Lecture Four Exercises!

Throughout, assume that you know the two theorems stated in class:

(1) Two representations are isomorphic iff $\chi_U = \chi_V$, and
(2) There are as many isomorphism classes of irreducible G-representations as there are conjugacy classes of G.

Throughout, take $k = \mathbb{C}$ to be the base field. All representations are assumed finite-dimensional.

1. Cyclic groups

Let C_n be a cyclic group of order n, with n finite. Knowing how many irreducible representations there are of C_n, write down (up to isomorphism) every irreducible representation of C_n.

2. S_3

Write down every irreducible representation of S_3, the permutation group of a set of 3 elements. The last representation might be hard to think of; try studying the representation induced by the action of S_3 on a set of 3 elements.

3. D_{2n}

Let D_{2n} be the plane symmetries of the regular n-gon, which has $2n$ elements (rotations and reflections). Write down every irreducible representation of D_{2n}. Note that the parity of n plays a role.

4. Optional

These are much harder, but definitely fun to try, especially with the poverty of technology at this point in the class. It helps one appreciate the power of the orthonormality we'll see next week.

(1) Classify all irreducible representations of S_4, the symmetric group on 4 letters.
(2) Classify all irreducible representations of A_4, the kernel of the sign representation $S_4 \to C_2$.