Math 223a : Algebraic Number Theory

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Meeting Time: TTh 1-2:30 **Location:** Science Center 222

Course Assistant: TBA **Section Time and Location:** TBA

Course Website: `canvas.harvard.edu/courses/30968`

Course Description

Math 223 is a graduate-level course in algebraic number theory. Math 223a will cover local class field theory and Math 223b global class field theory.

Tentative List of Topics

Brief overview of local and global class field theory. Theory of local fields, extensions and ramification; Galois cohomology; local class field theory; Lubin-Tate theory. (Brauer groups if time permits.)

Prerequisites

There are two main prerequisites for this class.

The first is a previous class on number fields, e.g. Math 129 at Harvard. Chapter I of Neukirch's *Algebraic Number Theory* (see below) is a good source for the relevant prerequisite material. This semester we'll mostly be developing the theory of local fields, so this background won't be strictly necessary much of the time, but will provide important context.

The second is exposure to homology/cohomology in some other context (simplicial/singular, de Rham, etc). We'll be developing the theory of Galois cohomology as an important tool, and I'll expect that you've previously seen chain complexes, the snake lemma, and such.

Textbook and References

The textbook for both semesters of Math 223 is *Algebraic Number Theory*, eds. Cassels and Frohlich.

Additionally, I will post PDF lecture notes on the Canvas website immediately after each class.

There are many good references for the course material: my recommendations are below. (Hyperlinks go to the SpringerLink website, which allows users on the Harvard network to download free PDFs of the books for free. Harvard affiliates can also download the book off-campus using EZProxy.)

Other recommended references:

- *Algebraic Number Theory* by Jürgen Neukirch.
  `https://link.springer.com/book/10.1007/978-3-662-03983-0`
  This is a really good overview of algebraic number theory from the basics through class field theory.

- *Class Field Theory: the Bonn Lectures* by Jürgen Neurkich.
  `https://link.springer.com/book/10.1007/978-3-642-35437-3`
  More focused on the proofs of class field theory than Neukirch's other book. The treatment of Galois cohomology is more in-depth.

- *Local Fields* by Jean-Pierre Serre.
  `https://link.springer.com/book/10.1007/978-1-4757-5673-9`
  The classic text for local fields. Excellent but a little terse and dry.

- *Class Field Theory course notes* by James Milne,
  `http://www.jmilne.org/math/CourseNotes/CFT310.pdf`
  I haven't used these much, but James Milne writes very good and comprehensive course notes.

- *Primes of the form* $x^2 + ny^2$ by David Cox.
  Not so relevant to the material we'll be covering in the fall, but this is one of the best places to go for statements of the basic theorems of global class theory and some of their major applications.

Homework and Grading Policies

If you are taking the class for a grade, you final class grade will be based 80% on weekly homework and 20% on the final paper.

Homeworks will be assigned weekly, and will be due on Tuesdays. You may submit them online before the start of class or bring a hard copy to class. All homeworks will be weighted equally and the lowest two homework grades will be dropped.

The final assessment for this class will be a 5-10 page paper on a topic related to the course material.

You are encouraged to discuss the homework problems with your classmates, but you must write them up independently. You should acknowledge everyone you worked with in your homework writeups, as well as any external sources you consulted.