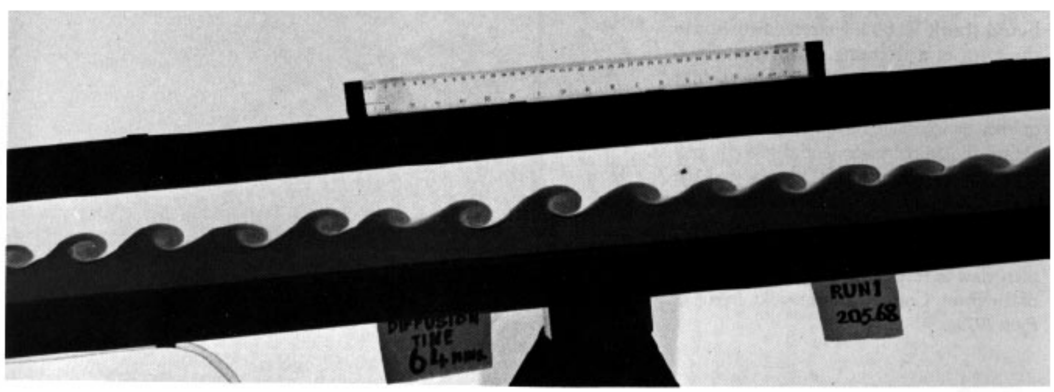
Turbulence and planetary boundary layer

Except very close to the surface (~1mm) where molecular diffusivity makes the flow laminar, the atmosphere is almost always turbulent near the surface. First consider the case where the atmospheric stratification is neutral. In the atmosphere, there is always some motion but at the surface the velocity goes to zero. The horizontal velocity being zero at the surface is called the no-slip condition, and arises from the fact that the surface is not perfectly smooth. Therefore horizontal winds are different at different heights (i.e. there is vertical wind shear). This can give rise to shear instability. One criterion for the onset of the instability is the Reynolds number: $\mathrm{Re} = \frac{UL}{\nu}$. The kinematic viscosity for air is ~$10^{-5}m^2/s$, so with a 1m/s wind at 1 meter height, we have a Reynolds number of 1.e5, exceeding the threshold for onset of shear turbulence, which is typically a few thousands. When the fluid is stably stratified, one could still get the Kelvin-Helmholtz instability when the shear is sufficiently strong, a measure of which is the Richardson number $Ri=N^2/(dU/dz)^2$. The flow is stable when $Ri > ¼$.

<u>Kelvin-Helmholtz Instability</u>



145. Kelvin-Helmholtz instability of stratified shear flow. A long rectangular tube, initially horizontal, is filled with water above colored brine. The fluids are allowed to diffuse for about an hour, and the tube then quickly tilted six degrees, setting the fluids into motion. The brine accelerates uniformly down the slope, while the water above similarly accelerates up the slope. Sinusoidal instability of the interface occurs after a few seconds, and has here grown nonlinearly into regular spiral rolls. *Thorpe 1971*

van Dyke, p. 85

[Photograph courtesy of Brooks Martner, NOAA.]

Therefore, we expect close to the surface, air will be turbulent, and the fluxes will be turbulent fluxes. In this case, how do we estimate the surface fluxes of momentum and heat? One way is to use eddy covariance measurements, which however are expensive. Some knowledge of turbulence can help us derive the bulk aerodynamic formula that we talked briefly before. In discussing turbulence, it's often useful to employ the Reynolds decomposition.

Reynolds Decomposition

We are not interested in how exactly the turbulent flow varies from one second to another. Instead, we are interested in the collective effect of this turbulent flow on the relatively slow varying aspects of the flow. It's often useful to decompose a field into a slowly varying mean component and a fast varying component:

$u = \overline{u} + u'$

so that one can write equations for the slowly varying component. As an example, let us consider incompressible flow, and look at the horizontal momentum equation in 2D

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + F_x .$$

For incompressible flow, we can write this as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + u\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + F_x$$

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uw}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + F_x$$

This is the flux form. Perform the decomposition, and take the average to get the equation for the slowly varying mean component:

$$\frac{\partial \bar{u}}{\partial t}+\frac{\partial \bar{u}\bar{u}}{\partial x}+\frac{\partial \bar{u}\bar{w}}{\partial z}=-\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x}+\bar{F}_x-\frac{\partial \overline{u'u'}}{\partial x}-\frac{\partial \overline{u'w'}}{\partial z}$$

The last two terms on the right hand side are due to turbulent eddies and are much larger than the effect of molecular viscosity F at the scales that we are interested in. We have not added any physics here. All that we have done are things like this:

$$\overline{uw}=\overline{(\bar{u}+u')(\bar{w}+w')}=\overline{(\bar{u}\bar{w}+\bar{u}w'+\bar{w}u'+u'w')}$$
$$=\bar{u}\bar{w}+\overline{u'w'}$$

where we have defined decomposition so that, u'_bar and w'_bar=0.

If the surface momentum flux is $\left(-\overline{u'w'}\right)_0=u_*^2$ and surface buoyancy flux is $B_0=\left(\overline{w'b'}\right)_0$, where subscript 0 denotes the surface, and u* is called the frictional velocity. One can define the Obukov length: $L=-u_*^3/kB_0$, where k is the von Karman constant and has a value of ~0.4. L varies from say a few hundred meters during the nighttime to negative values (a few meters) during the day. The Obukov length can be viewed as the height above the surface where the buoyancy effect becomes comparable to the effect of the shear. Very close to the surface (z<< |L|), the buoyancy effect becomes small.

The near surface downward momentum flux is $-\overline{u'w'}$ (multiply this by the density, one gets the surface drag). If the slowly varying component can be viewed as steady and horizontally homogeneous, then this momentum flux should be a constant with height (vertical advection by the slowly varying mean flow is in general small). Therefore, the velocity scale defined earlier $u_*^2=-\overline{u'w'}$ is constant with height, and the vertical shear at height z, based on dimensional reasoning should be

$$\frac{du}{dz}=\frac{u_*}{kz}$$

where k is again the von Karman constant. The reasoning here is that the flow is turbulent so that the molecular viscosity doesn't matter. The stratification is neutral, so gravity doesn't matter. The only things that matter appear to be the velocity scale of the eddy and the distance from the surface, and the only way to combine them to get du/dz is by u*/z. One way to look at this is in terms of the mixing length theory, where the eddy diffusion is proportional to the eddy velocity scale times the eddy length scale: $K_m \propto u_* z$ so that

$$\left(\overline{u'w'}\right)_0=-K_m\frac{d\bar{u}}{dz}$$

$$u_*^2=ku_*z\frac{d\bar{u}}{dz}$$

Integrating, we get the logarithmic velocity profile law:

$$\bar{u}(z)=u_*k^{-1}\ln(z/z_0) \qquad (1)$$

The integration constant z0 is called the roughness length and defines the surface where u_bar=0 for a non-smooth surface, and is loosely related to the typical height of closely spaced surface obstacles, often called roughness elements (e.g. water waves, trees, buildings, blades of grass). Below are some typical values for z0.

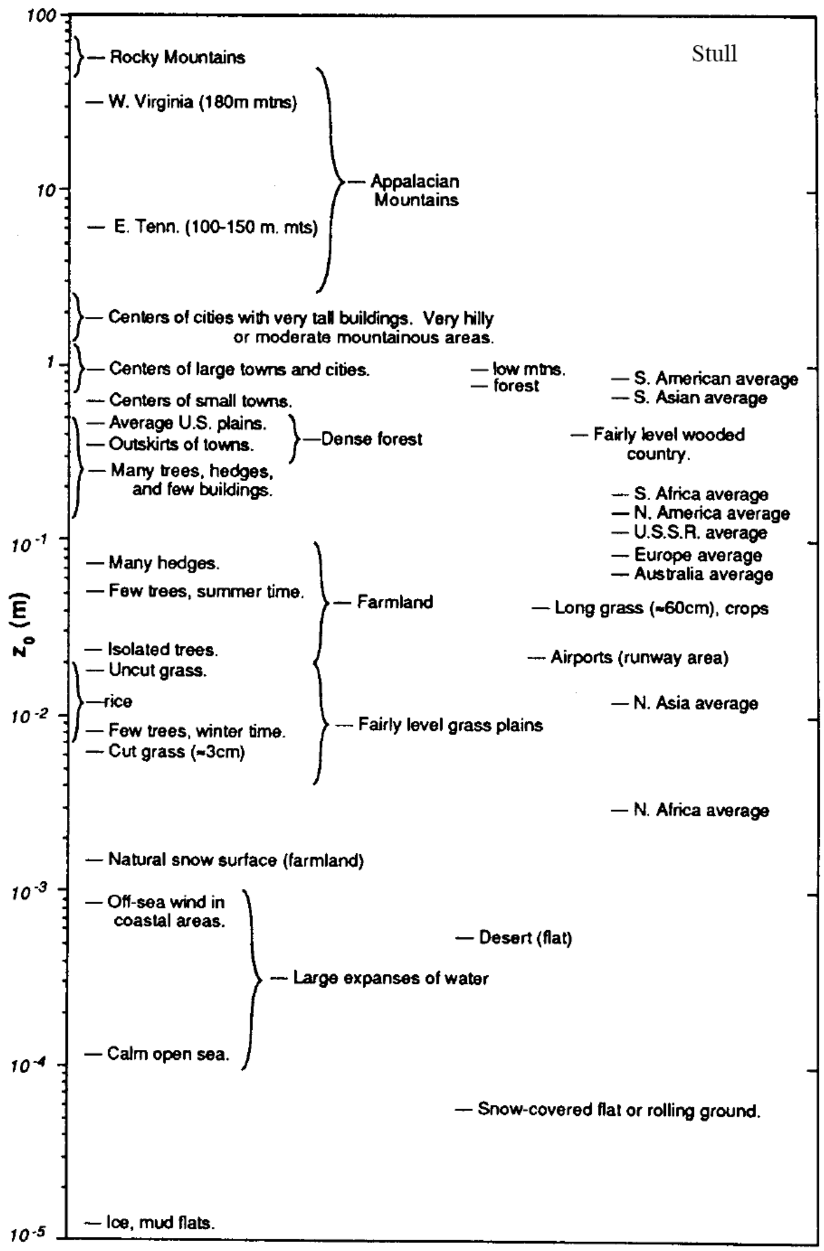


Fig. 9.6 Aerodynamic roughness lengths for typical terrain types. (After Garratt 1977, Smedman-Högström & Högström 1978, Kondo & Yamazawa 1986, Thompson 1978, Napo 1977, and Hicks 1975).

Now if we have measurements of u at some reference height z_R, we can calculate what u_* is from (1), and get the surface drag

$$-\rho_0 \overline{u'w'} = \rho_0 u_*^2 = \rho_0 C_{DN} \overline{u}^2(z_R)$$

$$C_{DN} = k^2 / \left[\ln(z_R / z_0)\right]^2$$

C_{DN} is the drag coefficient under neutral stratification. With this formula, we can compute the drag if we know u at some height above the surface. Note that C_{DN} depends on z_R. So for measurement of u at different heights, different drag coefficients should be used. We can do the same for temperature and moisture, which gives

$$\rho_0 \overline{w'a'} = \rho_0 C_{aN} \overline{u}(z_R)\left[a(0) - a(z_R)\right]$$
$$C_{aN} = k^2 / [\ln(z_R / z_0)\ln(z_R / z_{a0})] ,$$

where a is a scalar and may be specific humidity or potential temperature. These are called the bulk aerodynamic drag formulas. The same dependence on z_R also applies to C_{aN}. Traditionally z_R is 2m for temperature and moisture and 10m for wind. They are on the order of 10^{-3} and may be several times larger over land than over the ocean. The drag coefficients can be different for momentum and for scalars because of the role of pressure gradient force on momentum transport. This difference can be important in understanding the strength of hurricanes.

An example to put things together: Life cycle of the nocturnal inversion

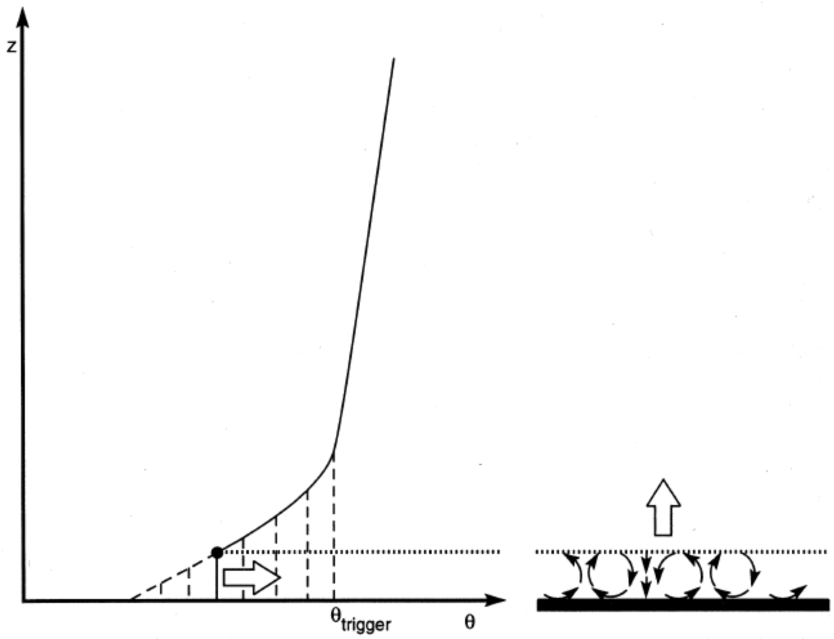


Figure 7.13 Schematic illustrating the breakdown of the nocturnal inversion, which forms during night. Following sunrise, surface heating and heat transfer to the overlying air destabilize a shallow layer adjacent to the ground. Convection then develops spontaneously and, through vertical mixing, restores the thermal structure inside that layer to neutral stability. Continued heat transfer from the ground requires θ to be mixed over a progressively deeper layer, which erodes the nocturnal inversion from below. When convection has advanced through the entire layer of strong stability, the nocturnal inversion has been destroyed. Deep convection can then penetrate into weakly stable air overhead, to disperse pollutants that were previously trapped near the surface in stable air.

An example from observations:

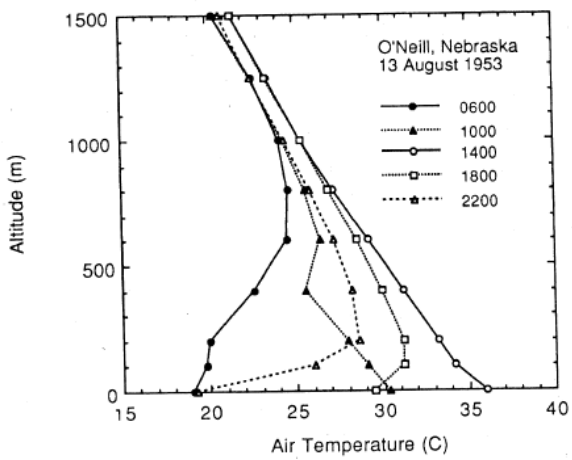


Fig. 4.8 Plot of air temperature at various local times in the lowest 1500 m of the atmosphere at O'Neill, Nebraska on August 13, 1953. Times are given using a 24-hour clock so that 1800 = 6 PM, etc. [Data from Lettau and Davidson (1957).]

The vertical wind profile over a diurnal cycle:

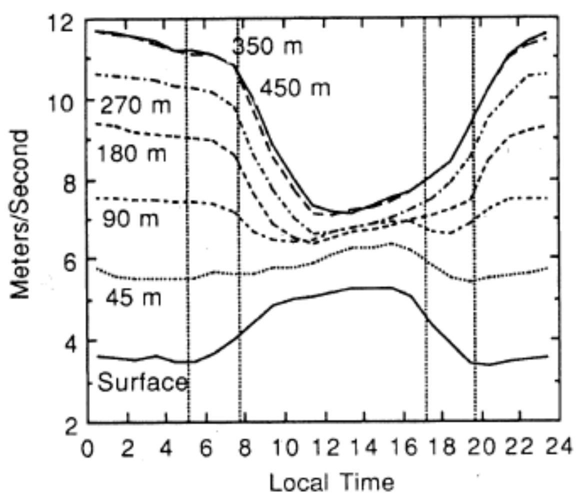


Fig. 4.9 Diurnal cycle of wind speed as a function of height measured from a tower in Oklahoma City and averaged over the period June 1966 to May 1967. [Adapted from Crawford and Hudson (1973). Reprinted with permission from the American Meteorological Society.]

Over the tropical ocean, there are observations when there are variations in the ocean surface temperature over relatively short distances (say a few hundred kilometers), surface winds are stronger when the ocean temperature is higher. This can be explained by the arguments above.

We have now learned how the climate works in a 1D sense. The earth of course is not just 1D. What happens when we introduce horizontal gradients?