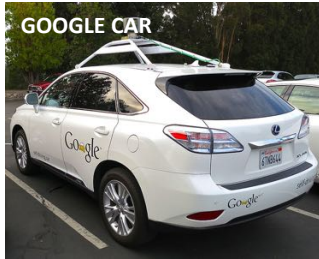




## Agenda

- **Lecture: Robot Navigation -> MAPPING!**
- Demo Time:
  - [LAB4 \(Extended Kalman Filter\\*\)](#)
  - [Then help TFs take robots down to MD B127 \(Pset 5 test arena\)](#)
- Upcoming:
  - **Pset 5: Autonomous Mapper due next week**
  - **Start ASAP! (uses Lab 4, in MD B127)**
- References:
  - This lecture is partially based on "Introduction to AI Robotics", chapter 11, Robin Murphy, 2000,
  - For SLAM, see online theory tutorial paper "SLAM: Part 1 The Essential Algorithms", by Durrant-Whyte et al, 2006 and online practical tutorial paper "SLAM for Dummies" S. Riisgaard, and M. Blas. (2005)

## Today: Robots Navigating the World



### Scenarios

- Hospital Helper (e.g. Diligent, Tugs)
- Office security or mail-delivery (e.g. Cobal, Savioke)
- Tour Guide robot in a museum (Minerva)
- Autonomous Car with GPS and Nav system

### Biological analogies:

Humans, bees and ants, migrating birds, herds

## Today: Robots Navigating the World

### Second Part of CS189: High-level reasoning

From finite state machines to complex representation and memory

➤ Path Planning: How to I get to my Goal?

➤ Localization: Where am I?

➤ Mapping: Where have I been?

➤ Exploration: Where haven't I been?

# Mapping and Exploration

## ➤ Question:

You are roaming around in an unknown space, what can you learn about it?

## ➤ Two parts of the problem:

- **Mapping:** As you roam around the world, how do you build a memory of the shape of the space you have moved through?
- **Exploration:** (coverage of unknown space) Given that you don't know the shape or size of the environment, how do make sure you covered all of it?

## ➤ Both have many uses:

- Searching for objects, Mapping a collapsed mine or building.
- Mowing a golf course or cleaning a room efficiently.

## ➤ Mapping and Exploration are also “collections of algorithms”

- E.g. Many representations of a “map” can be
- E.g. Random walks are a form of exploration that does come with guarantees
- We will focus on “Occupancy Grid” algorithms

# Today's topics

## ➤ Mapping and Exploration Algorithms

- **Occupancy Grids and Sensor Models**
- A First-cut Simple Mapping Algorithm

## ➤ Three Improvements

- **Exploration strategies**
  - Frontier based exploration (guaranteed coverage)
- **Managing sensor uncertainty**
  - Probabilistic algorithms for Occupancy Grid mapping (Bayes Rule)
- **Managing motion uncertainty**
  - Briefly: Simultaneous Localization and Mapping (SLAM)

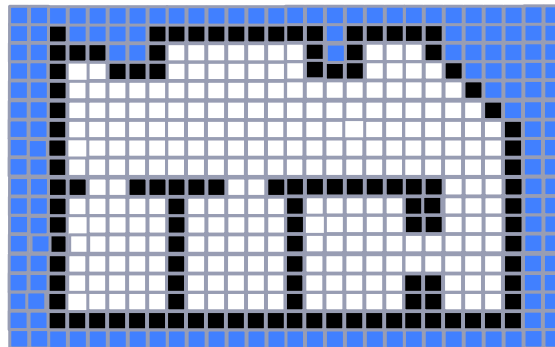
## ➤ Pset 5: Your Autonomous OG Mapper!

## What is an Occupancy Grid?

- A way of representing a map as a gridded world where each cell is either “occupied” or “empty” or “unknown”.

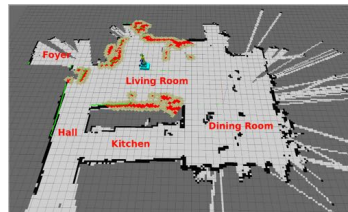
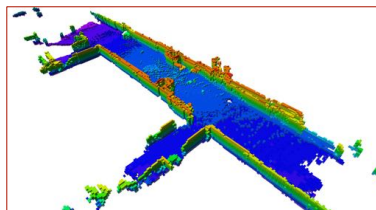
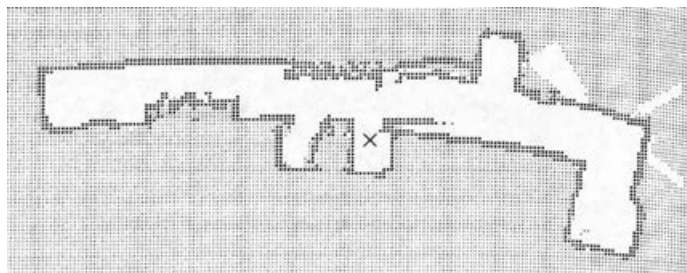


Your World



Grid generated by a Robot => boundary shape

## Examples



## What is a Sensor Model?

### ➤ Step1: Constructing a Sensor Model

- A sensor measures *raw values* in an environment
- You have to map that into a Grid Cell Value.
- Robots can have very different sensors and configurations
- Examples:
  - Think about LIDAR/Depth Camera
  - Vs. a 360 degree vision/ranging system

## Constructing a Sensor Model

### Example: Depth Sensor Model

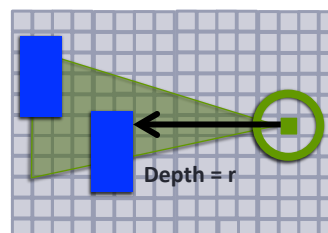
$R$  = maximum range,  $B$  = maximum angle

Let say the sensor at point  $p$  returns **depth** = " $r$ "

Region 1 (dist  $< r$ , grid cell probably empty)

Region 2 (dist  $= r$ , grid cell probably obstacle)

Region 3 (dist  $> r$ , grid cell unknown/obscured)



## Constructing a Sensor Model

### Example: Depth Sensor Model

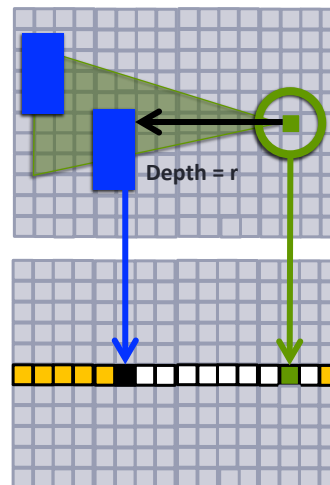
$R$  = maximum range,  $B$  = maximum angle

Let say the sensor at point  $p$  returns **depth** = " $r$ "

Region 1 (dist  $< r$ , grid cell probably empty)

Region 2 (dist =  $r$ , grid cell probably obstacle)

Region 3 (dist  $> r$ , grid cell unknown/obscured)



## Constructing a Sensor Model

### Example: Depth Sensor Model

$R$  = maximum range,  $B$  = maximum angle

Let say the sensor at point  $p$  returns **depth** = " $r$ "

Region 1 (dist  $< r$ , grid cell probably empty)

Region 2 (dist =  $r$ , grid cell probably obstacle)

Region 3 (dist  $> r$ , grid cell unknown/obscured)

#### Simple Model:

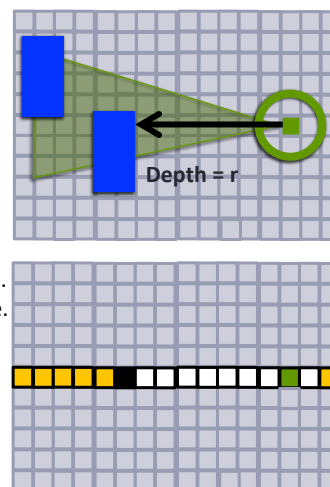
Set region 1 cells as empty, region 2 cells as occupied.  
Pick a Maximum Range/Angle where depth is reliable.

#### More Complex Model:

For a cell at distance  $r$  and angle  $a$

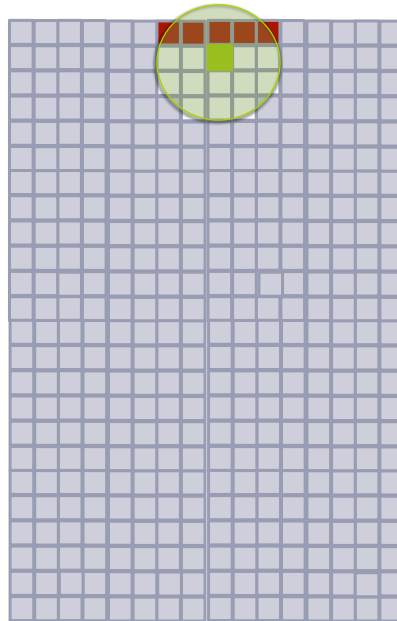
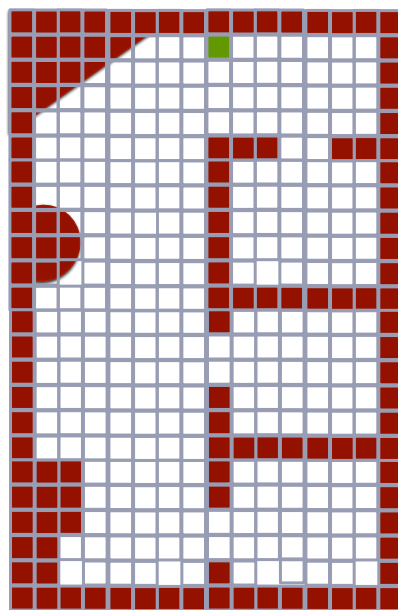
$$P(\text{correctness}) = [(R-r/R) + (B-a/B)]/2$$

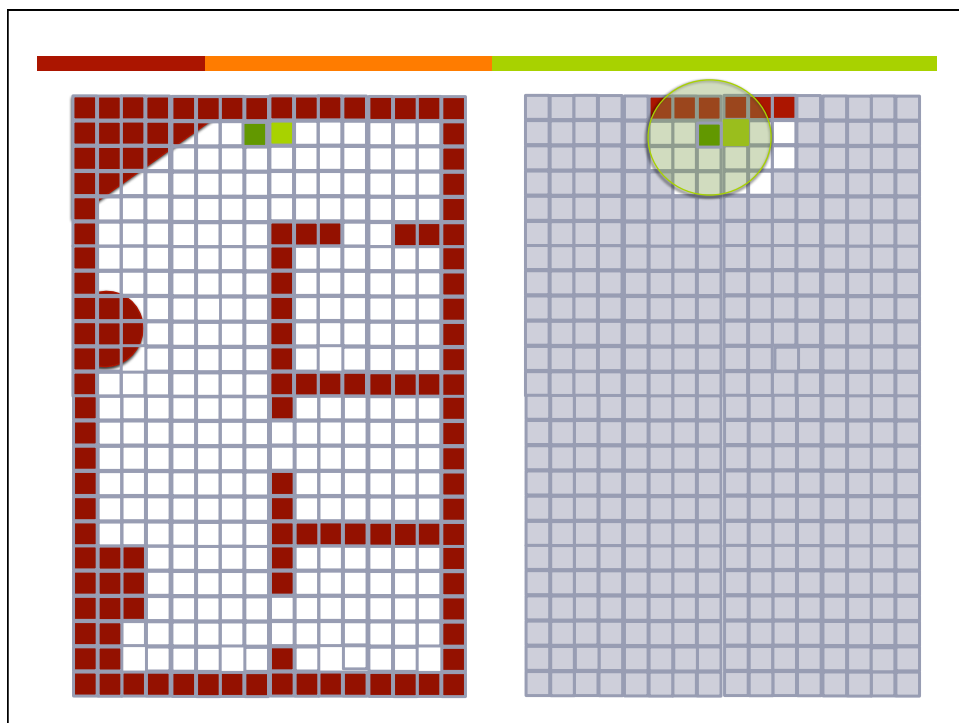
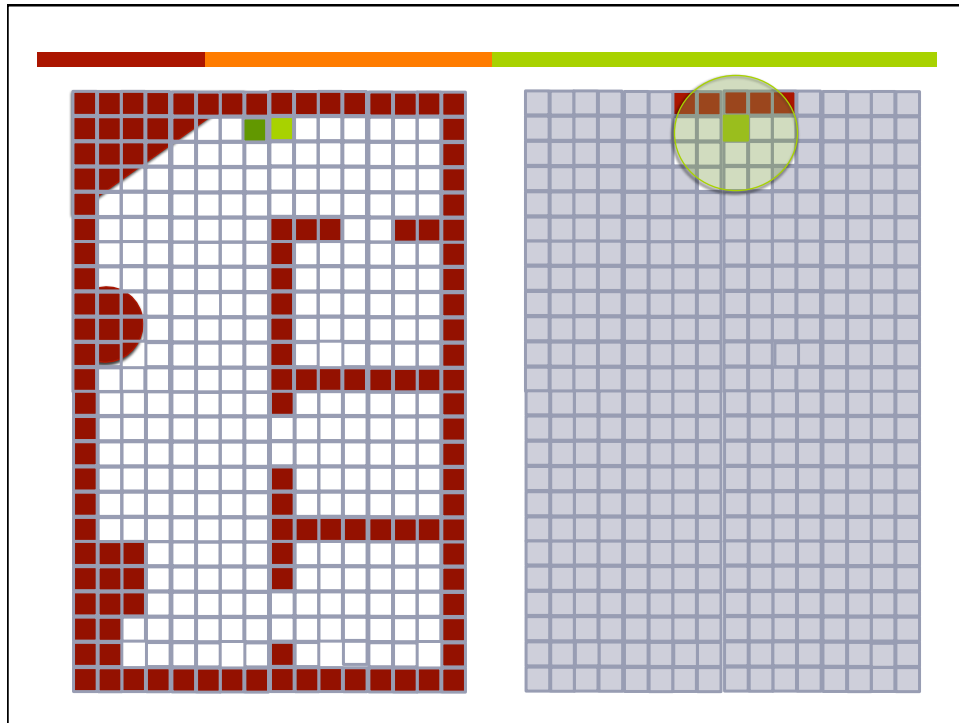
*i.e. Uncertainty in my assessment grows with distance and angle from the centerline*



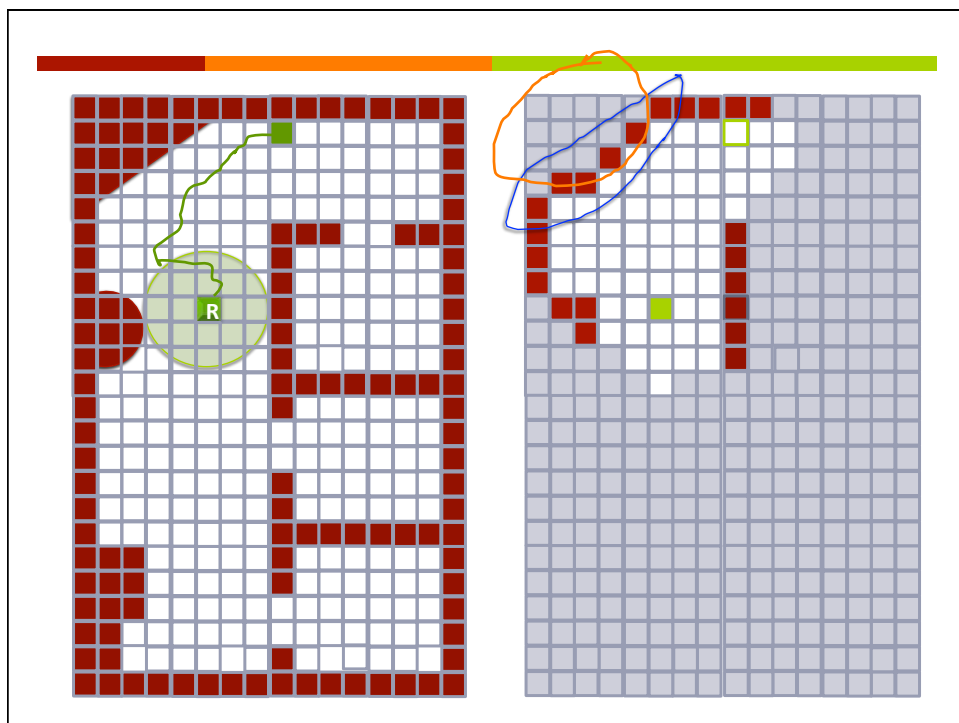
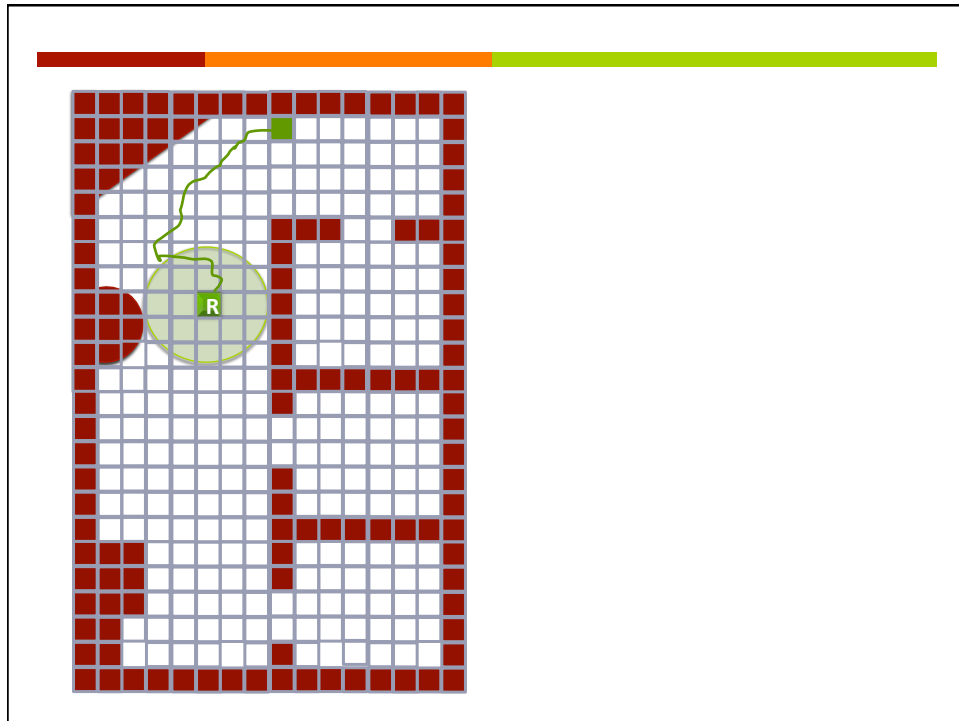
## A Simple OG Mapping Algorithm

1. **Initialize a Grid**
  - Set all locations as "unknown", pick a start location and orientation
2. **Update the Grid**
  - Mark your current grid position as "empty"
  - Using your simple sensor model,  
Mark all visible grid locations as "empty" or "occupied"
3. **Pick a Next Move**
  - Look at neighboring grid positions in your map
  - Pick a neighboring grid location that is empty (randomly)
  - Move to it and update your current position in the Grid
4. **Loop forever**
  - Keep moving and updating the grid (unless you are "done")









## A Simple Mapping Algorithm

1. **Initialize Grid**
2. **Update the Grid**
  - Mark your current position as “empty”
  - Mark sensed nearby grid locations  
As “empty” or “occupied”
3. **Pick a Next Move**
  - Look at neighboring grid positions
  - Choose a random empty direction
  - Move and update your position in the Grid
4. **Loop forever**

### Improvement 1: Exploration Strategy

Better to systematically  
and (hopefully)  
efficiently cover the  
space.

Also would be good to  
know when you are  
done.

## Exploration

- **Basic Concept in Math: Random Walks in bounded 2D**
  - With Probability=1 you will *eventually* visit every spot
- **Basic Concept in CS: Systematic Graph Coverage**
  - You are given a “graph” with  $V$  nodes  
Write an algorithm that visits all of the nodes
  - BFS, DFS, Time Complexity:  $O(V+E)$
- **Basic Concept in Robotics: Traversing a GRID Graph is different**
  - DFS works, but will still make a robot retrace steps
  - Ideal: Visit every node exactly once (Hamiltonian path, NP-complete!)
  - **Better choice: Frontier Based Exploration**

# Exploration in Grid Worlds

## ➤ Frontier Based Exploration

➤ A common technique for building maps

➤ **Key Idea:**

➤ Identify the “frontiers” between known and unknown

*Frontier cell = a unknown cell with at least one empty cell nbr*

➤ Pick a frontier cell (e.g. the closest)

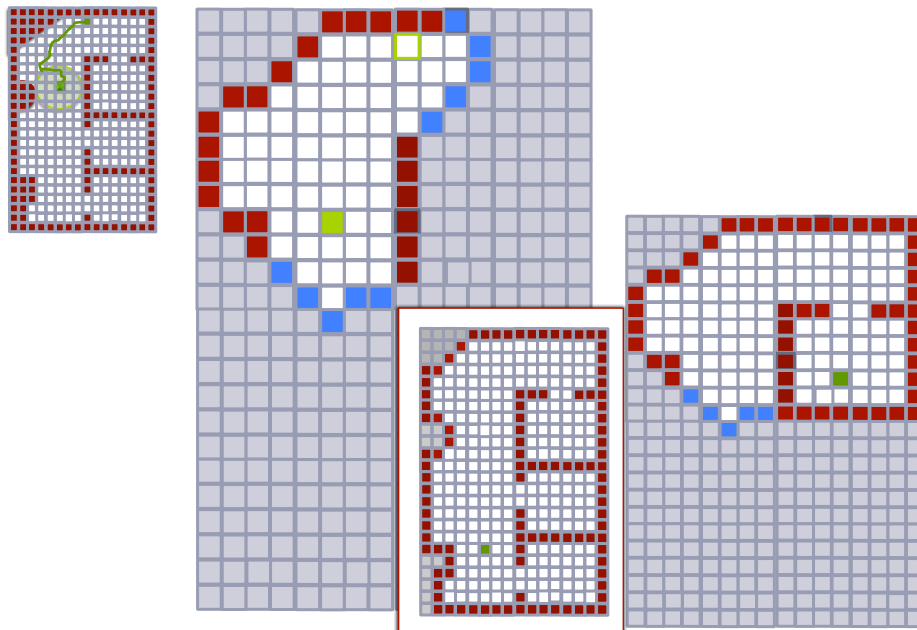
➤ Plan a path to go explore it.

➤ **Done Condition:**

➤ No more frontier nodes left!

*If finite world, then any algorithm that systematically explores frontier nodes is guaranteed to cover the whole world.*

*Can use this condition by itself to determine if your map is complete.*



## A Less Simple Mapping Algorithm

1. **Initialize Grid**
2. **Update the Grid**
  - Mark your current position as "empty"
  - Mark sensed nearby grid locations As "empty" or "occupied"
3. **Pick a Next Move**
  - Identify *frontier cells*
  - Pick one (e.g. maybe the closest)
  - Plan a path\* to the *nbr empty cell*.
  - Go to that location using this path (and keep track of your position as you move)
4. **Loop until no frontier nodes are left**

Improvement 2:  
Sensors aren't perfect

Take advantage of the fact that you are often retracing steps

And taking measurements multiple times of the same location

\* We covered path planning two lectures ago

## Bayesian Mapping

For every grid location (i,j), store a probability value

**P(Occupied)** = Probability this grid location is Occupied

$0 \leq P(\text{Occupied}) \leq 1$

**P(Empty)** =  $1 - P(\text{Occupied})$

A More Complex Sensor Model

**P(s|Occupied)**

Probability that you sense value **s** given that a grid location is occupied.

*Determine this experimentally*

Mapping

**P(Occupied|s)**

Probability that a grid location is occupied given that you sensed value **s**

**We can compute this!**

**Bayes Rule**

$$P(\text{Occupied} | s) = \frac{P(s | \text{Occupied}) P(\text{Occupied})}{P(s | \text{Occupied}) P(\text{Occupied}) + P(s | \text{Empty}) P(\text{Empty})}$$

**Bayes Update Rule**

$$P(\text{Occupied} | s_n) = \frac{P(s_n | \text{Occupied}) P(\text{Occupied} | s_{n-1})}{P(s_n | \text{Occupied}) P(\text{Occupied} | s_{n-1}) + P(s_n | \text{Empty}) P(\text{Empty} | s_{n-1})}$$

## Bayesian Mapping

- In the beginning of time,
  - $P(\text{Occupied}) = P(\text{Empty}) = 0.5$
- For grid(i,j), lets say  $s=6$  (depth sensor value)
  - $P(s=6 | \text{Occupied}) = 0.62$
  - $P(s=6 | \text{Empty}) = 0.38$
  - $P(\text{Occupied}) = P(\text{Empty}) = 0.5$
  - $P(\text{Occupied} | s=6) = (0.62 * 0.5) / (0.62 * 0.5 + 0.38 * 0.5) = 0.62$   
Which is what you'd expect because we have no better knowledge
- Later if we observe location grid (i,j) again, we have *prior* knowledge
  - We now think  $P(\text{Occupied}) = 0.62$   $P(\text{empty}) = 0.38$
  - New sensor reading  $P(s=s' | \text{Occupied}) = x$
  - $P(\text{Occupied} | s=s') = (x * 0.62) / (x * 0.62 + (1-x) * 0.38) = \text{new confidence}$

## Probabilistic Mapping

- **Overarching idea**
  - Store *probabilities* of occupancy rather than binary values.
- But you periodically must turn probability into Occupied/Empty!
  - Otherwise, how do you move?
    - Use some threshold to decide
      - $P(\text{occupied}) > 0.7$  and  $P(\text{empty}) < 0.3$ , rest is "unknown".
    - Then do frontier exploration and path planning on your deterministic map.

## A Probabilistic OG Mapping Algorithm

1. **Initialize Grid to 0.5**
2. **Update the Grid**
  - Mark your current position as high probability “empty”
  - Use your sensor model and Bayes rule to update grid
3. **Pick a Next Move**
  - Threshold your map into empty, occupied, unknown
  - Identify frontier nodes, and pick one
  - Plan a path to the clear node nearest frontier
  - Go to that location and update position
4. **Loop until no frontier nodes are left**

## Probabilistic Mapping

- **Overarching idea**
  - Store *probabilities* of occupancy rather than binary values.
- This is great! So what can go wrong?
  - Motion uncertainty!!!
  - (1 Lecture back: Kalman Filter and Particle Filter)

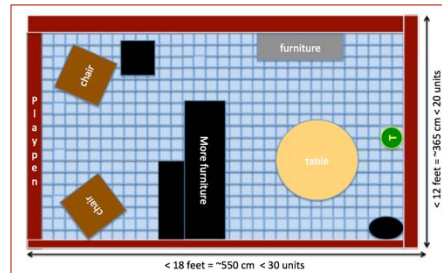
## Pset 5: The Autonomous OG Mapper

### Digression ---- Mapping A Fake Office!

*Generate a map for navigation in the office (B127)*

- Setup an Occupancy Grid (deterministic)
- Construct a Depth Sensor Model for the Turtlebot
- Use a Simple Exploration strategy (random)
- Use EKF (lab 4) for localization.
- Output the map.

*Optional: If you get the above working really well, then take a video and map of your work, and try more complex ideas from this lecture.*



## Probabilistic Localization and Mapping

### ➤ Probabilistic Localization

- $P(x_t | Z_{0:t} U_{0:t} \text{map})$
- Where am I? Given that I took the noisy actions  $U$  and noisy observations  $Z$  of things in my perfect map/landmarks.

1 lecture ago:  
[Kalman Filters](#)  
[Particle Filters](#)

# Probabilistic Localization and Mapping

## ➤ Probabilistic Localization

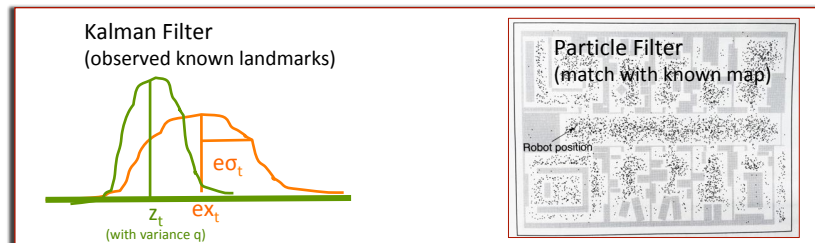
➤  $P(x_t | Z_{0:t} U_{0:t} \text{map})$

- Where am I? Given that I took the noisy actions  $U$  and noisy observations  $Z$  of things in my perfect map/landmarks.

1 lecture ago:

*Kalman Filters*

*Particle Filters*



# Probabilistic Localization and Mapping

## ➤ Probabilistic Localization

➤  $P(x_t | Z_{0:t} U_{0:t} \text{map})$

- Where am I? Given that I took the noisy actions  $U$  and noisy observations  $Z$  of things in my perfect map/landmarks.

1 lecture ago:

*Kalman Filters*

*Particle Filters*

## ➤ Probabilistic Mapping

➤  $P(\text{map} | Z_{0:t}, U_{0:t})$

- What is my map like? Given that I made noisy observations  $Z$  as I walked along my perfect path dictated by  $U$

Today:

*Bayesian*

*Occupancy Grids*



## Probabilistic Localization and Mapping

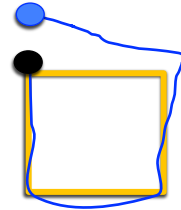
- **Probabilistic Localization:**  $P(x_t | Z_{0:t} U_{0:t} \text{ map})$
- **Probabilistic Mapping:**  $P(\text{map} | Z_{0:t} U_{0:t})$
- **Probabilistic SLAM (“Simultaneous”)**
  - $P(x_t, \text{map} | Z_{0:t} U_{0:t})$
  - Where am I and what is my map?
  - Given **noisy actions**  $U$  and made **noisy observations**  $Z$
  - Distribution of a huge space! (all possible positions and maps)
- **Many Methods**
  - EKF-SLAM (Kalman Filter) and Fast-SLAM (Particle Filters)

## Extended Kalman Filter SLAM

- In original EKF, robot position is a Gaussian ( $x_t \sigma_t$ )
- In EKF-SLAM, robot and all landmark positions are Gaussians
  - **State** =  $\{x_t, m1, m2, m3 \dots mn\}$  (number of landmarks grows!)
  - **Co-variance** =  $(n+1) \times (n+1)$  **matrix** (uncertainty in correlated!)
  - *Good news!: Correlations can help you converge faster and better!*
- **Basic Procedure is similar (w much more math! Often offline)**
  - **Alternate Three Steps**
    - **Motion Step:** Update  $P(x_t, \text{map} | Z_{0:(t-1)} U_{0:t})$  based on action  $U_t$
    - **Observation Step:** Update  $P(x_t, \text{map} | Z_{0:t} U_{0:t})$  based on  $Z_t$
    - **Add New landmarks to the State**
  - Important – these steps update the Whole Map (not just what you see)
    - Captures correlations between landmarks, with high certainty!
    - Many advances (FastSLAM) to make the process real-time.

## More About SLAM

- Data Association and Loop Closure
  - We don't really have landmarks
    - Instead we have depth camera "features"
    - Local matching is easier than long term matching
- Practical Implementations
  - These algorithms are theoretically well-grounded
  - But practical implementation still requires significant work, including understanding environment (constructing sensor/motion models)
- References (online)
  - SLAM Part 1: The Essential Algorithms, Durrant et al, 2006 (in theory)
  - SLAM for Dummies, Riisgaard et al 2005 (in practice)
  - Gmapping in ROS! (PRR chapter 9 = offline map making)



## Conclude: Robots Navigating the World

### Second Part of CS189: High-level reasoning

From finite state machines to complex representation and memory

- **PathPlanning:** *How to I get there?*
- **Localization:** *Where am I?*
- **Mapping:** *Where have I been?*
- **Exploration:** *Where haven't I been?*

*Preview  
of Rest of Term*

Lab 4 and Pset 5  
**Mapping**  
 Final Project  
**Warehouse**  
 Upcoming lectures  
**Automation Ethics**  
**Darpa Challenge**  
**Multi-Robot systems**