Forced Rossby waves

Start with the barotropic vorticity equation:

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\nabla^2\psi+\beta\frac{\partial\psi}{\partial x}=-\frac{f_0}{H}U\frac{\partial h_T}{\partial x} \tag{1.1}$$

where U is the mean zonal wind, and the mean meridional wind is assumed to be zero. H is the depth of the fluid, f0 is the Coriolis parameter at the latitude of interest and β=df/dy. ψ is the streamfunction so that $v=\partial\psi/\partial x$ and vorticity $\zeta=\nabla^2\psi$. h_T is the height of topography. We should assume h_T is uniform in y (the meridional direction).

Since this is a linear equation with constant coefficients, the response ψ has the same frequency and wavenumbers as the forcing h_T. Decompose h_T into the different Fourier components and we can treat them separate. Denote the component of h_T with zonal wavenumber k and frequency ω as

$$\hat{h}_0\exp\left[i(kx-\omega t)\right]$$

and response as

$$\hat{\psi}\exp\left[i(kx-\omega t)\right]$$

then equation (1.1) for this Fourier component becomes:

$$\left[i\left(\omega-Uk\right)k^2+ik\beta\right]\hat{\psi}=-\frac{f_0U\hat{h}_0}{H}ik$$

or:

$$\hat{\psi}=-\frac{f_0U\hat{h}_0}{Hk^2\left[\frac{\omega}{k}-\left(U-\frac{\beta}{k^2}\right)\right]}$$

which is the solution. For this, we can see whenever the phase speed of the forcing, which is given by ω/k, matches that of a free Rossby wave mode with a zonal wavenumber k, we have resonance. For orographic forcing, the frequency is zero, and resonance happens when $U=\beta/k^2$.