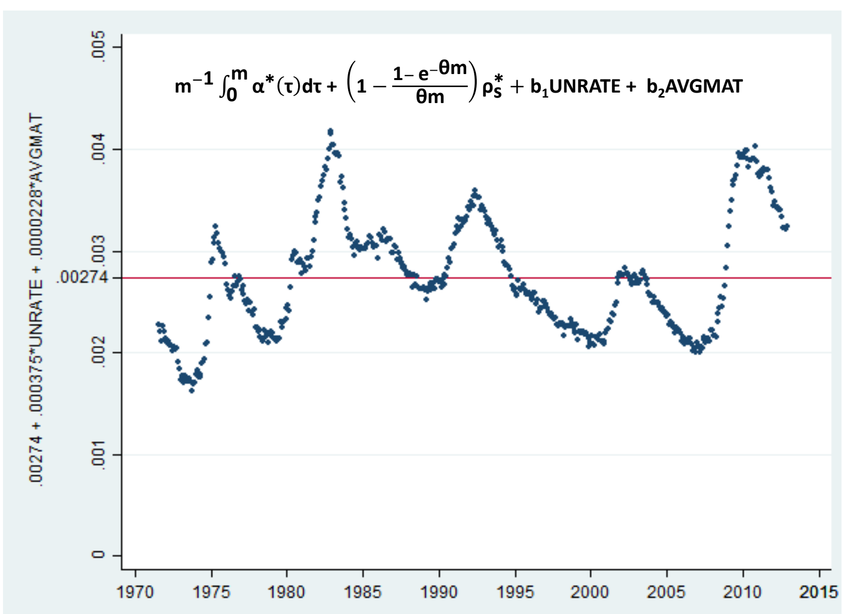


Figure 11

the constant element of the illiquidity premium $\int_0^m \alpha^*(\tau)d\tau$ from the normal-rate term $-a_1\rho_s^*$ (the constant 0.0274 is the sum of the two terms), but the fluctuations over time of the expression

$$m^{-1}\int_0^m \alpha^*(\tau)d\tau + \left(1 - \frac{1-e^{-\theta m}}{\theta m}\right)\rho_s^* + b_1\text{UNRATE(t)} + b_2\text{AVGMAT(t)}$$

$$= .00274 + .000375 \text{ UNRATE} + .0000228 \text{ AVGMAT}$$

involve only the illiquidity premium if we assume that the normal rate is unchanging.[17] Observe that AVGMAT acts overall as a counterweight to UNRATE; as Figure 12 shows,

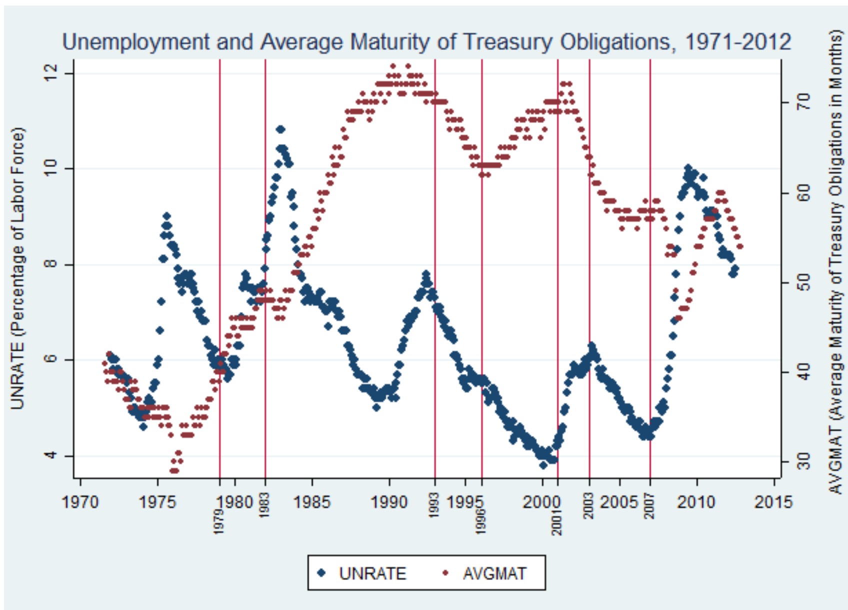


Figure 12

average maturity peaks about the same time that the unemployment reaches its floor. But over certain intervals of time, UNRATE and AVGMAT reinforce each other in terms of their effects on the yield premium. This is the case in 1979-1983, 1993-1996 and 2003-2007. These are also, it turns out, periods in which the changes in the illiquidity premium reinforce the effects of changes in the short rate. In 2003-2007, for example, changes in UNRATE and AVGMAT together account for almost ¼ of the total fall in the yield premium and these changes partly explain the mysterious failure of long yields to respond to the dramatic increase in short rates over this period. (The other part of the mystery is explained by expectations that short rates were, in Hicks's phrase, "abnormally high" and were therefore expected to fall—as indeed they did.) Figure 13, which graphs the short rate, the yield premium, and the "illiquidity

[17] To avoid repeating the cumbersome title of Figure 11, I shall refer to this sum of the illiquidity premium and a constant term as the "illiquidity premium" inside quotation marks.

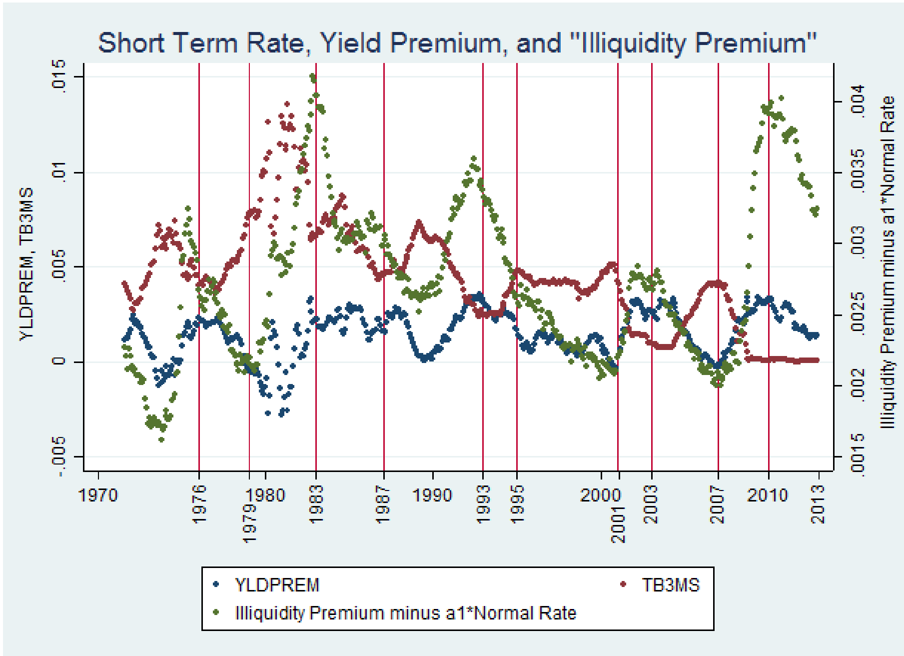


Figure 13

premium" over time, shows this.

But there are other periods, 1983-1987 and 1995-2001, in which the changes in the illiquidity premium have opposite effects and swamp the impact of the short rate, so that the short rate and the yield premium move in the same direction. Since 2010 the yield premium has fallen even as the short rate has hovered near the zero lower bound; the illiquidity premium has fallen as the economy has improved.

Bill Clinton's tenure as President of the United States illustrates both the tendency for changes in the illiquidity premium to reinforce and to counteract the effect of the short rate on the yield premium. Over the eight years of the Clinton Administration, the yield premium fell by more than 0.003 on a monthly basis, almost .04 on an annual basis. Unemployment—proxying for bond-price variability—appears to have been driving the fall in the illiquidity premium of approximately .0015, or 0.018 on an annual basis. But half of the reduction in both the illiquidity premium *and* the yield premium took place in the first two years of Clinton's tenure, when the reduction in average maturity reinforced the fall in the unemployment rate, rather than, as in the period 1995-2001, when the two variables moved in opposite directions.

Table 2 summarizes the relevant data.

Table 2. Short Rate, Yield Premium, Unemployment and Average Maturity, Selected Months								
	Levels				Changes			
Date	TB3MS	YLDPREM	UNRATE	AVGMAT	ΔTB3MS	ΔYLDPREM	ΔUNRATE	ΔAVGMAT
Jan-76	0.00406	0.00228	7.9	29				
Jan-79	0.00779	-0.00051	5.9	39	0.00373	-0.00279	-2	10
Jan-83	0.00655	0.00226	10.4	48	-0.00124	0.00277	4.5	9
Jan-87	0.00453	0.00164	6.6	64	-0.00203	-0.00063	-3.8	16
Jan-93	0.00250	0.00334	7.3	70	-0.00203	0.00171	0.7	6
Jan-95	0.00476	0.00171	5.6	65	0.00226	-0.00163	-1.7	-5
Jan-96	0.00417	0.00064	5.6	62	-0.00059	-0.00108	0	-3
Jan-01	0.00429	0.00019	4.2	69	0.00013	-0.00045	-1.4	7
Jan-03	0.00098	0.00274	5.8	64	-0.00332	0.00255	1.6	-5
Jan-07	0.00415	-0.00016	4.6	58	0.00318	-0.00290	-1.2	-6
Jan-10	0.00005	0.00334	9.7	54	-0.00410	0.00350	5.1	-4
Dec-12	0.00006	0.00140	7.9	54	0.00001	-0.00194	-1.8	0

How Does Inflation Bear on Interest-Rate Forecasts?

The results reported in this appendix depend not only on a theory of how the illiquidity premium is determined but also on a theory of how short-term rates are forecast, particularly how inflation is incorporated into projections of the future course of interest rates. The data permit at least a limited test of my assumption about inflation, namely, that inflation works its way into long rates by a progressive ramping up of the short-term rate according to the formula

$$\hat{\rho}_s(\tau) = (1 - e^{-\theta(t-\tau)})\rho_s^* + e^{-\theta(t-\tau)}\rho_s(t)$$

An alternative is that inflation-induced changes in the short rate are immediately incorporated into the nominal normal rate, which is to say that these changes are treated as permanent. (In this alternative a 2 percent increase in the current rate of inflation translates into the expectation that inflation will be 2 percent higher indefinitely.) The propagation of inflation in expected short-term rates is different under this assumption, with expected rates higher at each point in time by the amount of today's inflation. Denoting inflation at time t by INF(t) and real rates by the superscript R, we have real normal and current rates given by

$$\rho_s^{R*} = \rho_s^* - \mathrm{INF}(t)$$

$$\hat{\rho}_s^R(\tau) = \hat{\rho}_s(\tau) - \mathrm{INF}(t)$$

In real terms the relationship between the expected short rate, the current rate, and the normal rate is now

$$\hat{\rho}_s^R(\tau) = (1 - e^{-\theta(t-\tau)})\rho_s^{R*} + e^{-\theta(t-\tau)}\rho_s^R(t)$$

and the nominal relationship becomes

$$\hat{\rho}_s(\tau) = (1 - e^{-\theta(t-\tau)})\rho_s^{R*} + e^{-\theta(t-\tau)}\rho_s^R(t) + \mathrm{INF}(t) = (1 - e^{-\theta(t-\tau)})\rho_s^* + e^{-\theta(t-\tau)}\rho_s(t)$$

Since the real yield to maturity on the long bond is given by

$$\rho^R_m(m,t) = \rho_m(m,t) - INF(t)$$

the *real* yield premium and the *real* short rate are now driven by the same equation as in the regression reported earlier

$$\rho^R_m(m,t) - \rho^R_s(t) = \rho_m(m,t) - \rho_s(t) = m^{-1}\int_0^m \alpha^*(\tau)d\tau + b_1(UNRATE(t) + b_2(AVGMAT(t))$$

$$+ \left(1 - \frac{1-e^{-\theta m}}{\theta m}\right)\rho^{R*}_s + \left(\frac{1-e^{-\theta m}}{\theta m} - 1\right)\rho^R_s(t)$$

except that now the nominal interest rates on the right-hand side are replaced by real rates. On the left-hand side, the spread is the same whether expressed in real or nominal terms since inflation now affects the short rate and the long-bond yield equally.

We can rewrite the spread equation as

$$\rho_m(m,t) - \rho_s(t) = m^{-1}\int_0^m \alpha^*(\tau)d\tau + b_1 UNRATE(t) + b_2 AVGMAT(t)$$

$$+ \left(1 - \frac{1-e^{-\theta m}}{\theta m}\right)\rho^{R*}_s + \left(\frac{1-e^{-\theta m}}{\theta m} - 1\right)\rho_s(t) - \left(\frac{1-e^{-\theta m}}{\theta m} - 1\right) INF(t)$$

$$= a_o + a_1\rho_s(t) + a_2 INF(t) + b_1 UNRATE(t) + b_2 AVGMAT(t)$$

This leads to a straightforward test of the two hypotheses about how inflation affects the relationship between the short rate and the yield premium. On the hypothesis that the normal rate is independent of the rate of inflation and that inflation affects the adjustment to normal (H1), the test statistic is the coefficient a_2: the hypothesis is $a_2 = 0$. On the alternative hypothesis, inflation is immediately incorporated into forecasts, so that the expected real short rate is a weighted average of today's real rate and the normal real rate (H2), the test statistic is sum of the coefficients: according to H2, we expect $a_2 + a_1 = 0$. The results of running a two-stage least-squares regression augmented by the addition of the variable INF are

$$\underset{}{TB3MS} = \underset{(.0002)}{.000974} + \underset{(.044)}{.661}\, INF + \underset{(.071)}{.453}\, DUMINF - \underset{(.00006)}{.00176}\, UNRATE - \underset{(.000009)}{.0000137}\, AVGMAT$$

$$R^2 = .58$$

$$YLDPREM = \underset{(.00009)}{.00269} - \underset{(.058)}{.219}\, TB3MS + \underset{(.00002)}{.000374}\, UNRATE + \underset{(.000004)}{.0000187}\, AVGMAT - \underset{(.049)}{.0712}\, INF$$

$$R^2 = .63$$

Robust Standard Errors in Parentheses

Observations: 497 F(4, 492) = 115.34

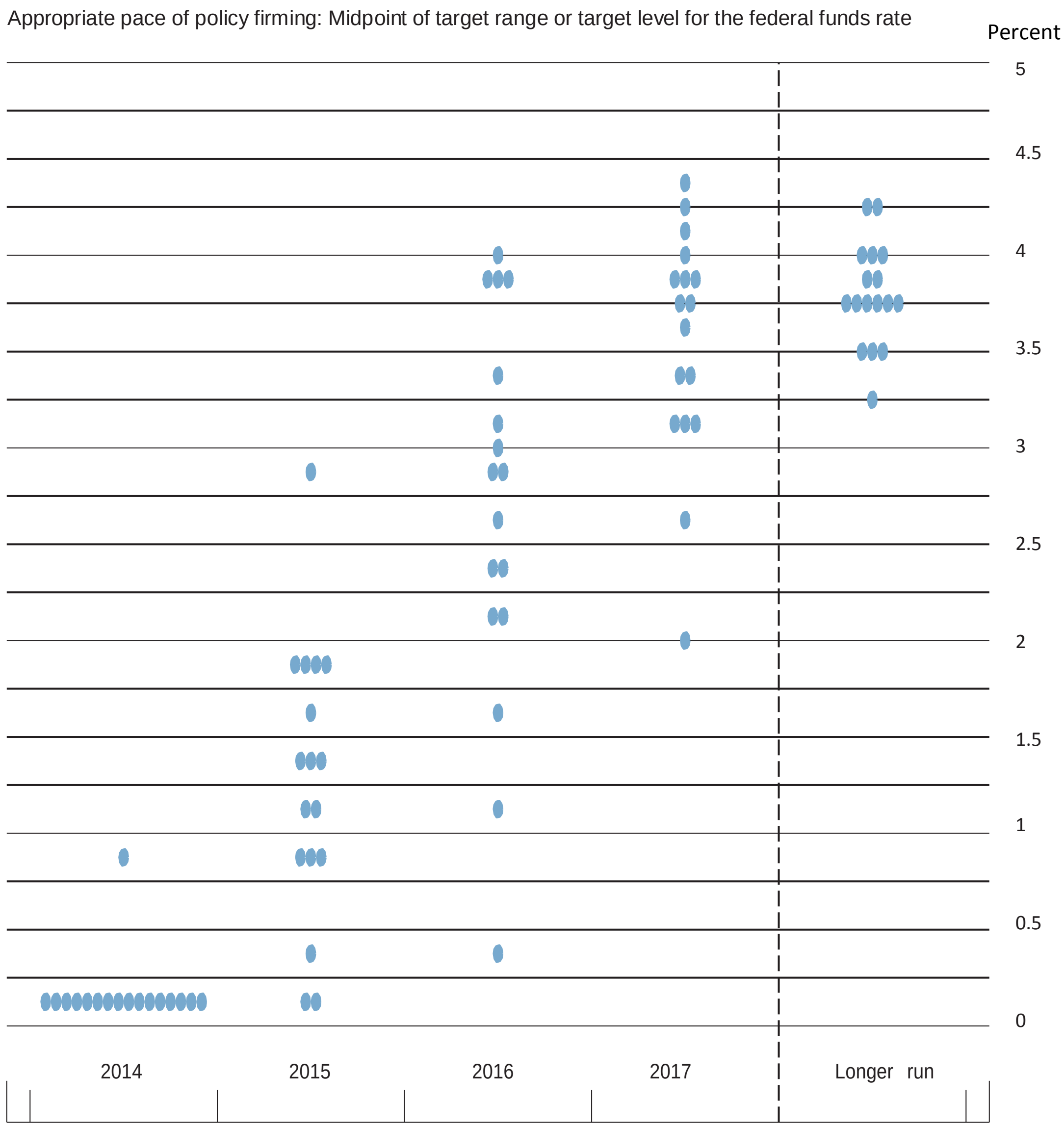
The standard errors of the estimated coefficient are consistent with H1 but not with H2. The data do not reject $a_2 = 0$ since the coefficient is .071 and the standard error is .05. But the data do reject $a_2 + a_1 = 0$ since the sum of these two coefficients is –.29 and the standard error of this sum is .02 (already estimated in the previous regression).

To summarize: the data are consistent with a theory of interest rate spreads in which both normal reversion and risk aversion matter. Additionally, the data support the view that the illiquidity premium is sensitive both to relative supplies of bonds and bills and economic conditions, the second of these two influences being proxied by the unemployment rate. It is important to bear in mind that economic conditions enter the picture not because they affect default risk, as would be expected for private obligations, but because economic conditions correlate with the volatility of bond prices. Finally, of two models of how inflation affects interest-rate forecasts, the data are consistent with a model in which the normal rate is fixed in nominal terms but reject the alternative model in which reversion is determined by real rates.

The data also reinforce the common-sense view of the balance between normal reversion and risk aversion. Figure 9 suggests that when short-term rates are abnormally high or abnormally low, normal reversion matters a lot. But it is the nature of abnormality that it be relatively rare.

Normal Reversion Matters a Lot, But Only Some of the Time

Experience says that most of the time, even if individual agents have strong views, there is not generally a strong consensus. But sometimes—like during the Great Depression, or like right now—agents have strong opinions *and* there is a strong consensus. As I write this (in spring of 2015), there is general agreement that short rates are going up—they have nowhere to go but up—but there is considerable disagreement about how rapidly interest rates will move. Even the members of the Federal Open Market Committee, who are presumably in a good position to know, and who have been uniform in their view that short-term rates will rise over the next few years, diverge widely with respect to the pace of the anticipated change. (They have been polling themselves at regular intervals since 2012.) Figure 14 is an example, emerging from the FOMC meeting in September, 2014, showing both the uniformity of



NOTE: Each shaded circle indicates the value (rounded to the nearest 1/8 percentage point) of an individual participant's judgment of the midpoint of the appropriate target range for the federal funds rate or the appropriate target level for the federal funds rate at the end of the specified calendar year or over the longer run.

Figure 14

views with respect to the *direction* of change and the divergence with respect to the *pace* of change. The so-called "dot plot," as the note to the figure explains, tells us where individual members of the FOMC believe the Federal-Funds rate will be at year's end from 2014 through 2017, as well as in an unspecified "longer run," a period in which the normal rate might be expected to come into its own.[18]

[18] There is also disagreement about what that the normal rate might be. In Europe, with the yield on the German 10 year Treasury bond near zero (April, 2015), there is evidently a widespread belief that the new normal is significantly lower than historical experience would suggest. In mid-2014 the economist-journalist Anatole

Both the central tendency and the variation are interesting.   Starting from the (September) 2014 rate of (near) zero, the median of the forecast for (December) 2015 is an annual rate of 1.375 percent; for (December) 2017 it is 3.75 percent. But while everybody agrees that short-term rates will rise, there is a wide band around the median. The Federal Reserve is committed to keeping interest rates low until labor-market conditions improve, but views on what constitutes improvement and how quickly improvement will take place differ. And it is evident that the Fed is hardly unanimous with regard to its perceptions of the implications of an improving labor market for price stability. Or, for that matter, its perceptions of the relative importance of the two elements of it dual mandate. Given these differences it is hardly surprising that the variability in individual forecasts increases as the time horizon lengthens.

The most interesting thing about the chart is its very existence. I don't have in mind the transparency of the FOMC, though that in itself reflects a historical sea change in the conduct of central banking, but rather that members have views about the course of future interest rates. If you had polled the FOMC in 2004, you well might have got similar answers to the polls in 2014; at that time too short rates had nowhere to go but up. But in 1996 or 2006 I imagine that the FOMC would not have had any view at all, certainly not a firm view, as to where interest rates were going. Lesser mortals perhaps did, but they were paid to have firm views; nobody shells out good money for a forecaster to say "I don't have a clue which way interest rates will move."   It is no wonder that for most of the observations in Figure 9—399 out of 497—the level of the short rate has relatively little predictive power with respect to the yield premium. But that when short rates are abnormally high *or* abnormally low (as they are today)—the other 98 observations—the short rate has considerable weight.

Default Risk

We have taken the hurdle rate to be the yield on investment-grade corporate bonds rather than Treasuries because private-sector loans typically factor in default risk, a consideration that is absent in the analysis of Treasury debt. If we follow the logic of liquidity preference, we now have a separate argument for the illiquidity premium on corporate bonds. Arvind Krishnmurthy and Annette Vissing-Jorgensen have argued that the spread between corporate and Treasury bonds reflects relative supplies

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Kaletsky posted this question on his blog: (http://blogs.reuters.com/anatole-kaletsky/2014/06/06/now-may-not-be-the-time-to-buy-bonds/, accessed 08/19/2014)

> What accounts for the rock-bottom levels not only of the overnight interest rates that central banks set directly, but also the long-term rates that depend on the willingness of pension funds, insurers and private investors to tie up their savings for 10 years or more in government bonds?

In answering his question, Kaletsky offered as one possibility that the normal rate has fallen:

> If investors were absolutely confident that short-term rates set by the central banks would remain near zero for many years ahead, then the seemingly paltry returns—varying from 2.6 percent down to 0.6 percent—on 10-year bonds issued by the U.S., European and Japanese governments would seem generous. Rational investors would be happy to lock up their money for a decade at these rates.

In other words, reversion to normal is alive and well, but the normal rate itself is not what it used to be.

as well as the default risk on corporate bonds ("The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120:233-267, 2012). It is also possible that price variability will affect corporate bonds differently at different points in the cycle, and that the unemployment rate will once again proxy for this effect. These considerations lead to positing the illiquidity premium for corporate bonds as

$$\alpha_C(m, \text{UNRATE}, \text{AVGMAT}, \text{DEFTRAIL}, \text{DELDEF}) =$$

$$\alpha_C^*(m) + d_1\text{UNRATE}(t) + d_2\text{AVGMAT}(t) + d_3\text{DEFTRAIL}(t) + d_4\text{DELDEF}(t)$$

where DEFTRAIL is the trailing 12-month default rate for all US corporate bond issues (calculated by Moody's Analytics), and DELDEF is the first difference of this series, that is the year-on-year change in the default rate.[19] The coefficients d_1 and d_2 on UNRATE and AVGMAT are conceptually the same but numerically different from the corresponding coefficients b_1 and b_2 in the linear decomposition of the illiquidity premium for Treasuries.

The integral equation for the corporate-bond yield premium is

$$\rho_{mCORP}(m,t) = m^{-1}\left(\int \hat{\rho}_s(t+m)\, dm + \int \alpha_C^*(m)\, dm\right) + d_1\text{UNRATE}(t) + d_2\text{AVGMAT}(t) + d_3\text{DEFTRAIL}(t) + d_4\text{DELDEF}(t) + m^{-1} c$$

where ρ_{mCORP} is the yield to maturity on corporate bonds that mature in m years. Once again we invoke limiting values of the left- and right-hand sides to fix c, and once again we have $c = 0$. Alas, direct estimation of the coefficients is problematic because of the lack of a series of zero-coupon bonds comparable to the series that McCullogh originally developed for Treasuries, a series which Gurkaynak, Sack, and Wright have re-estimated and updated on a continuous basis. In principle we could estimate the relationship between the yield to maturity of a coupon bond on the one hand and the short rate and illiquidity premium on the other. Or we could construct a zero-coupon corporate bond following the procedures that have been applied in the case of Treasuries.

Given that both of these procedures would require considerable resources, resources that I do not have at my disposal, I have chosen instead to estimate the coefficients of interest by subtracting the equation for Treasuries

$$\rho_m(m,t) = m^{-1}\left(\int \hat{\rho}_s(t+m)\, dm + \int \alpha^*(m)\, dm\right) + b_1\text{UNRATE}(t) + b_2\text{AVGMAT}(t) + m^{-1} c$$

from the equation for corporates to obtain

$$\rho_{mCORP}(m,t) - \rho_m(m,t) = m^{-1}\int \alpha_C^*(m)\, dm + d_1\text{UNRATE}(t) + d_2\text{AVGMAT}(t) + d_3\text{DEFTRAIL}(t) + d_4\text{DELDEF}(t) -$$

$$m^{-1}\int \alpha^*(m)\, dm - b_1\text{UNRATE}(t) - b_2\text{AVGMAT}(t)$$

or

$$\rho_{mCORP}(m,t) - \rho_m(m,t) = m^{-1}\int \left(\alpha_C^*(m) - \alpha^*(m)\right) dm + (d_1 - b_1)\,\text{UNRATE}(t) + (d_2 - b_2)\,\text{AVGMAT}(t) + d_3\text{DEFRISK}(t) + d_4\text{DELDEF}(t)$$

[19] These data were made available by Moody's Analytics by special agreement, and the results using these data are reproduced here by permission (Moody's Agreement No. 00043372.0).

For this equation to generate unbiased estimates of the relevant parameters, it is necessary to assume that the difference between yields to maturity on zero-coupon and coupon bonds are the same, up to a random error, for Treasuries as for corporates. Although there is no way to test this hypothesis short of constructing a time-series of zero-coupon corporate bonds and estimating the yields, it seems plausible.

The regression-data set now includes the yields on 10-year corporate bonds from the Treasury "High Quality Market" Corporate Bond Yield Curve (http://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/Pages/Corp-Yield-Bond-Curve-Papers.aspx accessed 09/07/2014). The regression results are

$$\rho_{mCORP}(m,t) - \rho_m(m,t) = \underset{(.000045)}{.000847} + \underset{(.000036)}{.0000131}\,UNRATE(t) - \underset{(.000011)}{.0000364}\,AVGMAT(t) + \underset{(.0037)}{.0112}\,DEFTRAIL(t) + \underset{(.020)}{.0672}\,DELDEF(t)$$

$$R^2 = .46$$

Newey-West Standard Errors in Parentheses

Variable	Obs	Mean	Std. Dev.	Min	Max
$\rho_{mCORP}(m,t) - \rho_m(m,t)$	348	0.000847	0.000517	0.000195	0.003652
UNRATE	348	6.148	1.507	3.8	10
AVGMAT	348	63.138	6.789	46	74
DEFTRAIL	348	0.0222	0.0158	0.0052	0.0843
DELDEF	348	0.0000267	0.00224	-0.0102	0.0112

The means are the means of the raw numbers; in the regressions the independent variables are measured as deviations from their respective means so that the constant term is equal to the mean of the yield premium on corporate bonds relative to Treasuries of the same maturity.

Three things stand out. First, the coefficient of UNRATE, which is an estimate of the difference $d_1 - b_1$ is two orders of magnitude lower than the coefficient of UNRATE in the Treasury yield-premium regressions, and it is statistically insignificant. This suggests that if this variable is indeed a proxy for price variability, corporate-bond price variability should exhibit the same correlation with unemployment that shows up for Treasuries in Figure 10. But the data show no such correlation, as Figure 15 shows. Except for the two outliers in the 6 to 8 percent unemployment interval, the pattern of

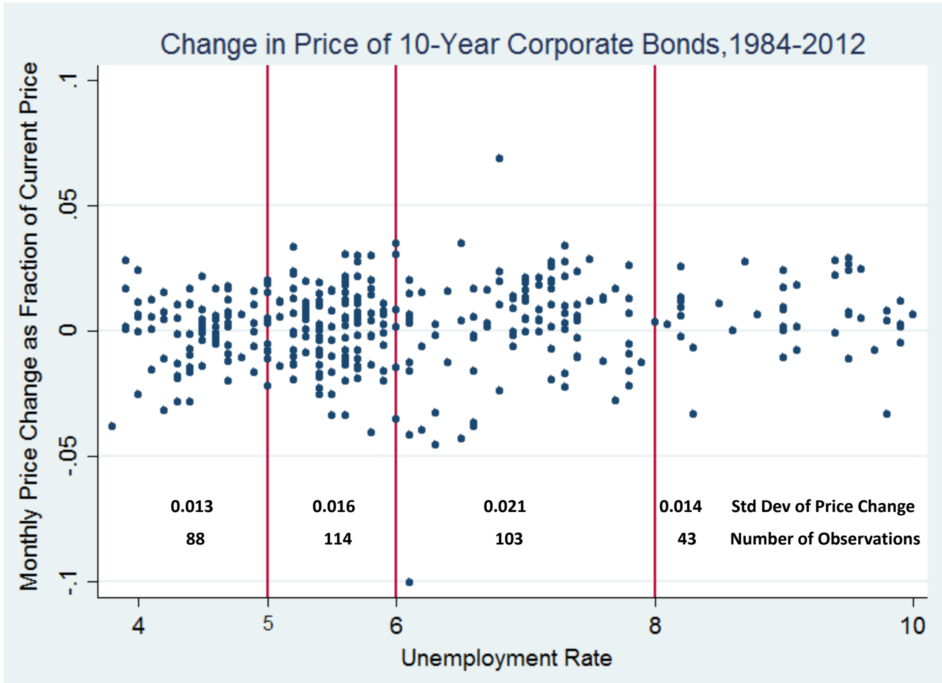

Figure 15

corporate-bond price changes does not show any sensitivity to the rate of unemployment. It may be that this is due to the truncated sample of 10-year corporate-bond yields. The 497 observations in the Treasury regressions include the high unemployment years of the 1970s and 1980s. In contrast, the data for 10-year corporates begins in 1984.

Second, the coefficient on AVGMAT is negative, indicating, unsurprisingly, that the impact of the relative supplies of Treasury bills and bonds has more impact on the prices and yields of Treasuries than on the prices and yields of corporate bonds. The difference $\hat{d}_2 - \hat{b}_2 = -0.0000364$, coupled with the estimate $\hat{b}_2 = 0.000187$, implies $\hat{d}_2 = 0.00015$.

Finally, observe that DEFTRAIL and DELDEF are both statistically significant. Each variable, the average of the preceding year and the year-on-year change in the default rate, captures a piece of the perceived default risk.

What Do the Numbers Mean?

The regressions in this appendix provide a quantitative estimate of the contribution of various factors—the short rate, unemployment, average maturity, default risk—to the liquidity premium attaching to short-term Treasury bills relative to longer Treasury obligations and to corporate bonds. Of particular interest, given the focus of Keynes's *General Theory,* are the consequence for the liquidity trap, that is, the spread between the short rate and the corporate-bond yield when the when the short rate approaches the zero lower bound.

In Table 3 we compare predicted and actual spreads when the independent variables are at their sample

Table 3. Estimated Spreads with Varying Unemployment and Default Risk

	Sample Means	October, 2007	October, 2008	January, 2009	October, 2009	Cumlative: October, 2007, to October, 2009
TB3MS = $\rho_s(t)$ (Percent, Annual)	5.26	3.9	0.67	0.13	0.07	
UNRATE (Percent)	6.43	4.70	6.50	7.80	10.00	
AVGMAT (Months)	56	58	46	47	51	
DEFRISK (Index)	0.017	0.006	0.022	0.035	0.084	
DELDEFRISK (Index)	0.000037	-0.000937	0.0014	0.0044	0.0017	
	Actual (Percent, Annual)					
YLDPREM = $\rho_m(m,t) - \rho_s(t)$	1.76	0.75	3.77	3.06	3.71	
$\rho_{mCORP}(m,t) - \rho_m(m,t)$	2.10	1.85	4.62	3.59	2.66	
$\rho_{mCORP}(m,t) - \rho_s(t)$	3.86	2.60	8.40	6.65	6.36	
	Estimates (Percent, Annual)					
YLDPREM = $\rho_m(m,t) - \rho_s(t)$	1.76	1.43	2.85	3.62	4.73	
$\rho_{mCORP}(m,t) - \rho_m(m,t)$	1.02	0.69	1.64	2.03	2.34	
$\rho_{mCORP}(m,t) - \rho_s(t)$	2.77	2.12	4.49	5.65	7.07	
	Decomposition of Estimated Changes (Percent, Annual)					
$\Delta[\rho_m(m,t) - \rho_s(t)]$			1.42	0.77	1.12	3.31
Contribution of ΔTB3MS			0.94	0.16	0.02	1.11
Contribution of ΔUNRATE			0.810	0.585	0.989	2.384
Contribution of Δ AVGMAT			-0.328	0.027	0.109	-0.192
$\Delta[\rho_{mCORP}(m,t) - \rho_m(m,t)]$			0.95	0.39	0.31	1.65
Contribution of ΔUNRATE			0.03	0.02	0.03	0.08
Contribution of ΔAVGMAT			0.52	-0.04	-0.17	0.31
Contribution of ΔDEFRISK			0.22	0.17	0.67	1.05
Contribution of ΔDELDEFRISK			0.19	0.24	-0.22	0.21

mean with spreads before the financial crisis unleashed by the fall of Lehman Brothers in September of 2008 with three points in time after the crisis began to unfold. The short-term rate falls from almost 4 percent per year in October, 2007, to 0.67 percent in October, 2008, and then to almost 0, where it has remained until the Federal Reserve began to raise rates in December, 2015. Nonetheless, both the spread between the 10-year Treasury and the 3-month bill and the spread between the 10-year corporates and 10-year Treasuries widen in this period. Unemployment and the related price volatility is the main driver of the spread between Treasury bills and bonds whereas default risk is the main driver of the corporate-Treasury spread. The decrease in the average maturity of Treasury obligations softens the blow but not by much.

It is noteworthy that the residual is very large at the beginning of the crisis. Both the actual default rates over the 12 months leading up to the crisis and the year-on-year change in October, 2008, appear to underestimate the perceived risk of default: the estimate of the yield premium on 10-year corporate bonds relative to Treasuries of the same maturity is barely 1/3 of the actual premium, 1.64 percent annually vs 4.62 percent. The model does better by January, but the estimate is remains hardly half of the actual.

The estimates of the premium of 10-year corporates over 3-month Treasuries do better than the estimates of the corporate premium relative to the 10-year Treasury, especially if we leave out the chaotic month of October, 2008. The overall change in the yield premium on 10-year corporates relative to 3-month Treasuries over the two years between October, 2007, and October, 2009, was close to 400 basis points while the model predicted an increase of 500 basis points. Exactly 2/3 of the estimated change is accounted for by the change in the yield premium on Treasury bonds over Treasury bills, the remainder by the change in the premium of long Treasuries over long corporates.

It is surprising that more than half of the total change in the yield premium on corporate bonds relative to the 3-month Treasury bill rate is accounted for by the change in the unemployment rate, acting through expected price volatility rather than as a proxy for default risk. It is also surprising that the change in the short-term rate itself did not have more of an impact, especially since the abrupt change from boom to bust would appear to be just the right circumstance for agents to believe in the abnormality of interest-rate levels, and therefore just the right time to believe in normal reversion.