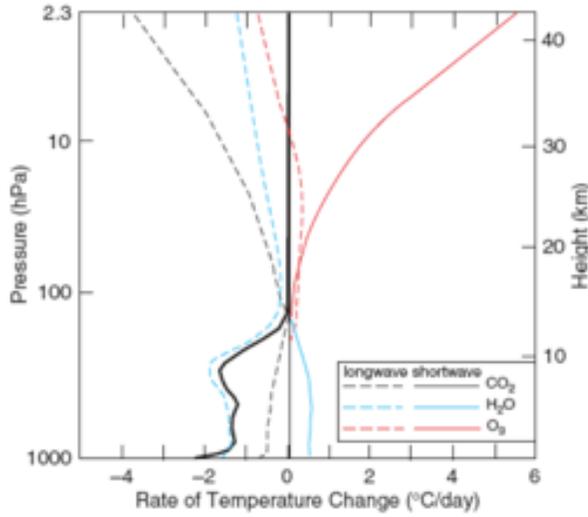


## Surface energy balance

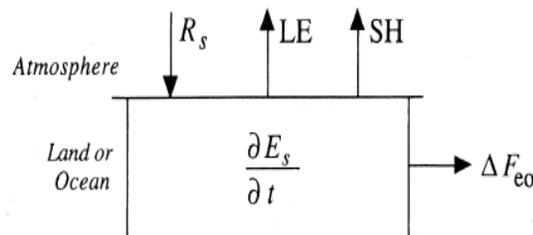
We learned that the radiative equilibrium profile is hydrostatically unstable and convection will adjust the profile to that of radiative-convective equilibrium. Now what about the surface? The radiative-convective equilibrium profile is warmer in the upper troposphere than the radiative equilibrium profile so energy can escape more efficiently. Indeed, radiative transfer calculation shows that the net radiative effect in the troposphere is a cooling of  $\sim 1.5\text{K/day}$  (you can do this with the radiative transfer code from the homework). In equilibrium, this cooling is balanced by convective heating.



[Adapted from S. Manabe and R. F. Strickler, *J. Atmos. Sci.*, 21, p. 373 (1964).]

The energy comes from the surface, and must be included in considering the surface energy balance, which now includes both radiative heating (SW+LW) and surface heat fluxes (latent heat flux and sensible heat flux) and transport within the ocean or land.

$$C_{eff} \frac{\partial T_{eff}}{\partial t} = R - LE - SH - \Delta F$$



**Fig. 4.1** Diagram showing the relationship of the various terms in the surface energy balance ( $R_s$  = net radiation, LE = evaporative cooling, SH = sensible cooling,  $\partial E_s / \partial t$  = heat storage below the surface,  $\Delta F_{eo}$  = divergence of horizontal energy flux below the surface).

How do we calculate SH and LE? We know it's related to the temperature and moisture difference at air-surface interface. If the atmosphere were quiescent, then the near surface air temperature and moisture would become close to the surface values. However, the atmosphere near the surface (except the bottom mm or so) is in general turbulent, because of the low viscosity of the air. In this case, one could make direct estimates through eddy-covariance measurements but these are expensive. Alternatively, the surface fluxes can be estimated using the bulk aerodynamic formula. Those for momentum and scalars (e.g. potential temperature or moisture) under neutrally stratified conditions are:

$$-\overline{u'w'} = C_{DN}|U|u(z_R)$$

$$\overline{a'w'} = C_{aN}|U|[a(0) - a(z_R)]$$

We will defer their derivation to a later lecture on planetary boundary layers. With these formula, we can compute the surface fluxes of momentum, SH and LE, if we know  $u$ ,  $T$  (or  $\theta$ ),  $q$  at some height above the surface. Note that  $C_{DN}$ ,  $C_{aN}$  depend on  $z_R$ . So for measurements of  $u$ ,  $T$ ,  $q$  at different heights, different exchange coefficients should be used. Traditionally  $z_R$  is 2m for temperature and moisture and 10m for wind.  $C_{DN}$ ,  $C_{aN}$  are on the order of  $10^{-3}$  and may be several times larger over land than over the ocean. The exchange coefficients can be different for momentum and the scalars. This difference can be important in understanding the strength of hurricanes.

The ratio of SH/LE is called the Bowen ratio, which varies from 0.1 over tropical ocean to 10 over deserts.

**Table 4.5**  
Mean Latitudinal Values of the Components of the Energy Balance Equation for Earth's Surface

Latitude zone	Oceans				Land			Earth			
	$R_s$	LE	SH	$\Delta F_{eo}$	$R_s$	LE	SH	$R_s$	LE	SH	$\Delta F_{eo}$
80-90N	•	•	•	•	•	•	•	-12	4	-13	-3
70-80N	•	•	•	•	•	•	•	1	12	-1	-9
60-70N	31	44	21	-35	27	19	8	28	27	13	-12
50-60N	39	52	21	-35	40	25	15	40	37	19	-16
40-50N	68	70	19	-21	60	32	28	64	50	23	-9
30-40N	110	114	17	-21	80	31	49	97	78	32	-13
20-30N	150	139	12	-1	92	27	65	127	97	32	-1
10-20N	158	131	8	19	94	39	56	141	108	21	12
0-10N	153	106	5	41	96	64	32	139	96	15	29
0-10S	153	112	5	36	96	66	29	139	101	13	25
10-20S	150	138	7	5	97	54	42	138	119	15	4
20-30S	134	133	9	-8	93	37	56	125	110	21	-7
30-40S	109	106	11	-8	82	37	45	106	98	15	-7
40-50S	76	73	12	-9	54	28	27	74	70	13	-9
50-60S	37	41	13	-17	41	27	15	37	41	15	-19
60-70S	•	•	•	•	•	•	•	17	13	15	-11
70-80S	•	•	•	•	•	•	•	-3	4	-5	-1
80-90S	•	•	•	•	•	•	•	-15	0	-15	0
0-90N								96	73	21	1
0-90S								96	82	15	-1
Globe	109	98	11	0	65	33	32	96	78	18	0

Values in  $W m^{-2}$ . [Data from Sellers (1965). Reprinted with permission from the University of Chicago Press.]

Given the surface temperature, degree of saturation, and the surface air temperature and humidity and winds (at  $z_R$ ), we can calculate SH and LE, and from radiative transfer, we know R.  $\Delta F$  is negligible over land and can be calculated for the ocean if we know the circulation. Now, we can calculate how the (effective) temperature of land/ocean will change, if we know the effective heat capacity.

### Heat capacity of land.

Heat transfer through solid is by conduction.

$$C_s \frac{\partial T}{\partial t} = -\frac{\partial}{\partial z} \left( -K_T \frac{\partial T}{\partial z} \right)$$

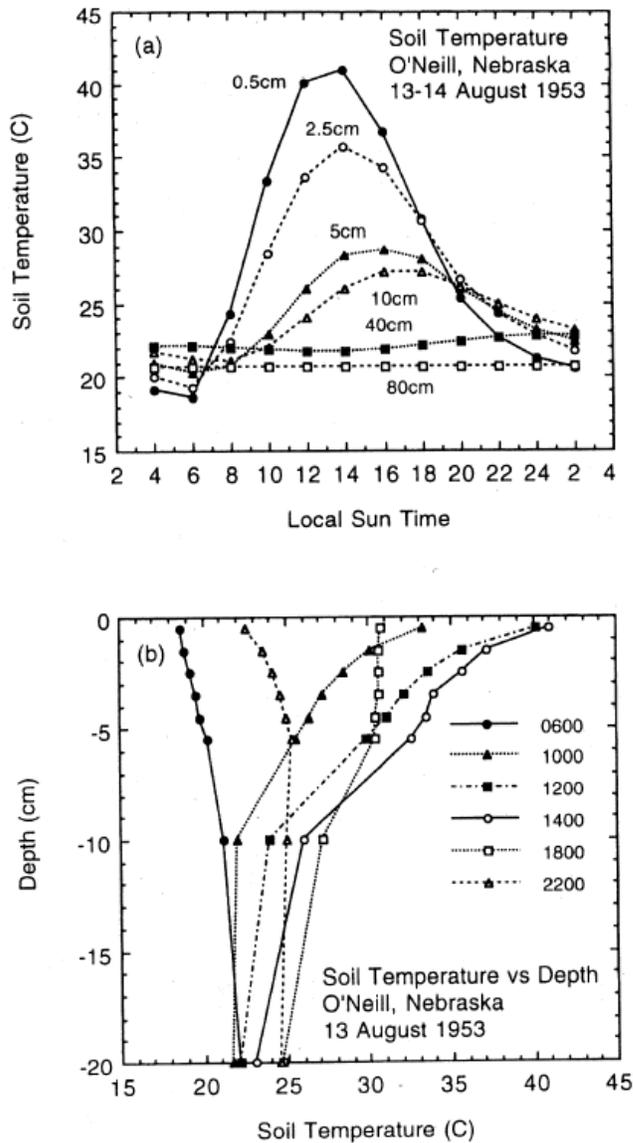
$$\frac{\partial T}{\partial t} = D_T \frac{\partial^2 T}{\partial z^2}$$

$$D_T = K_T / C_s$$

So the depth through which a temperature anomaly applied at the surface will penetrate in a given time is

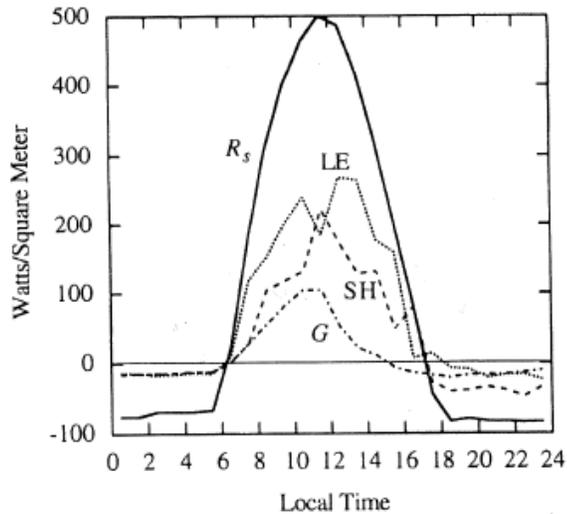
$$h_T = \sqrt{D_T \tau}$$

where  $\tau$  is the time scale of the periodic forcing at the surface. A typical value of soil diffusivity  $D_T$  is  $5 \times 10^{-7} \text{m}^2/\text{s}$ . So for diurnal forcing the penetration depth is  $\sim 10 \text{cm}$  and for annual forcing it is  $\sim 1-2 \text{m}$ , and it can be longer if the forcing is of longer timescale. Indeed, if we know the thermal conductivity and specific heat profiles, measured temperature profiles in a deep layer of Earth can be used to estimate past variations in surface temperature on time scales of hundreds to thousands of years. (Some are suggesting doing this for the moon to get how the solar irradiance has changed in the past!) Below is an example of the soil temperature over a diurnal cycle at various depths:



**Fig. 4.2** Soil temperature at various depths under a grass field at O'Neill, Nebraska on August 13, 1953: (a) temperature at various depths as a function of local time; (b) temperature as a function of depth at various times. Measured thermal diffusivities on the day illustrated range from  $2.5 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$  at 1 cm to  $6 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$ , at 5-cm depth in the soil. [Data from Lettau and Davidson (1957).]

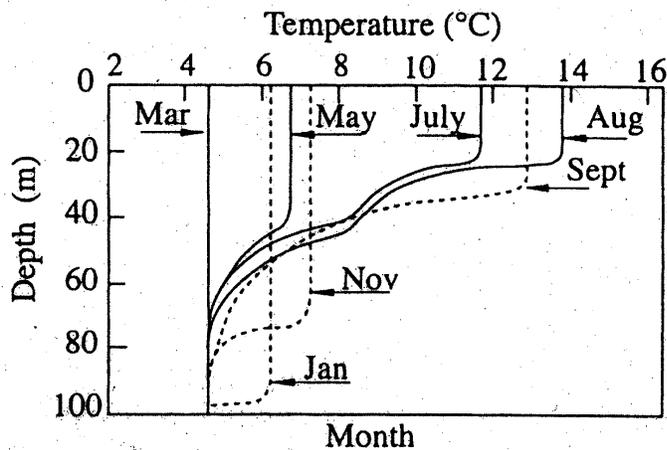
Thus, the effective heat capacity of the soil is that of a 10cm thick soil layer for a diurnal forcing. The number becomes a couple of meters for seasonal forcing.



**Fig. 4.14** Heat budget for a field of mature corn in Madison, Wisconsin, on September 4, 1952. [Data from Tanner (1960). Reprinted with permission from the Soil Science Society of America.]

### Heat capacity of the ocean.

In the ocean, heat is efficiently mixed through fluid motion and the effective heat capacity is much greater. It does depend on the strength of the mixing. In windy, cold days, the mixing is strong and the heat capacity is large. In calm and warm days, mixing may be limited to a shallow layer. But on average, it's tens of meters. Below is a plot of ocean temperature profile over the seasonal cycle in a north pacific location.



Combining the following table with the above discussion, we see that the effective heat capacity over land is much lower than that over the ocean for timescales of say a year. This is why ocean temperature varies much less over the year compare to that over land.

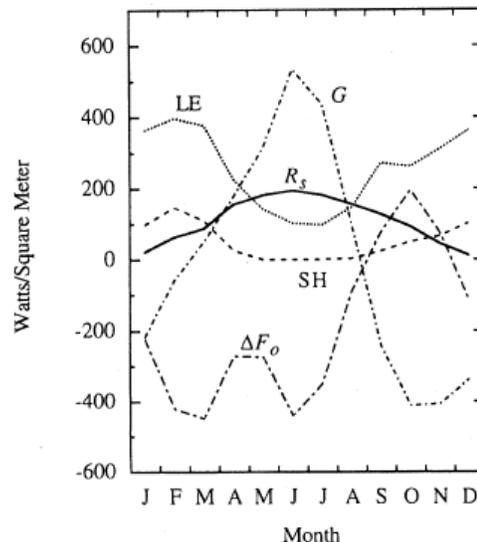
**Table 4.1**  
Properties of Soil Components at 293 K

	Specific heat ( $c_p$ ) ( $\text{J kg}^{-1} \text{K}^{-1}$ )	Density ( $\rho$ ) ( $\text{kg m}^{-3}$ )	$\rho c_p$ ( $\text{J m}^{-3} \text{K}^{-1}$ )
Soil inorganic material	733	2600	$1.9 \times 10^6$
Soil organic material	1921	1300	$2.5 \times 10^6$
Water	4182	1000	$4.2 \times 10^6$
Air	1004	1.2	$1.2 \times 10^3$

[After Brutsaert (1982). Reprinted with permission from Kluwer Academic Publishers.]

It is interesting to compare these numbers with the heat capacity of the atmosphere. The total mass of the atmosphere per unit area is  $p_s/g$ . This is roughly 10 meters of water. The specific heat is 4 times less, so the heat capacity of the atmosphere is roughly that of 2m of water, greater than that of land on sub-annual timescales but smaller than that of the ocean mixed layer.

Because of the large heat capacity of the ocean, large amounts of heat can be stored in the summer and released in the winter. This is part of the reason why winter over western coasts of Europe and America is quite mild.



**Fig. 4.17** Annual cycle of heat budget components for the Gulf Stream at 38°N, 71°W. (Adapted from Sellers, 1965. Reprinted with permission from the University of Chicago Press.)