

NOTE

These review materials are from the Fall 2009 version of Math E-8. Since E-8 changes from term to term, these review materials, while hopefully useful, are not necessarily an accurate reflection of the quizzes and exams you will take this term.

Quiz 1

Math E-8: College Algebra
September 3, 2009

Answer Problems 1–2 judging from this pattern of numbers: $2, 5, 8, 11, b, b + c, \dots$. Here, b and c are constants that fit the overall pattern.

1. Judging from the pattern, give reasonable values for b and c .
2. The instructions list the first six terms of the pattern. What is the seventh? Give both a numerical value and an expression in b and c .

Quiz 1 Solutions

Math E-8: College Algebra

September 3, 2009

1. The terms appear to go up by 3 each time, so the next two terms should be 14 and 17:

$$2, 5, 8, 11, \underbrace{b}_{14}, \underbrace{b+c}_{17}.$$

Since $b = 14$, we see that

$$b + c = 17$$

$$14 + c = 17$$

$$c = 3.$$

2. The terms appear to go up by 3 each time, so the first seven terms are 2, 5, 8, 11, 14, 17, 20. Thus the seventh term should be 20. Writing this in terms of b and c , we have

$$2, 5, 8, 11, \underbrace{b}_{14}, \underbrace{b+c}_{17}, \underbrace{b+c+3}_{20}.$$

Thus, the seventh term is $b + c + 3$. Since $b = 14$, we see that

$$b + c + 3 = 20$$

$$14 + c + 3 = 20$$

$$c = 3.$$

Thus, another way to write $b + c + 3$ is $b + c + c$ or $b + 2c$.

Quiz 1 Makeup

Math E-8: College Algebra

September 8, 2009

Find a and b so that the expressions in Problems 1–2 are equivalent to $5x + 7y$.

1. $x + y + ax + by$

2. $a(x + y) + by$

Quiz 1 Makeup Solutions

Math E-8: College Algebra

September 8, 2009

1. The expression $x + y + ax + by$ must equal $5x + 7y$. We see that

$$\begin{aligned}x + y + ax + by &= ax + x + by + y && \text{rearrange terms} \\&= \underbrace{(a + 1)}_5 x + \underbrace{(b + 1)}_7 y && \text{factor}\end{aligned}$$

Thus, $a = 4$ and $b = 6$.

2. The expression $x + y + ax + by$ must equal $5x + 7y$. We see that

$$\begin{aligned}a(x + y) + by &= ax + ay + by && \text{distribute} \\&= \underbrace{a}_5 x + \underbrace{(a + b)}_7 y. && \text{factor}\end{aligned}$$

Thus, $a = 5$ and $b = 2$.

Quiz 2

Math E-8: College Algebra
September 10, 2009

Say what must be true about α in order for the expressions in Problems 1–5 to be equivalent to $3x + 5$.

1. $3x + 5 + \alpha$

2. $\alpha(3x + 5)$

3. $3(x + \alpha) + 5 - 3\alpha$

4. $\frac{\alpha + 1}{\alpha - 1} \cdot (3x + 5)$

5. $\frac{2x - 1}{2x + 1} \cdot (3x + 5) + \frac{\alpha - \pi}{2x - 1}$

Quiz 2 Solutions

Math E-8: College Algebra
September 10, 2009

1. Adding anything other than 0 to an expression changes its value, so α must equal 0:

$$\underbrace{3x + 5 + \alpha}_{\text{provided } \alpha = 0} \text{ is equivalent to } \underbrace{3x + 5}$$

provided $\alpha = 0$.

2. Multiplying an expression by anything other than 1 changes its value, so α must equal 1:

$$\alpha \underbrace{(3x + 5)}_{\text{provided } \alpha = 1} \text{ is equivalent to } \underbrace{3x + 5}$$

provided $\alpha = 1$.

3. We have:

$$\begin{aligned} 3(x + \alpha) + 5 - 3\alpha &= 3x + 3\alpha + 5 - 3\alpha && \text{distribute} \\ &= 3\alpha + 5. && \text{collect like terms} \end{aligned}$$

We see that $3(x + \alpha) + 5 - 3\alpha$ is equivalent to $3\alpha + 5$ regardless the value of α . Thus, any value of α makes these two expressions equivalent.

4. Multiplying an expression by anything other than 1 changes its value, so the fraction must equal 1:

$$\underbrace{\frac{\alpha + 1}{\alpha - 1}}_1 \cdot (3x + 5).$$

However, the numerator of this fraction can not possibly equal its denominator, so the fraction can not equal 1. Thus, no value of α makes this expression equivalent to $3x + 5$.

5. We see that

$$\begin{aligned} \underbrace{\frac{2x - 1}{2x - 1}}_1 \cdot (3x + 5) + \frac{\alpha - \pi}{2x - 1} &= 1 \cdot (3x + 5) + \frac{\alpha - \pi}{2x - 1} \\ &= 3x + 5 + \frac{\alpha - \pi}{2x - 1}. \end{aligned}$$

Since adding anything other than 0 to an expression changes its value, the value of the fraction must equal 0:

$$3x + 5 + \underbrace{\frac{\alpha - \pi}{2x - 1}}_0$$

A fraction equals 0 only if its numerator equals 0, so:

$$\begin{aligned} \alpha - \pi &= 0 \\ \alpha &= \pi. \end{aligned}$$

Checking our answer, we let $\alpha = \pi$:

$$\begin{aligned} \underbrace{\frac{2x - 1}{2x - 1}}_1 \cdot (3x + 5) + \frac{\overbrace{\alpha}^{\pi} - \pi}{2x - 1} &= 1 \cdot (3x + 5) + \frac{\pi - \pi}{2x - 1} \\ &= 3x + 5 + \underbrace{\frac{0}{2x - 1}}_0 = 3x + 5, \end{aligned}$$

as required.

Quiz 2 Makeup

Math E-8: College Algebra

September 15, 2009

For each pair of equations in Problems 1–4, say whether the equations are equivalent.

1. $\begin{cases} x + 4 = 9 \\ \sqrt{x + 4} = 3 \end{cases}$

2. $\begin{cases} x + 4 = 9 \\ \frac{1}{x} + \frac{1}{4} = \frac{1}{9} \end{cases}$

3. $\begin{cases} x + 4 = 9 \\ \frac{1}{x+4} = \frac{1}{9} \end{cases}$

4. $\begin{cases} x + 4 = 9 \\ (x + 4)^2 = 81 \end{cases}$

Quiz 2 Makeup Solutions

Math E-8: College Algebra

September 15, 2009

1. The solution to the first equation is given by

$$\begin{aligned}x + 4 &= 9 \\x &= 5.\end{aligned}$$

We see that $x = 5$ is also a solution to the second equation:

$$\text{Letting } x = 5, \text{ we have } \sqrt{x+4} = \sqrt{5+4} = 3.$$

Since no number aside from 9 has a square root of 3, this equation has no other solutions. Thus, these equations are equivalent.

2. The solution to the first equation is given by

$$\begin{aligned}x + 4 &= 9 \\x &= 5.\end{aligned}$$

However, this is not a solution to the second equation:

$$\text{Letting } x = 5, \text{ we have } \frac{1}{5} + \frac{1}{4} = \frac{9}{20} \neq \frac{1}{9}.$$

Since these equations have different solutions, they are not equivalent.

3. The solution to the first equation is given by

$$\begin{aligned}x + 4 &= 9 \\x &= 5.\end{aligned}$$

We see that $x = 5$ is also a solution to the second equation:

$$\text{Letting } x = 5, \text{ we have } \frac{1}{x+4} = \frac{1}{9}.$$

Since no number aside from 9 has reciprocal $1/9$, this equation has no other solutions. Thus, these equations are equivalent.

4. The solution to the first equation is given by

$$\begin{aligned}x + 4 &= 9 \\x &= 5.\end{aligned}$$

We see that $x = 5$ is also a solution to the second equation:

$$\text{Letting } x = 5, \text{ we have } (x+4)^2 = (5+4)^2 = 81.$$

However, since $(-9)^2$ also equals 81, we see that another solution to the second equation is $x = -13$:

$$\text{Letting } x = -13, \text{ we have } (x+4)^2 = (-13+4)^2 = 81.$$

Thus, since they have different solutions, these equations are not equivalent.

Quiz 3

Math E-8: College Algebra
September 17, 2009

Evaluate and simplify the expressions in Problems 1–3 given that $f(z) = 9z - 1$.

1. $f(4)$

2. $2f(p + 1)$

3. $f(2p) + 1$

4. Let $f(A)$ give the cost in euros of A square meters of imported tile . Given that 1 m^2 equals 10.764 ft^2 , and that (as of September 17, 2009) 1 euro equals \$1.471, write an expression using f for the cost in dollars for B square feet of this tile.

Quiz 3 Solutions

Math E-8: College Algebra
September 17, 2009

1. We have

$$\begin{aligned}f(5) &= 9 \cdot 4 - 1 \\&= 36 - 1 = 35.\end{aligned}$$

2. We have

$$\begin{aligned}2f(p+1) &= 2(9(p+1) - 1) \quad \text{because } f(p+1) = 9(p+1) - 1 \\&= 2(9p + 9 - 1) \\&= 2(9p + 8) \\&= 18p + 16.\end{aligned}$$

3. We have

$$\begin{aligned}f(2p) + 1 &= (9(2p) - 1) + 1 \quad \text{because } f(2p) = 9(2p) - 1 \\&= (18p - 1) + 1 \\&= 18p.\end{aligned}$$

4. We know that

$$\text{Area (m}^2\text{)} = \frac{\overbrace{\text{Area (ft}^2\text{)}}^B}{10.764} = \frac{B}{10.764} = 0.0929B.$$

This means that

$$\text{Cost (euros) for tile} = f(\text{Area (m}^2\text{)}) = f(0.0929B).$$

Since 1 euro equals \$1.471, we have

$$\begin{aligned}\text{Cost (\$) for tile} &= 1.471 \times \text{Cost (euros) for tile} \\&= 1.471f(0.0929B).\end{aligned}$$

Quiz 3 Makeup

Math E-8: College Algebra
September 22, 2009

Evaluate and simplify $f(5)$ given the definitions of f in Problems 1–3.

1. $f(z) = 5z - 9$

2. $f(z) = \frac{5 - 9z}{5 - 9z^2}$

3. $f(z) = \sqrt{9z - z\sqrt{9 - z} + 1}$

4. Let $C(q)$ give the cost in Canadian dollars for q liters of gas. Given that (as of September 17, 2009) 1 dollar Canadian equals \$0.932 US, and that 1 gallon equals 3.785 liters, write an expression using C for the cost in \$ US for r gallons of gas.

Quiz 3 Makeup Solutions

Math E-8: College Algebra
September 22, 2009

1. We have

$$\begin{aligned}f(5) &= 5 \cdot 5 - 9 \\&= 25 - 9 \\&= 16.\end{aligned}$$

2. We have

$$\begin{aligned}f(5) &= \frac{5 - 9 \cdot 5}{5 - 9 \cdot 5^2} \\&= \frac{5 - 45}{5 - 225} \\&= \frac{-40}{-220} \\&= \frac{2}{11} = 0.1818.\end{aligned}$$

3. We have

$$\begin{aligned}f(5) &= \sqrt{9 \cdot 5 - 5\sqrt{9 \cdot 5} + 1} \\&= \sqrt{45 - 5\sqrt{4} + 1} \\&= \sqrt{45 - 5 \cdot 2 + 1} \\&= \sqrt{45 - 10 + 1} \\&= \sqrt{36} = 6.\end{aligned}$$

4. We know that

$$\text{Amount (liters)} = 3.785 \times \overbrace{\text{Amount (gallons)}}^r = 3.785r.$$

This means that

$$\text{Cost (\$ Canadian) for gas} = C(\text{Amount (liters)}) = C(3.785r).$$

Since \$1 Canadian equals \$0.932 US, we have

$$\begin{aligned}\text{Cost (\$ US)} &= 0.932 \times \text{Cost (\$ Canadian)} \\&= 0.932C(3.785r).\end{aligned}$$

Quiz 4

Math E-8: College Algebra
September 24, 2009

1. Let $P = f(t)$ give the population of a town in year t . The town grows at a steady rate of n people per year. Given that $f(20) = L$, evaluate $f(35)$. Your answer should involve the constants L and n .
2. The graph of w is a line intersecting the graph of $y = (x - 3)^2 - 7$ at $x = -3$ and $x = 8$. Find a possible formula for w .

Quiz 4 Solutions

Math E-8: College Algebra
September 24, 2009

1. One approach is to think about the problem conceptually.

- $f(20) = L$ means that in year $t = 20$ there are L people in the town.
- $f(35)$ is the number of people in year $t = 35$, or 15 years later.
- Since the town grows by n people every year, after 15 years it will grow by $15n$ people.
- We conclude that

$$\underbrace{\text{Number of people in year 35}}_{f(35)} = \underbrace{\text{Number of people in year 20}}_L + \underbrace{\text{Number of new people after 15 years}}_{15n} = L + 15n.$$

Another approach is algebraic.

- From the point-slope formula for a line, we know that $f(x) = y_0 + m(x - x_0)$.
- Here, instead of using x , we are using t for the year.
- We know m is the slope or rate of change, in this case n people per year.
- Since $f(20) = L$, this means that a point on the line is $(t_0, P_0) = (20, L)$.
- Putting this all together, we have

$$\begin{aligned} f(t) &= \underbrace{P_0}_L + \underbrace{m}_n (t - \underbrace{t_0}_{20}) \\ &= L + n(t - 20) \\ \text{so } f(35) &= L + n(35 - 20) \\ &= L + 15n, \end{aligned}$$

which is the same answer that we got before.

- 2.
- Since its graph is a line, w is a linear function, so $w(x) = b + mx$.
 - The graph intersects the graph of $y = (x - 3)^2 - 7$ at two points:

$$\begin{aligned} \text{At } x = -3: \quad y &= (-3 - 3)^2 - 7 \\ &= (-6)^2 - 7 \\ &= 36 - 7 = 29 \\ \text{At } x = 8: \quad y &= (8 - 3)^2 - 7 \\ &= 5^2 - 7 = 18. \end{aligned}$$

Thus, the graph of w contains the points $(-3, 29)$ and $(8, 18)$.

- The slope of w is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{18 - 29}{8 - (-3)} = \frac{-11}{11} = -1.$$

- Using the point-slope formula with $(x_0, y_0) = (8, 18)$, we have

$$\begin{aligned} w(x) &= 18 + (-1)(x - 8) \\ &= 18 - x + 8 \\ &= 26 - x. \end{aligned}$$

We can verify our answer using the other point, $(-3, 29)$. To do this, we need to show that, according to our formula, $w(-3) = 29$:

$$\begin{aligned} w(-3) &= 26 - (-3) \quad \text{because } w(x) = 26 - x \\ &= 29, \end{aligned}$$

as required.

Quiz 4 Makeup

Math E-8: College Algebra

September 29, 2009

1. Suppose $P = f(t)$ gives the population of a town in year t , and that $f(10) = L$, $f(30) = 5L$, $f(50) = 9L$, Assuming the pattern continues, evaluate $f(90)$ and say what L tells you about the town's growth rate.
2. The graph of g is a line containing the points $(0.72, 1.55)$ and $(0.97, 1.10)$. Find a possible formula for g .

Quiz 4 Makeup Solutions

Math E-8: College Algebra
September 29, 2009

1. We record the values of f in Table 1. We see that each time t grows by 20, P grows by $4L$. We surmise that $f(90) = 17L$, as shown in the table.

Table 1

t	P
10	L
30	$5L$
50	$9L$
70	$13L$
90	$17L$

Notice that

$$\begin{aligned}\text{Rate of change} &= \frac{\Delta P}{\Delta t} \\ &= \frac{4L}{20} \\ &= \frac{L}{5}.\end{aligned}$$

This means that $L/5$ (or $0.2L$) is the population's growth rate. Another way to say this is that the population grows by L people every 5 years.

2.
 - Since its graph is a line, g is linear function, so $g(x) = b + mx$.
 - The slope is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{1.10 - 1.55}{0.97 - 0.72} = -1.8.$$

- Using the point slope formula with $(x_0, y_0) = (0.72, 1.55)$, we have:

$$\begin{aligned}g(x) &= x_0 + m(y - y_0) \\ &= 1.55 + (-1.8)(x - 0.72) \\ &= 2.846 - 1.8x.\end{aligned}$$

We can verify our answer using the other point, $(0.97, 1.10)$. To do this, we need to show that, according to our formula, $g(0.97) = 1.10$:

$$\begin{aligned}g(0.97) &= 2.846 - 1.8(1.0.97) \quad \text{because } g(x) = 2.846 - 1.8x \\ &= 2.846 - 1.746 \\ &= 1.10,\end{aligned}$$

as required.

Quiz 5

Math E-8: College Algebra

October 1, 2009

Write the power functions in Problems 1–3 in the form $y = kx^p$. State the values of k and p .

1. $y = \frac{\sqrt{w^5}}{5}$

2. $y = \frac{2\sqrt{t}}{3t^4}$

3. $y = \left((2h^5)^3 \right)^2$

Quiz 5 Solutions

Math E-8: College Algebra
October 1, 2009

1. We have:

$$\begin{aligned}y &= \frac{\sqrt{w^3}}{5} \\&= \frac{1}{5} \cdot (w^5)^{1/2} && \text{rewrite radical as exponent} \\&= \frac{1}{5} \cdot w^{5/2}, && \text{exponent rule}\end{aligned}$$

so $k = 1/5, p = 5/2$.

2. We have:

$$\begin{aligned}y &= \frac{2\sqrt{t}}{3t^4} \\&= \frac{2}{3} \cdot \frac{\sqrt{t}}{t^4} \\&= \frac{2}{3} \cdot \frac{t^{1/2}}{t^4} && \text{rewrite radical as exponent} \\&= \frac{2}{3} \cdot t^{1/2-4} && \text{exponent rule} \\&= \frac{2}{3} \cdot t^{-7/2},\end{aligned}$$

so $k = 2/3, p = -7/2$.

3. We have:

$$\begin{aligned}y &= \left((2h^5)^3\right)^2 \\&= \left(2^3 (h^5)^3\right)^2 \\&= (8h^{15})^2 && \text{exponent rule} \\&= 8^2 (h^{15})^2 \\&= 64h^{30},\end{aligned}$$

so $k = 64, p = 30$.

Quiz 5 Makeup

Math E-8: College Algebra

October 6, 2009

Write the power functions in Problems 1–3 in the form $y = kx^p$. State the values of k and p . Your answers may involve the constant r .

1. $y = 3x^2 \cdot x^r$

2. $y = \frac{(r-1)x^{r+1}}{(r+1)x^{r-1}}$

3. $y = \sqrt{x}\sqrt{x^{4r}}$

Quiz 5 Makeup Solutions

Math E-8: College Algebra

October 6, 2009

1. We have:

$$\begin{aligned}y &= 3x^2 \cdot x^r \\ &= 3x^{2+r} \quad \text{exponent rule,}\end{aligned}$$

so $k = 3, p = 2 + r$.

2. We have:

$$\begin{aligned}y &= \frac{(r-1)x^{r+1}}{(r+1)x^{r-1}} \\ &= \frac{r-1}{r+1} \cdot \frac{x^{r+1}}{x^{r-1}} \\ &= \frac{r-1}{r+1} \cdot x^{r+1-(r-1)} \quad \text{exponent rule} \\ &= \frac{r-1}{r+1} \cdot x^2,\end{aligned}$$

so $k = (r-1)/(r+1)$ and $p = 2$.

3. We have:

$$\begin{aligned}y &= \sqrt{x}\sqrt{x^{4r}} \\ &= x^{\frac{1}{2}} \left(x^{4r}\right)^{\frac{1}{2}} \quad \text{rewrite radicals using exponents} \\ &= x^{\frac{1}{2}} \cdot x^{4r \cdot \frac{1}{2}} \quad \text{exponent rule} \\ &= x^{\frac{1}{2}} \cdot x^{2r} \\ &= 1 \cdot x^{2r+\frac{1}{2}},\end{aligned}$$

so $k = 1, p = 2r + 1/2$.

Quiz 6

Math E-8: College Algebra
October 8, 2009

Evaluate and simplify the expressions in Problems 1–2 given that $r(p) = 3p - 7$.

1. $7r(p - 3)$

2. $3r(7 - 3p) - 7$

3. *Kleiber's law* claims that an animal's metabolic rate r is a power function of its mass, M , where $r = f(M) = kM^{0.75}$.¹ Suppose one animal has mass M_0 and another has mass $100M_0$. Evaluate the following expression, and say what your answer tells you about these animals' metabolic rates:

$$\frac{f(100M_0)}{f(M_0)}.$$

¹ See http://en.wikipedia.org/wiki/Kleiber%27s_law, October 7, 2009.

Quiz 6 Solutions

Math E-8: College Algebra

October 8, 2009

1. We have

$$\begin{aligned} 7r(p-3) &= 7 \underbrace{(3(p-3)-7)}_{r(p-3)} \quad \text{because } r(p) = 3p-7 \\ &= 7(3p-9-7) \\ &= 7(3p-16) \\ &= 21p-112. \end{aligned}$$

2. We have

$$\begin{aligned} 3r(7-3p)-7 &= 3 \underbrace{(3(7-3p)-7)}_{r(7-3p)} - 7 \quad \text{because } r(p) = 3p-7 \\ &= 3(21-9p-7)-7 \\ &= 3(14-9p)-7 \\ &= 42-27p-7 \\ &= 35-27p. \end{aligned}$$

3. We have

$$\begin{aligned} \frac{f(100M_0)}{f(M_0)} &= \frac{k(100M_0)^{0.75}}{kM_0^{0.75}} \quad \text{because } f(M) = kM^{0.75} \\ &= \frac{k}{k} \cdot \frac{100^{0.75} \cdot M_0^{0.75}}{M_0^{0.75}} \\ &= 100^{0.75} \\ &= 31.623. \end{aligned}$$

This tells us that an animal 100 times as massive as another animal ($100M_0$ versus M_0) has a metabolic rate only 31.623 times as large.

Quiz 6 Makeup

Math E-8: College Algebra

October 13, 2009

Evaluate and simplify the expressions in Problems 1–2 given that $w(h) = \frac{3-h}{4-h}$.

1. $w(h-4)$

2. $\frac{w(3-h)}{w(4-h)}$

3. *Stevens' power law* asserts that the perceived sensation of a physical stimulus, ψ (the Greek letter *psi*), is related to the magnitude or size of the stimulus I , according to the formula $\psi = kI^a$. Here, k and a are constants that depend on the stimulus.¹ For instance, for the perceived sensation of the taste of sugar, $a = 1.3$. Suppose I_0 is the amount of sugar in a sample, and $10I_0$ is the amount of sugar in another sample. How many times sweeter is the second sample perceived to taste according to Stevens' power law?

¹http://en.wikipedia.org/wiki/Stevens%27_power_law, October 7, 2009.

Quiz 6 Makeup Solutions

Math E-8: College Algebra

October 13, 2009

1. We have

$$\begin{aligned}w(h-4) &= \frac{3-(h-4)}{4-(h-4)} \\&= \frac{3-h+4}{4-h+4} \\&= \frac{7-h}{8-h}.\end{aligned}$$

2. We have:

$$\begin{aligned}\frac{w(3-h)}{w(4-h)} &= \frac{\left(\frac{3-(3-h)}{4-(3-h)}\right)}{\left(\frac{3-(4-h)}{4-(4-h)}\right)} \\&= \frac{\left(\frac{3-3+h}{4-3+h}\right)}{\left(\frac{3-4+h}{4-4+h}\right)} \\&= \frac{\left(\frac{h}{h+1}\right)}{\left(\frac{h-1}{h}\right)} \\&= \frac{h}{h+1} \cdot \frac{h}{h-1} \\&= \frac{h^2}{(h-1)(h+1)}.\end{aligned}$$

Multiplying out, we can also write this answer as $\frac{h^2}{h^2-1}$.

3. The perceived sweetness of the first sample is $\psi = k(I_0)^{1.3}$. If the amount of sugar is increased tenfold, to $10I_0$, the new sweetness sensation is:

$$\begin{aligned}\psi &= k(10I_0)^{1.3} \\&= k \cdot 10^{1.3}(I_0)^{1.3} \\&= 19.953k(I_0)^{1.3} \\&= 19.953 \times \underbrace{\text{Original sweetness sensation}}_{k(I_0)^{1.3}}.\end{aligned}$$

In other words, if the sugar level is increased by a factor of 10, the perceived sweetness goes up by nearly a factor of 20.

Quiz 7

Math E-8: College Algebra

October 15, 2009

Evaluate and simplify the following expressions given that $f(x) = 3 - 2x$ and $g(x) = x^2 + 2x$.

1. $f(g(x))$

2. $g(f(x))$

3. $f(f(x))$

Quiz 7 Solutions

Math E-8: College Algebra

October 15, 2009

1. We have:

$$\begin{aligned}f(g(x)) &= f(x^2 + 2x) && \text{because } g(x) = x^2 + 2x \\&= 3 - 2(x^2 + 2x) && \text{because } f(x) = 3 - 2x \\&= 3 - 2x^2 - 4x \\&= -2x^2 - 4x + 3.\end{aligned}$$

2. We have:

$$\begin{aligned}g(f(x)) &= g(3 - 2x) && \text{because } f(x) = 3 - 2x \\&= (3 - 2x)^2 + 2(3 - 2x) && \text{because } g(x) = x^2 + 2x \\&= (3 - 2x)(3 - 2x) + 2(3 - 2x) \\&= 9 - 12x + 4x^2 + 6 - 4x \\&= 4x^2 - 16x + 15.\end{aligned}$$

3. We have:

$$\begin{aligned}f(f(x)) &= f(3 - 2x) && \text{because } f(x) = 3 - 2x \\&= 3 - 2(3 - 2x) && \text{because } f(x) = 3 - 2x \\&= 3 - 6 + 4x && \text{distribute} \\&= -3 + 4x.\end{aligned}$$

Quiz 7 Makeup

Math E-8: College Algebra

October 20, 2009

Evaluate and simplify the following expressions given that $v(t) = \frac{1}{1-t}$ and $w(t) = 2t - 4$.

1. $v(w(t))$

2. $w(w(t))$

3. $v(v(t))$

Quiz 7 Makeup Solutions

Math E-8: College Algebra

October 20, 2009

1. We have

$$\begin{aligned}v(w(t)) &= v(2t - 4) && \text{because } w(t) = 2t - 4 \\&= \frac{1}{1 - (2t - 4)} && \text{because } v(t) = \frac{1}{1-t} \\&= \frac{1}{1 - 2t + 4} \\&= \frac{1}{5 - 2t}.\end{aligned}$$

2. We have:

$$\begin{aligned}w(w(t)) &= w(2t - 4) && \text{because } w(t) = 2t - 4 \\&= 2(2t - 4) - 4 && \text{because } w(t) = 2t - 4 \\&= 4t - 8 - 4 \\&= 4t - 12.\end{aligned}$$

3. We have:

$$\begin{aligned}v(v(t)) &= v\left(\frac{1}{1-t}\right) && \text{because } v(t) = \frac{1}{1-t} \\&= \frac{1}{1 - \frac{1}{1-t}} && \text{because } v(t) = \frac{1}{1-t} \\&= \frac{1}{\frac{1-t}{1-t} - \frac{1}{1-t}} && \text{find common denominator} \\&= \frac{1}{\frac{1-t-1}{1-t}} && \text{add numerators} \\&= \frac{1}{\frac{-t}{1-t}} && \text{simplify} \\&= \frac{1-t}{-t} && \text{take reciprocal} \\&= -\frac{1-t}{t}.\end{aligned}$$

This expression can also be written as $\frac{t-1}{t}$ or as $1 - \frac{1}{t}$.

Quiz 8

Math E-8: College Algebra
October 22, 2009

Find the domain for each function in Problems 1–3.

1. $f(w) = \frac{1}{4w - 12}$

2. $g(t) = \sqrt{t - 5} + \sqrt{8 - t}$

3. $h(r) = \sqrt{9 + r^2}$

Quiz 8 Solutions

Math E-8: College Algebra
October 22, 2009

1. This function is undefined if the denominator equals zero. To find where this happens, we solve:

$$4w - 12 = 0$$

$$4w = 12$$

$$w = 3.$$

Thus, the domain of this function is all values of w except $w = 3$.

2. The function g is undefined if either square root is undefined.

- The first square root is undefined if $t - 5$ is negative:

$$t - 5 < 0$$

$$t < 5$$

- The second square root is undefined if $8 - t$ is negative:

$$8 - t < 0$$

$$8 < t,$$

or $t > 8$.

Thus, g is undefined if either $t < 5$ or $t > 8$. This means t must equal or lie between 5 and 8, and we write $5 \leq t \leq 8$.

3. The value of h is undefined if the square root is undefined, which happens where $9 + r^2$ is negative. But $9 + r^2$ is the sum of a positive number, 9, and a nonnegative number, r^2 , so it must itself be positive. Thus, h is defined everywhere, and its domain is all values of r .

Quiz 8 Makeup

Math E-8: College Algebra

October 27, 2009

Find the range for each function in Problems 1–3.

1. $y = 5 + \sqrt{x}$

2. $y = 3 + x^3$

3. $y = 3(x^2 + 2)^3$

Quiz 8 Makeup Solutions

Math E-8: College Algebra

October 27, 2009

1. The value of \sqrt{x} can not be negative, but it can be zero or positive. Thus the value of $y = 5 + \sqrt{x}$ can be as small as 5 but no smaller, and has no upper limit, so the range is $y \geq 5$.
2. The value of x^3 can be anything—large positive, large negative, zero, or any other number. Thus $y = 3 + x^3$ can also equal any value, so the range is all values of y .
3. Breaking this down into steps, we see that:
 - The value of x^2 can't be negative, but it can be zero or positive.
 - Thus, the value of $x^2 + 2$ can't be less than 2, but it can be 2 or larger.
 - This means the value of $(x^2 + 2)^3$ can't be less than $2^3 = 8$, but it can be 8 or larger.
 - This means the value of $y = 3(x^2 + 2)^3$ can't be less than $y = 3(8) = 24$, but it can be 24 or larger.

We conclude that $y \geq 24$.

Quiz 9

Math E-8: College Algebra

October 29, 2009

- Find $f^{-1}(x)$ given that $f(x) = \frac{7x - 4}{2 - 3x}$.
- Write in vertex form and state the vertex: $y = x^2 + 20x + 7$.
- Say which of the following equations has no solutions, which has only one solution, and which has two solutions: **(a)** $3x^2 + 8x + 4 = 0$ **(b)** $3x^2 + 6x + 4 = 0$
(c) $4x^2 + 8x + 4 = 0$

Quiz 9 Solutions

Math E-8: College Algebra
October 29, 2009

1. Solving $y = f(x)$ for x gives:

$$\begin{aligned}y &= \frac{7x - 4}{2 - 3x} \\y(2 - 3x) &= 7x - 4 && \text{multiply by denominator} \\2y - 3xy &= 7x - 4 && \text{distribute} \\-3xy - 7x &= -4 - 2y && \text{collect } x\text{-terms} \\x(-3y - 7) &= -4 - 2y && \text{factor} \\x &= \frac{-4 - 2y}{-3y - 7} && \text{divide} \\&= \frac{-(4 + 2y)}{-(3y + 7)} && \text{simplify} \\&= \frac{2y + 4}{3y + 7}.\end{aligned}$$

Since $x = f^{-1}(y)$, this means that a formula for $f^{-1}(x)$ is given by $f^{-1}(x) = \frac{2x + 4}{3x + 7}$..

2. We have:

$$\begin{aligned}y &= x^2 + 20x + 7 \\y - 7 &= x^2 + 20x \\y - 7 + (10)^2 &= x^2 + 20x + (20/2)^2 && \text{complete square} \\y + 93 &= (x + 10)^2 && \text{factor right-hand side} \\y &= (x + 10)^2 - 93.\end{aligned}$$

Writing this as $y = (x - (-10))^2 - 93$, we see that the vertex is $(h, k) = (10, -93)$.

3. We can use the discriminant, $b^2 - 4ac$, of each equation to find how many solutions there are.
- (a) Here, the discriminant is $b^2 - 4ac = 8^2 - 4(3)(4) = 16$. Since the discriminant is positive, there are two solutions.
 - (b) Here, the discriminant is $b^2 - 4ac = 6^2 - 4(3)(4) = -12$. Since the discriminant is negative, there are no solutions.
 - (c) Here, the discriminant is $b^2 - 4ac = 8^2 - 4(4)(4) = 0$. Since the discriminant is zero, there is only one solution.

Quiz 9 Makeup

Math E-8: College Algebra

November 3, 2009

1. Find $g^{-1}(x)$ given that $g(x) = 2 - (3 - \sqrt{x})^3$.
2. Write in vertex form and state the vertex: $y = 3x^2 - 24x - 7$.
3. What must be true about a if there is exactly one solution to the equation $ax^2 + 6x + 5 = 0$?

Quiz 9 Makeup Solutions

Math E-8: College Algebra

November 3, 2009

1. Solving $y = g(x)$ for x gives:

$$\begin{aligned}y &= 2 - (3 - \sqrt{x})^3 \\(3 - \sqrt{x})^3 &= 2 - y \\3 - \sqrt{x} &= (2 - y)^{1/3} \\\sqrt{x} &= (2 - y)^{1/3} - 3 \\x &= ((2 - y)^{1/3} - 3)^2.\end{aligned}$$

Since $x = g^{-1}(y)$, this means that a formula for $g^{-1}(x)$ is given by $g^{-1}(x) = ((2 - x)^{1/3} - 3)^2$.

2. We have:

$$\begin{aligned}y &= 3x^2 - 24x - 7 \\y + 7 &= 3x^2 - 24x \\\frac{y + 7}{3} &= x^2 - 8x \\\frac{y + 7}{3} + 16 &= x^2 - 8x + (-8/2)^2 \\\frac{y + 7}{3} + 16 &= (x - 4)^2 \\\frac{y + 7}{3} &= (x - 4)^2 - 16 \\y + 7 &= 3(x - 4)^2 - 48 \\y &= 3(x - 4)^2 - 55.\end{aligned}$$

We see that the vertex is $(h, k) = (4, -55)$.

3. Since there is exactly one solution, the discriminant must equal zero:

$$\begin{aligned}b^2 - 4ac &= 0 \\6^2 - 4a(5) &= 0 \\36 - 20a &= 0 \\20a &= 36 \\a &= \frac{36}{20} = 1.8.\end{aligned}$$

Quiz 10

Math E-8: College Algebra

November 5, 2009

Write the exponential functions in Problems 1–3 in the form $Q = ab^t$, and identify the values of a and b .

1. $Q = 0.031(0.882)^t$

2. $y = 900 \left(\frac{2}{3}\right)^{x+2}$

3. $P = 70 \cdot 2^{t/5}$

Quiz 10 Solutions

Math E-8: College Algebra

November 5, 2009

1. This is already in the required form with $a = 0.031, b = 0.882$.
2. We have

$$\begin{aligned}y &= 900 \left(\frac{2}{3}\right)^{x+2} \\&= 900 \left(\frac{2}{3}\right)^2 \left(\frac{2}{3}\right)^x && \text{exponent rule} \\&= 900 \cdot \frac{4}{9} \left(\frac{2}{3}\right)^x \\&= \underbrace{400}_a \underbrace{\left(\frac{2}{3}\right)^x}_b,\end{aligned}$$

so $a = 400, b = 2/3$.

3. We have

$$\begin{aligned}P &= 70 \cdot 2^{t/5} \\&= 70 \cdot 2^{\frac{1}{5} \cdot t} \\&= 70 \left(2^{1/5}\right)^t && \text{exponent rule} \\&= 70(1.1487)^t,\end{aligned}$$

so $a = 70, b = 1.1487$.

Quiz 10 Makeup

Math E-8: College Algebra

November 10, 2009

Write the exponential functions in Problems 1–3 in the form $Q = ab^t$, and identify the values of a and b .

1. $Q = \frac{37.3}{1.17^t}$

2. $N = 350(0.8)^{2s+1}$

3. $w = 40e^{-0.035t}$, where $e = 2.718$

Quiz 10 Makeup Solutions

Math E-8: College Algebra

November 10, 2009

1. We have

$$\begin{aligned}Q &= \frac{37.3}{1.17^t} \\&= 37.3 (1.17^t)^{-1} && \text{exponent rule} \\&= 37.3 (1.17^{-t})^t && \text{exponent rule} \\&= 37.3 (1.17^{-1})^t && \text{exponent rule} \\&= 37.3(0.8547)^t,\end{aligned}$$

so $a = 37.3, b = 0.8547$.

2. We have

$$\begin{aligned}N &= 350(0.8)^{2s+1} \\&= 350(0.8)^1 \cdot (0.8)^{2s} && \text{exponent rule} \\&= 350(0.8) ((0.8)^2)^s && \text{exponent rule} \\&= 280(0.64)^t,\end{aligned}$$

so $a = 280, b = 0.64$.

3. We have

$$\begin{aligned}w &= 40e^{-0.035t} \\&= 40 (e^{-0.035})^t && \text{exponent rule} \\&= 40 (2.718^{-0.035})^t && \text{since } e = 2.718 \\&= 40(0.9656)^t,\end{aligned}$$

so $a = 40, b = 0.9656$.

Quiz 11

Math E-8: College Algebra

November 12, 2009

1. Find a formula for the exponential function f given that $f(30) = 100$ and $f(70) = 300$.
2. Atmospheric pressure is $P_0 = 29.53$ inches of mercury at sea level. It drops by half for every 3.37 miles of altitude.¹ Find a formula for $P = f(s)$, the pressure at altitude s miles.

¹http://en.wikipedia.org/wiki/Exponential_decay, page accessed November 19, 2008.

Quiz 11 Solutions

Math E-8: College Algebra

November 12, 2009

1. Since f is exponential, we know that $f(t) = ab^t$. Taking ratios, we have

$$\begin{aligned}\frac{f(70)}{f(30)} &= \frac{ab^{70}}{ab^{30}} \\ \frac{300}{100} &= b^{70-30} \\ b^{40} &= 3 \\ b &= 3^{1/40},\end{aligned}$$

or approximately $b = 1.0278$. We can use the point $(30, 100)$ to solve for a :

$$\begin{aligned}ab^{30} &= 100 \\ a(3^{1/40})^{30} &= 100 && \text{using the exact value for } b \\ a &= \frac{100}{3^{30/40}} \\ &= 43.869.\end{aligned}$$

Thus, $f(t) = 43.869(1.0278)^t$.

2. We see that

$$\underbrace{\text{Atmospheric pressure}}_P = \underbrace{\text{Sea level pressure}}_{29.53} \times \underbrace{\frac{1}{2} \times \frac{1}{2} \cdots \times \frac{1}{2}}_{\text{number of 3.37-mile increases}}.$$

In s miles, the number of 3.37-mile increases is $s/3.37$. Thus:

$$\begin{aligned}\underbrace{\text{Atmospheric pressure}}_P &= \underbrace{\text{Sea level pressure}}_{29.53} \times \underbrace{\frac{1}{2} \times \frac{1}{2} \cdots \times \frac{1}{2}}_{s/3.37 \text{ times}} \\ P &= 29.53 \left(\frac{1}{2}\right)^{s/3.37}.\end{aligned}$$

Quiz 11 Makeup

Math E-8: College Algebra

November 17, 2009

1. Find a formula for the exponential function g given that its graph contains the points $(-25, 18)$ and $(30, 5)$.
2. Suppose N_0 houses are for sale in a given community on day $t = 0$, and that of these houses, half are sold every 70 days. Find a formula for $N = w(t)$, the number remaining on the market after t days.

Quiz 11 Makeup Solutions

Math E-8: College Algebra

November 17, 2009

1. Since its graph contains the points $(-25, 18)$ and $(30, 5)$, we know $g(-25) = 18$ and $g(30) = 5$. Since g is exponential, we know that $g(t) = ab^t$. Taking ratios, we have

$$\begin{aligned}\frac{g(30)}{g(-25)} &= \frac{ab^{30}}{ab^{-25}} \\ \frac{5}{18} &= b^{30-(-25)} \\ b^{55} &= \frac{5}{18} \\ b &= \left(\frac{5}{18}\right)^{1/55},\end{aligned}$$

or approximately $b = 0.977$. We can use the point $(30, 5)$ to solve for a :

$$\begin{aligned}ab^{30} &= 5 \\ a\left(\left(\frac{5}{18}\right)^{1/55}\right)^{30} &= 5 && \text{using the exact value for } b \\ a &= \frac{5}{\left(\frac{5}{18}\right)^{30/55}} \\ &= 10.056.\end{aligned}$$

Thus, $g(t) = 10.056(0.977)^t$.

2. The number of houses remaining for sale drops by half every 70 days, so:

$$\underbrace{\text{Number remaining}}_N = \underbrace{\text{Initial number}}_{N_0} \times \underbrace{\frac{1}{2} \times \frac{1}{2} \cdots \times \frac{1}{2}}_{\text{Numbr of 70-day periods}}.$$

We see that

$$\text{Number of 70-day periods in } t \text{ days} = \frac{t}{70}.$$

For instance, in 140 days there are two 70-day periods. Thus,

$$\begin{aligned}\underbrace{\text{Number remaining}}_N &= \underbrace{\text{Initial number}}_{N_0} \times \underbrace{\frac{1}{2} \times \frac{1}{2} \cdots \times \frac{1}{2}}_{t/70 \text{ times}} \\ N &= N_0 \left(\frac{1}{2}\right)^{t/70}.\end{aligned}$$

Quiz 12

Math E-8: College Algebra

November 19, 2009

1. Solve exactly: $3000(1.047)^t = 5000(1.038)^t$.
2. Many calculators are unable to evaluate $\log 2^{1,000,000}$. Given that $\log 2 = 0.301029996$, approximate this expression's value to the nearest whole number.

Quiz 12 Solutions

Math E-8: College Algebra

November 19, 2009

1. We have:

$$\begin{aligned} 3000(1.047)^t &= 5000(1.038)^t \\ \frac{1.047^t}{1.038^t} &= \frac{5000}{3000} && \text{divide} \\ \left(\frac{1.047}{1.038}\right)^t &= \frac{5}{3} && \text{exponent rule} \\ t \log\left(\frac{1.047}{1.038}\right) &= \log \frac{5}{3} && \text{take logs} \\ t &= \frac{\log(5/3)}{\log(1.047/1.038)}. \end{aligned}$$

The approximate value of t is $t = 59.2$.

2. We have:

$$\begin{aligned} \log 2^{1,000,000} &= 1,000,000 \log 2 \\ &= 1,000,000(0.301029996) \\ &= 301029.996 \\ &= 301030. \end{aligned}$$

Quiz 12 Makeup

Math E-8: College Algebra

December 1, 2009

1. Solve exactly: $25 \cdot 2^{t/7} = 1000$.

2. Which number is larger, 5^{3000} or 3^{5000} ?

Quiz 12 Makeup Solutions

Math E-8: College Algebra

December 1, 2009

1. We have

$$25 \cdot 2^{t/7} = 1000$$

$$2^{t/7} = 40 \quad \text{divide by 25}$$

$$\log(2^{t/7}) = \log 40 \quad \text{take logs}$$

$$\frac{t}{7} \cdot \log 2 = \log 40 \quad \text{log property}$$

$$t = \frac{7 \log 40}{\log 2}. \quad \text{isolate } t$$

The approximate value of t is $t = 37.3$.

2. Many calculators are unable to evaluate these two expressions. However, taking logs, we see that:

$$\log 5^{3000} = 3000 \log 5 = 2096.9$$

$$\log 3^{5000} = 5000 \log 3 = 2385.6.$$

We conclude that 3^{5000} is the larger expression, since its log is larger.

Quiz 13

Math E-8: College Algebra
December 3, 2009

1. Given that $q = \log v$ and $r = \log w$, write the following expression in the form $aq + br$, where a and b are constants: $\log(w^3 \sqrt[5]{v})$.

The functions in Problems 2–3 describe the value of an investment in year t . Write the function in the required form. State the value of any parameters, and say what the parameters tell you about the investment. *Example.* Write $Q = 5000 \cdot 2^{t/15}$ in the form $Q = a(1 + r)^t$. *Solution.* We have $Q = 5000 \left(2^{1/15}\right)^t = 5000(1.04729)^t$, so $a = 5000$ and $r = 0.04729$. This tells us the investment begins at \$5000 and grows by 4.729% each year.

2. $Q = 3000 \left(1 + \frac{3\%}{12}\right)^{12t}$ in the form $Q = ab^t$.

3. $Q = 2500(1.088)^t$ in the form $Q = a \cdot 2^{t/k}$.

Quiz 13 Solutions

Math E-8: College Algebra
December 3, 2009

1. We have:

$$\begin{aligned}\log(w^3 \sqrt[5]{v}) &= \log(w^3) + \log \sqrt[5]{v} && \text{log property} \\ &= 3 \log w + \log(v^{0.2}) && \text{because } \sqrt[5]{v} = v^{1/5} = v^{0.2} \\ &= 3 \log w + 0.2 \log v && \text{log property} \\ &= 0.2q + 3r. && \text{because } q = \log v, r = \log w\end{aligned}$$

This is in the form $aq + br$, where $a = 0.2, b = 3$.

2. We have

$$\begin{aligned}Q &= 3000 \left(1 + \frac{3\%}{12}\right)^{12t} \\ &= 3000 \left((1.0025)^{12}\right)^t \\ &= 3000(1.03042)^t,\end{aligned}$$

so $a = 3000$ and $b = 1.03042$. This tells us the investment begins at \$3000 and grows by a factor of 1.03042 each year, or by 3.042% each year.

3. We need to write

$$Q = 2500(1.088)^t \quad \text{as} \quad Q = a \cdot 2^{t/k}.$$

Letting $a = 2500$, we have

$$\begin{aligned}2500(1.088)^t &= 2500 \cdot 2^{t/k} \\ 1.088^t &= 2^{t/k} && \text{divide by 2500} \\ \log(1.088^t) &= \log(2^{t/k}) && \text{take logs} \\ t \log 1.088 &= \frac{t}{k} \log 2 && \text{log property} \\ k &= \frac{\log 2}{\log 1.088} && \text{solve for } k.\end{aligned}$$

Note that in the last step, we assume $t \neq 0$. An approximate value for k is $k = 8.218$, so we have $Q = 2500 \cdot 2^{t/8.218}$. This tells us the doubling time of the investment is 8.218 years.

Quiz 13 Makeup

Math E-8: College Algebra

December 8, 2009

Providing brief explanations, say whether the expressions in Problems 1–4 are equivalent to

$\log\left(\frac{ab}{\sqrt{c}}\right)$, where a, b, c are positive constants.

1. $\log a + \log b - 0.5 \log c$

2. $\log(ab) + 0.5 \log \frac{1}{c}$

3. $\log b + 0.5 \log \frac{a}{c}$

4. $0.5 (\log(a^2b^2) - \log c)$

Quiz 13 Makeup Solutions

Math E-8: College Algebra

December 8, 2009

1. We have

$$\begin{aligned}\log a + \log b - 0.5 \log c &= (\log a + \log b) - \log (c^{0.5}) \\ &= \log(ab) - \log(\sqrt{c}) \\ &= \log\left(\frac{ab}{\sqrt{c}}\right),\end{aligned}$$

so the expressions are equivalent.

2. We have

$$\begin{aligned}\log(ab) + 0.5 \log \frac{1}{c} &= \log(ab) + \log\left(\left(\frac{1}{c}\right)^{0.5}\right) \\ &= \log(ab) + \log\sqrt{\frac{1}{c}} \\ &= \log\left(ab \cdot \frac{1}{\sqrt{c}}\right) \\ &= \log\left(\frac{ab}{\sqrt{c}}\right),\end{aligned}$$

so the expressions are equivalent.

3. We have

$$\begin{aligned}\log b + 0.5 \log \frac{a}{c} &= \log b + \log\left(\left(\frac{a}{c}\right)^{0.5}\right) \\ &= \log b + \log\left(\sqrt{\frac{a}{c}}\right) \\ &= \log\left(b\sqrt{\frac{a}{c}}\right) \\ &= \log\left(\frac{b\sqrt{a}}{\sqrt{c}}\right),\end{aligned}$$

so the expressions are not equivalent.

4. We have

$$\begin{aligned}0.5 (\log (a^2 b^2) - \log c) &= 0.5 \log \left(\frac{a^2 b^2}{c}\right) \\ &= \log \left(\left(\frac{a^2 b^2}{c}\right)^{0.5}\right) \\ &= \log \left(\frac{\sqrt{a^2 b^2}}{\sqrt{c}}\right) \\ &= \log \left(\frac{ab}{\sqrt{c}}\right),\end{aligned}$$

so the expressions are equivalent.

Quiz 14

Math E-8: College Algebra
December 10, 2009

1. State values of n , a_n , and a_0 for the following polynomial:

$$y = 5x^2(3x^2 + 2x + 1)^2(2x^3 + 2x^2 - 3)^3.$$

Note that you need not write the polynomial in standard form.

2. Write the fourth-degree polynomial P in standard form given that its coefficients a_0, a_1, \dots, a_4 are determined by the formula

$$a_i = (-1)^i \cdot \frac{2i + 1}{3i + 2}.$$

Quiz 14 Solutions

Math E-8: College Algebra

December 10, 2009

1. Focusing on the leading and constant terms of each factor, we have

$$\begin{aligned} y &= 5x^2 (3x^2 + \cdots + 1)^2 (2x^3 + \cdots - 3)^3 \\ &= 5x^2 (3x^2 + \cdots + 1)(3x^2 + \cdots + 1)(2x^3 + \cdots - 3)(2x^3 + \cdots - 3)(2x^3 + \cdots - 3) \\ &= \underbrace{5x^2 \cdot 3x^2 \cdot 3x^2 \cdot 2x^3 \cdot 2x^3 \cdot 2x^3}_{\text{leading terms of each factor}} + \cdots + \underbrace{1 \cdot 1 \cdot (-3)(-3)(-3)}_{\text{constant terms of each factor}} \end{aligned}$$

$$\begin{aligned} \text{so Leading term} &= 5x^2 \cdot 3x^2 \cdot 3x^2 \cdot 2x^3 \cdot 2x^3 \cdot 2x^3 \\ &= 360x^{15} \end{aligned}$$

$$\begin{aligned} \text{and Constant term} &= 1 \cdot 1 \cdot (-3) \cdot (-3) \cdot (-3) \\ &= -27. \end{aligned}$$

Thus, $n = 15$, $a_n = 360$, and $a_0 = -27$.

2. We have

$$\begin{aligned} a_0 &= (-1)^0 \cdot \frac{2 \cdot 0 + 1}{3 \cdot 0 + 2} = \frac{1}{2} \\ a_1 &= (-1)^1 \cdot \frac{3 \cdot 1 + 1}{2 \cdot 1 + 1} = -\frac{3}{5} \\ a_2 &= (-1)^2 \cdot \frac{3 \cdot 2 + 1}{2 \cdot 2 + 1} = \frac{5}{8} \\ a_3 &= (-1)^3 \cdot \frac{3 \cdot 3 + 1}{2 \cdot 3 + 1} = -\frac{7}{11} \\ a_4 &= (-1)^4 \cdot \frac{3 \cdot 4 + 1}{3 \cdot 4 + 2} = \frac{9}{14}, \end{aligned}$$

so

$$P = \frac{1}{2} - \frac{3}{5}x + \frac{5}{8}x^2 - \frac{7}{11}x^3 + \frac{9}{14}x^4.$$

Quiz 14 Makeup

Math E-8: College Algebra

December 15, 2009

Let f be a polynomial with leading term $a_n x^n$ and constant term a_0 , and let g be another polynomial with leading term $b_n x^n$ and constant term b_0 . Assuming $a^n \neq b^n$, find the degree, the leading coefficient, and the constant term of the polynomials in Problems 1–2.

1. $y = f(x)g(x)$

2. $y = f(x) + g(x)$

Quiz 14 Makeup Solutions

Math E-8: College Algebra

December 15, 2009

1. We know that

$$\begin{aligned}f(x) &= a_n x^n + \cdots + \text{lower-powered terms} + a_0 \\g(x) &= b_n x^n + \cdots + \text{lower-powered terms} + b_0 \\ \text{so } y &= f(x)g(x) \\&= (a_n x^n + \cdots + \text{lower-powered terms} + a_0)(b_n x^n + \cdots + \text{lower-powered terms} + b_0) \\&= a_n x^n \cdot b_n x^n + \text{lower-powered terms} + a_0 b_0 \\&= a_n b_n x^{2n} + \text{lower-powered terms} + a_0 b_0.\end{aligned}$$

Thus, the degree is $2n$, the leading coefficient is $a_n b_n$, and the constant term is $a_0 b_0$.

2. We know that

$$\begin{aligned}f(x) &= a_n x^n + \cdots + \text{lower-powered terms} + a_0 \\g(x) &= b_n x^n + \cdots + \text{lower-powered terms} + b_0 \\ \text{so } y &= f(x) + g(x) \\&= a_n x^n + b_n x^n + \text{lower-powered terms} + a_0 + b_0 \\&= (a_n + b_n)x^n + \text{lower-powered terms} + a_0 + b_0.\end{aligned}$$

Thus, the degree is n , the leading coefficient is $a_n + b_n$, and the constant term is $a_0 + b_0$.

Exam 1

Math E-8: College Algebra
October 1, 2009

Rewrite the expressions in Problems 1–4 as specified. Simplify your answers. *Example.* Rewrite $x/2 + b$ without using division. *Solution.* We have $x/2 + b = 0.5x + b$, an expression that does not involve division.

1. $ax + b$ without using addition
2. $\frac{9x + 6b}{3}$ without using division.
3. $(x + b)(x + 2)$ without using parentheses.
4. $3x + 5b$ using only addition.

Evaluate and simplify $g(7)$ given the definitions of g in Problems 5–7.

5. $g(z) = z(z - 4)^2(z^2 - 4) - 4$
6. $g(v) = 2 + \frac{v + 8}{5 - \frac{v + 1}{v - 3}}$
7. $g(r) = \sqrt{r^2 + 24^2} - \sqrt{25^2 - r^2}$

Evaluate and simplify the expressions in Problems 8–10 given that $w(s) = 4 - 7s$.

8. $3w(s - 5)$
9. $w(s^2 - 2) + 2$
10. $4w(7 - 4s) - 7$

Write the functions in Problems 11–12 in the form $y = b + mx$. State the values of m and b . Your answers may involve the constant k .

11. $y = \frac{3x + 5}{k}$
12. $y = 3k(2 - k(t - 5))$
13. Find a formula for f , a linear function, given: $f(-30) = 20$ and $f(20) = -50$.

14. Let $P = g(t)$ give the population of a town in year t . Given that $g(12) = P_0 + 3N$ and $g(16) = P_0 + 4N$, evaluate $g(24)$, assuming that the population grows at a steady rate. Your answer should involve the constants N and P_0 .

15. Let $f(t)$ give the amount (in cubic meters) of water in a tank after t hours. Write an expression involving f for the amount of water in kilograms after n minutes. Note that one cubic meter of water has a mass of 1000 kg.

16. Solve:
$$\begin{cases} 3x + 2y = 5 \\ 2x - y = 8 \end{cases}$$

Answer Problems 17–19 judging from the pattern followed by this list of five expressions:

$$a, a + b, a + 2b + c, a + 3b + 2c, a + 4b + 3c, \dots$$

Here, a , b , and c are constants. You should assume that the pattern continues indefinitely.

17. What is the sixth member of this pattern?
18. What must r and s be if the seventh member of the pattern is $a + rb + sc$?
19. Suppose that $a = 3$, $b = 5$, $c = 2$. Find the numerical value of the twentieth member of this pattern.
20. Say whether these equations are equivalent, and explain your reasoning:

$$\begin{cases} (x - 3)^2 + (y - 4)^2 = 9 \\ (x - 6)^2 + (y - 7)^2 = 9. \end{cases}$$

Use the fact that solutions to the first equation include $x = 6, y = 4$; $x = 3, y = 7$; and $x = 3, y = 1$.

Exam 1 Solutions

Math E-8: College Algebra

October 1, 2009

1. We have $ax + b = ax - (-b)$.

2. We have

$$\begin{aligned}\frac{9x + 6b}{3} &= \frac{1}{3}(9x + 6b) \\ &= 3x + 2b.\end{aligned}$$

3. We have

$$\begin{aligned}(x + b)(x + 2) &= (x + b) \cdot x + (x + b) \cdot 2 \\ &= x \cdot x + b \cdot x + x \cdot 2 + b \cdot 2 \\ &= x^2 + bx + 2x + 2b.\end{aligned}$$

4. We have

$$\begin{aligned}3x + 5b &= \underbrace{x + x + x}_3 + \underbrace{b + b + b + b + b}_5 \\ &= x + x + x + b + b + b + b + b.\end{aligned}$$

5. We have

$$\begin{aligned}g(7) &= 7(7 - 4)^2(7^2 - 4) - 4 \\ &= 7(3)^2(49 - 4) - 4 \\ &= 7(9)(45) - 4 \\ &= 2831.\end{aligned}$$

6. We have:

$$\begin{aligned}g(7) &= 2 + \frac{7 + 8}{5 - \frac{7 + 1}{7 - 3}} \\ &= 2 + \frac{15}{5 - \frac{8}{4}} \\ &= 2 + \frac{15}{5 - 2} \\ &= 2 + 5 = 7.\end{aligned}$$

7. We have:

$$\begin{aligned}g(7) &= \sqrt{7^2 + 24^2} - \sqrt{25^2 - 7^2} \\ &= \sqrt{49 + 576} - \sqrt{625 - 49} \\ &= \sqrt{625} - \sqrt{576} \\ &= 25 - 24 \\ &= 1.\end{aligned}$$

8. We have

$$\begin{aligned} w(s-5) &= 4 - 7(s-5) \quad \text{because } w(s) = 4 - 7s \\ &= 4 - 7s + 35 \\ &= 39 - 7s. \end{aligned}$$

Thus,

$$\begin{aligned} 3w(s-5) &= 3 \underbrace{(39 - 7s)}_{w(s-5)} \\ &= 117 - 21s. \end{aligned}$$

9. We have

$$\begin{aligned} w(s^2 - 2) &= 4 - 7(s^2 - 2) \quad \text{because } w(s) = 4 - 7s \\ &= 4 - 7s^2 + 14 \\ &= 18 - 7s^2. \end{aligned}$$

Thus,

$$\begin{aligned} w(s^2 - 2) + 2 &= 18 - 7s^2 + 2 \quad \text{because } w(s^2 - 2) = 18 - 7s^2 \\ &= 20 - 7s^2. \end{aligned}$$

10. We have

$$\begin{aligned} w(7 - 4s) &= 4 - 7(7 - 4s) \quad \text{because } w(s) = 4 - 7s \\ &= 4 - (49 - 28s) \\ &= 4 - 49 + 28s \\ &= -45 + 28s. \end{aligned}$$

Thus,

$$\begin{aligned} 4w(7 - 4s) - 7 &= 4(-45 + 28s) - 7 \quad \text{because } w(7 - 4s) = -45 + 28s \\ &= -180 + 112s - 7 \\ &= -187 + 112s. \end{aligned}$$

11. We have

$$\begin{aligned} y &= \frac{3x + 5}{k} \\ &= \frac{3x}{k} + \frac{5}{k} \quad \text{split numerator} \\ &= \underbrace{\frac{5}{k}}_b + \underbrace{\frac{3}{k}}_m \cdot x, \end{aligned}$$

so $b = 5/k$ and $m = 3/k$.

12. We have:

$$\begin{aligned} y &= 3k(2 - k(t - 5)) \\ &= 3k \cdot 2 - 3k \cdot k(t - 5) \quad \text{distribute} \\ &= 6k - 3k^2(t - 5) \\ &= 6k - (3k^2 \cdot t - 3k^2 \cdot 5) \end{aligned}$$

$$\begin{aligned}
&= 6k - (3k^2t - 15k^2) \\
&= 6k - 3k^2t + 15k^2 \\
&= \underbrace{6k + 15k^2}_b + \underbrace{(-3k^2)}_m t,
\end{aligned}$$

so $b = 6k + 15k^2$ and $m = -3k^2$.

13. • The graph of f is a line containing the points $(-30, 20)$ and $(20, -50)$.
• The slope is given by

$$\begin{aligned}
m &= \frac{\Delta y}{\Delta x} \\
&= \frac{-50 - 20}{20 - (-30)} \quad \text{using the points } (-30, 20), (20, -50) \\
&= \frac{-70}{50} \\
&= -1.4.
\end{aligned}$$

- Using the point-slope formula and the point $(x_0, y_0) = (20, -50)$, we have:

$$\begin{aligned}
f(x) &= y_0 + m(x - x_0) \\
&= -50 - 1.4(x - 20) \quad \text{since } m = -1.4, x_0 = 20, y_0 = -50 \\
&= -50 - 1.4x + 28 \\
&= -22 - 1.4x.
\end{aligned}$$

- We can verify our answer using the fact that $f(-30) = 20$:

$$\begin{aligned}
f(x) &= -22 - 1.4x. \\
f(-30) &= -22 - 1.4(-30) \quad \text{let } x = -30 \\
&= -22 + 42 = 20,
\end{aligned}$$

as required.

14. The population grows at a steady rate, so g is a linear function. We see that between year 12 and year 16, the population grew by N people, which means it grows by $N/4$ people every year. Thus, between year 16 and year 20, it should grow by N people, from $P_0 + 4N$ to $P_0 + 5N$. Likewise, between year 20 and year 24 it should grow by N more people, from $P_0 + 5N$ to $P_0 + 6N$. We conclude that $g(24) = P_0 + 6N$.

Another approach to solving this problem is to find the slope:

$$\begin{aligned}
m &= \frac{\Delta P}{\Delta t} \\
&= \frac{g(16) - g(12)}{16 - 12} \\
&= \frac{\overbrace{P_0 + 4N}^{g(16)} - \overbrace{(P_0 + 3N)}^{g(12)}}{4} \\
&= \frac{P_0 + 4N - P_0 - 3N}{4} \\
&= \frac{N}{4} = 0.25N.
\end{aligned}$$

Solving for the starting value b gives:

$$\begin{aligned}
g(t) &= b + mt \\
&= b + 0.25Nt \quad \text{because } m = 0.25N \\
g(12) &= b + 0.25N(12) \quad \text{let } t = 12
\end{aligned}$$

$$\underbrace{P_0 + 3N}_{g(12)} = b + 3N \quad \text{because } g(12) = P_0 + 3N,$$

$$b = P_0.$$

Thus, a formula for this function is $g(t) = P_0 + 0.25t$. This gives

$$\begin{aligned} g(24) &= P_0 + 0.25N(24) \\ &= P_0 + 6N, \end{aligned}$$

which is the same answer that we got before.

15. • There are 60 minutes in an hour, so n minutes equals $n/60$ hours. For instance, $n = 180$ minutes is the same as $t = 180/60 = 3$ hours.
- There are 1000 kg of water in one cubic meter, so W cubic meters of water equals $1000W$ kg of water. For instance, $W = 5$ cubic meters has a mass of $1000W = 5000$ kg.
- We conclude that $f(n/60)$ gives the amount of water in cubic meters after n minutes, and that $1000f(n/60)$ gives the amount of water in kg after n minutes.
16. One approach is to multiply the second equation by 2:

$$\begin{aligned} 2(2x - y) &= 2 \cdot 8 \\ 4x - 2y &= 16. \end{aligned}$$

Now add the resulting equation to the original first equation:

$$\begin{aligned} 3x + 2y + (4x - 2y) &= 5 + 16 \\ 7x &= 21 \\ x &= 3. \end{aligned}$$

Having solved for x , we solve for y using either of the original equations:

$$\begin{aligned} 3x + 2y &= 5 && \text{original equation} \\ 3 \cdot 3 + 2y &= 5 && \text{because } x = 3 \\ 9 + 2y &= 5 \\ 2y &= -4 \\ y &= -2. \end{aligned}$$

We check our answer in the other equation:

$$\begin{aligned} 2x - y &= 8 && \text{original equation} \\ 2 \cdot 3 - (-2) &= 8 && \text{because } x = 3, y = -2 \\ 6 + 2 &= 8, \end{aligned}$$

which is what we wanted.

17. Judging from the pattern, each member after the second term adds an additional b and an additional c to the previous member. Thus,

$$\begin{aligned} \text{Sixth member} &= \underbrace{\text{Fifth member}}_{a+4b+3c} + b + c \\ &= a + 4b + 3c + b + c \\ &= a + 5b + 4c. \end{aligned}$$

18. Judging from the pattern, each member after the second term adds an additional b and an additional c to the previous member. Thus,

$$\begin{aligned}
 \text{Sixth member} &= \underbrace{\text{Fifth member}}_{a+4b+3c} + b + c \\
 &= a + 4b + 3c + b + c \\
 &= a + 5b + 4c \\
 \text{Seventh member} &= \underbrace{\text{Sixth member}}_{a+5b+4c} + b + c \\
 &= a + 5b + 4c + b + c \\
 &= a + 6b + 5c.
 \end{aligned}$$

In order for $a + 6b + 5c$ to equal $a + rb + sc$, we have $r = 6, s = 5$.

19. We see that

$$\begin{aligned}
 \text{Member 3} &= a + 2b + 1 \cdot c \\
 \text{Member 4} &= a + 3b + 2c \\
 \text{Member 5} &= a + 4b + 3c \\
 &\vdots \\
 \text{Member 20} &= a + 19b + 18c.
 \end{aligned}$$

We see there are 2 cs and 3 bs for member 4; 3 cs and 4 bs for member 5; and so on. We surmise there are 18 cs and 19 bs for member 20, so $a + 19b + 18c$ is the twentieth member. Letting $a = 3, b = 5, c = 2$, we have

$$\begin{aligned}
 \text{Value of twentieth member} &= a + 19b + 18c \\
 &= 3 + 19 \cdot 5 + 18 \cdot 2 \\
 &= 134.
 \end{aligned}$$

20. We can verify that $x = 3, y = 1$ is a solution to the first equation:

$$\begin{aligned}
 \text{the following } (x-3)^2 + (y-4)^2 &= 9 \quad \text{first equation} \\
 (3-3)^2 + (1-4)^2 &= 9 \quad \text{let } x = 3, y = 1 \\
 0^2 + (-3)^2 &= 9 \quad \text{a true statement.}
 \end{aligned}$$

Since letting $x = 3$ and $y = 1$ makes the first equation true, this is a solution to the first equation. We can confirm the other two solutions using the same approach.

To decide if these equations are equivalent, we need to see if solutions to the first equation are also solutions to the second equation. As you can check for yourself, both $x = 6, y = 4$, and $x = 3, y = 7$ work for both equations. However, although $x = 3, y = 1$ is a solution for the first equation, it isn't for the second:

$$\begin{aligned}
 (x-6)^2 + (y-7)^2 &= 9 \quad \text{second equation} \\
 (3-6)^2 + (1-7)^2 &= 9 \quad \text{let } x = 3, y = 1 \\
 (-3)^2 + (-6)^2 &= 9 \quad \text{simplify} \\
 9 + 36 &= 9. \quad \text{not a true statement}
 \end{aligned}$$

Since the values $x = 3, y = 1$ make the first equation but not the second equation, this is not a solution to both equations. Therefore, the equations are not equivalent.

Exam 2

Math E-8: College Algebra
November 5, 2009

1. Find the domain of $y = \frac{3}{2 - \sqrt{x-1}}$.
 2. Find the range of $y = \left(2 + (3 + x^2)^2\right)^2$.
 3. Given $f(x) = 2x - 1$, $g(x) = \sqrt{x+1}$, find $f(g(x))$ and $g(f(x))$.
 4. Find $q(z)$ given $p(x) = 3x - 2$ and $p(q(z)) = \frac{6}{2z-1} - 2$.
 5. Evaluate $w(5)$ and solve $w(t) = 5$ given that $w(t) = 2t^2 + 1$.
 6. Evaluate $r(17)$ and $r^{-1}(10)$ given $r(x) = 20 - 2\sqrt{x-1}$.
 7. Find $p^{-1}(x)$ given that $p(x) = \frac{5 - \sqrt{x}}{3 + 2\sqrt{x}}$.
 8. Write the quadratic function in standard form and state the values of a, b, c :
 $Q = 3(4t - 5t(3t - 7(2t - 3))) - 6t - 2$.
 9. Find a possible equation for an upward-opening parabola with vertex $(3, 7)$ and y -intercept $y = 20$.
- Find a point on the graphs of the functions in Problems 10–11 given that (p, q) is a point on the graph of $y = f(x)$. *Example.* If $g(x) = f(x) + 4$, then the graph of g is the graph of f shifted up 4 units. Since (p, q) is a point on the graph of f , we see that $(p, q + 4)$ must be a point on the graph of g .
10. $v(x) = -4f(-x)$
 11. $w(x) = f(x - 3) - 8$
 12. Write in factored form and state the values of a, r, s : $y = 2(3x - 6)(4x + 12)$.
 13. What must be true about r and s if the graphs of the following quadratic functions have the same vertex?
$$f(x) = 5(x - 3r)^2 + r - 8$$
$$g(x) = -2(x + 12)^2 - 3s$$
 14. Put into vertex form and state the values of a, h, k : $y = 3x^2 - 48x + 20$.
 15. A quadratic function f has zeros at $x = 2, x = -3$. Does this mean $f(x) = (x - 2)(x + 3)$? Explain your reasoning.
 16. Solve for v : $2v^2 - 3v - 6 = 0$
 17. Describe the graph of the quadratic function $y = h(f(g(x)))$ given that
$$f(x) = 2(x - 3)^2 + 5$$
$$g(x) = x - 3$$
$$h(x) = x + 2.$$

State the vertex and y -intercept.
 18. Given $g(t) = 3t^2 + 5t + 3$, (a) evaluate $g(7)$, and (b) solve $g(t) = 7$.
 19. Letting $f(x) = 3x + 1$ and $g(x) = x - 3$, solve: $f(x)g(x) = f(g(x))$.
 20. Say what must be true about r in order for both of the following equations to have exactly one solution:
$$2x^2 + rx + r = 0$$
$$4x^2 + rx + 4 = 0.$$

Exam 2 Solutions

Math E-8: College Algebra

November 5, 2009

1.
 - We see that $x - 1 \geq 0$ or otherwise the square root will be undefined. Thus, $x \geq 1$.
 - We see that $2 - \sqrt{x - 1} \neq 0$, or otherwise the denominator will be 0. Thus

$$\begin{aligned}2 - \sqrt{x - 1} &\neq 0 \\2 &\neq \sqrt{x - 1} \\x - 1 &\neq 4 \\x &\neq 5.\end{aligned}$$

Putting this together, we see that the domain is $x \geq 1, x \neq 5$.

2. Since $x^2 \geq 0$, we see that

$$\begin{aligned}y &= \left(2 + (3 + x^2)^2\right)^2 \\&= \left(2 + (3 + \text{A nonnegative number})^2\right)^2 \\&= \left(2 + (\text{A number no smaller than } 3)^2\right)^2 \\&= \left(2 + (\text{A number no smaller than } 9)\right)^2 \\&= (\text{A number no smaller than } 11)^2 \\&= \text{A number no smaller than } 121,\end{aligned}$$

so the range is $y \geq 121$.

3. We have

$$\begin{aligned}f(g(x)) &= f(\sqrt{x+1}) \\&= 2\sqrt{x+1} - 1 \quad \text{cannot be further simplified}\end{aligned}$$

and

$$\begin{aligned}g(f(x)) &= g(2x-1) \\&= \sqrt{(2x-1)+1} \\&= \sqrt{2x}.\end{aligned}$$

4. We know that $p(x) = 3x - 2$, so

$$p(q(z)) = 3q(z) - 2.$$

Thus, we have:

$$\begin{aligned}p(q(z)) &= \frac{6}{2z-1} - 2 \quad \text{given} \\3q(z) - 2 &= \frac{6}{2z-1} - 2 \\3q(z) &= \frac{6}{2z-1} \\q(z) &= \frac{1}{3} \cdot \frac{6}{2z-1} \\&= \frac{2}{2z-1}.\end{aligned}$$

5. Evaluating $w(5)$, we have:

$$\begin{aligned}w(5) &= 2(5)^2 + 1 \quad \text{because } w(t) = 2t^2 + 1 \\&= 2 \cdot 25 + 1 \\&= 51.\end{aligned}$$

Solving $w(t) = 5$, we have:

$$\begin{aligned} w(t) &= 5 \\ 2t^2 + 1 &= 5 && \text{because } w(t) = 2t^2 + 1 \\ 2t^2 &= 4 \\ t^2 &= 2 \\ t &= \pm\sqrt{2}. \end{aligned}$$

6. We have

$$\begin{aligned} r(17) &= 20 - 2\sqrt{17 - 1} \\ &= 20 - 2\sqrt{16} \\ &= 20 - 2 \cdot 4 \\ &= 12. \end{aligned}$$

To evaluate $r^{-1}(10)$, we need an input value that gives an output value of 10:

$$\begin{aligned} r(x) &= 10 && \text{solve for } x \\ 20 - 2\sqrt{x - 1} &= 10 \\ -2\sqrt{x - 1} &= -10 \\ \sqrt{x - 1} &= 5 \\ x - 1 &= 25 \\ x &= 26, \end{aligned}$$

so $r^{-1}(10) = 26$.

7. Letting $y = p(x)$, we know that $x = p^{-1}(y)$. Solving for x , we obtain

$$\begin{aligned} y &= \frac{5 - \sqrt{x}}{3 + 2\sqrt{x}} \\ y(3 + 2\sqrt{x}) &= 5 - \sqrt{x} && \text{multiply by denominator} \\ 3y + 2y\sqrt{x} &= 5 - \sqrt{x} && \text{distribute} \\ 3y - 5 &= -2y\sqrt{x} - \sqrt{x} && \text{collect } x\text{-terms} \\ 3y - 5 &= -\sqrt{x}(2y + 1) && \text{factor} \\ -\sqrt{x} &= \frac{3y - 5}{2y + 1} && \text{divide} \\ x &= \left(\frac{3y - 5}{2y + 1}\right)^2 && \text{square both sides} \\ \text{so } p^{-1}(x) &= \left(\frac{3x - 5}{2x + 1}\right)^2. \end{aligned}$$

8. We have

$$\begin{aligned} Q &= 3(4t - 5t(3t - 7(2t - 3))) - 6t - 2 \\ &= 3(4t - 5t(3t - (14t - 21))) - 6t - 2 \\ &= 3(4t - 5t(3t - 14t + 21)) - 6t - 2 \\ &= 3(4t - 5t(-11t + 21)) - 6t - 2 \\ &= 3(4t - (-55t^2 + 105t)) - 6t - 2 \\ &= 3(4t + 55t^2 - 105t) - 6t - 2 \\ &= 3(55t^2 - 101t) - 6t - 2 \end{aligned}$$

$$\begin{aligned}
&= 165t^2 - 303t - 6t - 2 \\
&= 165t^2 - 309t - 2,
\end{aligned}$$

so $a = 165, b = -309, c = -2$.

9. • The vertex is $(h, k) = (3, 7)$, so $y = a(x - 3)^2 + 7$.
 • The y -intercept is 20, which means $y = 20$ at $x = 0$. Solving for a gives

$$\begin{aligned}
a(0 - 3)^2 + 7 &= 20 & y = 20 \text{ at } x = 0 \\
a \cdot 9 &= 13 \\
a &= \frac{13}{9}.
\end{aligned}$$

Thus, $y = (13/9)(x - 3)^2 + 7$.

10. The graph of v is the graph of f flipped horizontally across the y -axis, then flipped vertically across the x axis, then stretched vertically by a factor of 4. Since (p, q) is a point on the graph of f , we see that $(-p, -4q)$ must be a point on the graph of v .
 11. The graph of w is the graph of f shifted to the right by 3 units and down by 8 units. Since (p, q) is a point on the graph of f , we see that $(p + 3, q - 8)$ must be a point on the graph of w .
 12. We have

$$\begin{aligned}
y &= 2(3x - 6)(4x + 12) \\
&= 2(3(x - 2))(4(x + 3)) \\
&= 2 \cdot 3 \cdot 4(x - 2)(x + 3) \\
&= 24(x - 2)(x - (-3)),
\end{aligned}$$

so $a = 24, r = 2, s = -3$.

13. Writing these in vertex form, we have:

$$\begin{aligned}
f(x) &= 5(x - 3r)^2 + r - 8 & \text{so } (h, k) &= (3r, r - 8) \\
g(x) &= -2(x - (-12))^2 + (-3s) & (h, k) &= (-12, -3s).
\end{aligned}$$

Since the graphs of these functions have the same vertex, we see that the h -values are the same, giving:

$$\begin{aligned}
3r &= -12 \\
r &= -4.
\end{aligned}$$

Likewise, the k -values are the same, giving:

$$\begin{aligned}
-3s &= r - 8 \\
-3s &= -4 - 8 & \text{because } r &= -4 \\
-3s &= -12 \\
s &= 4.
\end{aligned}$$

Thus, $r = -4$ and $s = 4$.

14. We have

$$\begin{aligned}
y &= 3x^2 - 48x + 20 \\
y - 20 &= 3x^2 - 48x \\
\frac{y - 20}{3} &= x^2 - 16x \\
\frac{y - 20}{3} + (-8)^2 &= x^2 - 16x + (-8)^2
\end{aligned}$$

$$\begin{aligned}
\frac{y-20}{3} + 64 &= (x-8)^2 \\
\frac{y-20}{3} &= (x-8)^2 - 64 \\
y-20 &= 3(x-8)^2 - 192 \\
y &= 3(x-8)^2 + (-172).
\end{aligned}$$

Thus, $a = 3, h = 8, k = -172$.

15. Not necessarily. For instance, the following functions all have zeros at $x = 2, x = -3$:

$$\begin{aligned}
y &= 2(x-2)(x+3) \\
y &= -5(x-2)(x+3) \\
y &= 0.1(x-2)(x+3).
\end{aligned}$$

16. Here, $a = 2, b = -3, c = -6$, so the discriminant is

$$\begin{aligned}
b^2 - 4ac &= (-3)^2 - 4(2)(-6) \\
&= 57.
\end{aligned}$$

Since the discriminant is positive, there are two solutions:

$$\begin{aligned}
v &= \frac{-b \pm \sqrt{57}}{2a} \\
&= \frac{-(-3) \pm \sqrt{57}}{2(2)} \\
&= \frac{3 \pm \sqrt{57}}{4}.
\end{aligned}$$

17. We see that

$$\begin{aligned}
f(g(x)) &= f(x-3) \\
&= 2((x-3)-3)^2 + 5 \\
&= 2(x-6)^2 + 5,
\end{aligned}$$

and that $h(f(g(x))) = h(2(x-6)^2 + 5)$

$$\begin{aligned}
&= (2(x-6)^2 + 5) + 2 \\
&= 2(x-6)^2 + 7.
\end{aligned}$$

Thus, the graph of this function is an upward-opening parabola with vertex $(h, k) = (6, 7)$ and y -intercept

$$\begin{aligned}
y &= 2(0-6)^2 + 7 \\
&= 2 \cdot 36 + 7 \\
&= 79.
\end{aligned}$$

18. (a) We have:

$$\begin{aligned}
g(7) &= 3 \cdot 7^2 + 5 \cdot 7 + 3 \\
&= 3 \cdot 49 + 35 + 3 \\
&= 185.
\end{aligned}$$

(b) We have:

$$\begin{aligned}
g(t) &= 7 \\
3t^2 + 5t + 3 &= 7
\end{aligned}$$

$$3t^2 + 5t - 4 = 0.$$

Using the quadratic formula with $a = 3, b = 5, c = -4$, we see that

$$\begin{aligned} t &= \frac{-5 \pm \sqrt{5^2 - 4(3)(-4)}}{2(3)} \\ &= \frac{-5 \pm \sqrt{25 + 48}}{6} \\ &= \frac{-5 \pm \sqrt{73}}{6}. \end{aligned}$$

Approximate solutions are $t = -2.257, 0.591$.

19. We have:

$$\begin{aligned} f(x)g(x) &= (3x + 1)(x - 3) = 3x^2 - 8x - 3 \\ f(g(x)) &= 3(x - 3) + 1 = 3x - 8. \end{aligned}$$

This means we can write:

$$\begin{aligned} f(x)g(x) &= f(g(x)) \\ 3x^2 - 8x - 3 &= 3x - 8 \\ 3x^2 - 11x + 5 &= 0. \end{aligned}$$

Using the quadratic formula with $a = 3, b = -11, c = 5$, we have:

$$\begin{aligned} x &= \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(5)}}{2(3)} \\ &= \frac{11 \pm \sqrt{121 - 60}}{6} \\ &= \frac{11 \pm \sqrt{61}}{6}. \end{aligned}$$

Approximate values for x are $x = 0.5316$ and $x = 3.135$.

20. A quadratic equation has exactly one solution provided the discriminant equals zero. For the first equation, this means

$$\begin{aligned} r^2 - 4(2)(r) &= 0 \\ r^2 - 8r &= 0 \\ r(r - 8) &= 0 \\ r &= 0, 8. \end{aligned}$$

For the second equation, this means

$$\begin{aligned} r^2 - 4(4)(4) &= 0 \\ r^2 &= 64 \\ r &= \pm 8. \end{aligned}$$

Thus, if $r = 8$, both equations have exactly one solution. To verify our answer, we see from the first equation that:

$$\begin{aligned} 2x^2 + 8x + 16 &= 0 \\ x^2 + 4x + 4 &= 0 \\ (x + 2)^2 &= 0 \\ x &= -2. \end{aligned}$$

From the second equation, we see that

$$\begin{aligned}4x^2 + 8x + 4 &= 0 \\x^2 + 2x + 1 &= 0 \\(x + 1)^2 &= 0 \\x &= -1.\end{aligned}$$

Practice Final

Math E-8: College Algebra
December 10, 2009

1. Letting $r = -2$ and $s = 5$, evaluate and simplify: $\frac{r - s - rs}{s - r - r(s - r)}$.

Answer Problems 2–3 exactly given that

$$f(t) = \frac{3t^2 + 1}{t + 2}.$$

2. Evaluate $f(-3)$. 3. Solve $f(t) = 5$.

Answer Problems 4–5.

4. Solve for q : $p\sqrt{q} + np = k$. 5. Solve for p : $p\sqrt{q} + np = k$.

6. Evaluate $g(2t + 1)$ if $g(t) = 4t - 2$.

7. Solve $w(2v + 7) = 3w(v - 1)$ if $w(v) = 3v + 2$.

Identify the family of each function in Problems 8–9, writing it in standard form and stating the values of all constants.

8. $f(t) = 4^{t+1} \cdot 5^{2t-1}$ 9. $g(t) = \frac{5}{3\sqrt[3]{t^2}}$

10. Let $f(r)$ give the number of people living within r miles of a city's center. Given that 1 km equals 0.62 miles, and assuming an average of 4 people per household, write an expression in terms of f for the number of households within R kilometers of the city's center.

11. Find a formula for the linear function w given that $w(0.3) = 0.07$ and $w(0.7) = 0.01$.

12. Find the exponential function f given that $f(19) = 47.2$ and $f(42) = 23.3$.

13. Find a formula for the exponential function $v(t)$ given that $v(10) = 200$ and that v has a half-life of 5 days.

14. An investment initially worth \$2550 earns 8.7% annual interest. When will the investment be worth \$3900?

15. A population starts at 6200 and grows by 3.1% per year. Another population starts at 4000 and doubles every 12 years. When are the two populations equal in size?

16. Solve:

$$\begin{cases} 3x + 4y + z = -1 \\ 2x - 3y - z = 8 \\ 5x - 2y = 16. \end{cases}$$

Hint: Cancel z from the first two equations.

17. Find the vertex and x - and y -intercepts of the graph of $y = 12x - 2x^2 - 8$.

18. Find a possible formula for $w(t)$ given

$$v(t) = \frac{t - 1}{2t - 3} \quad \text{and} \quad v(w(t)) = \frac{2t^2}{4t^2 - 1}.$$

19. Find a formula for q in terms of p given that the graph of q is the graph of p shifted right 2 units, then flipped vertically, then stretched vertically by a factor of 3, then shifted up 4 units.

20. Squaring both sides of Equation 1 yields Equation 2:

$$x = \sqrt{2x + 3} \quad (1)$$

$$x^2 = 2x + 3. \quad (2)$$

As you can check for yourself, $x = 3$ is a solution to both equations. Does this mean these equations are equivalent? Explain your reasoning.

- 21.** Rewrite this expression using only one log operation:

$$2 \log P - 3 \log Q + \log (2R) .$$

Answer Problems 22–23. **Hint:** Use what you know about the graphs of quadratic functions.

- 22.** Find the domain of g given that

$$g(x) = \sqrt{(x-4)(x+6)}.$$

- 23.** Find the range of h given that

$$h(x) = \sqrt{25 + (x-3)^2}.$$

- 24.** One of the following quadratic functions has no zeros, one has one zero, and one has two zeros:

$$f(x) = x^2 + nx + n^2$$

$$g(x) = x^2 - 2nx + n^2$$

$$h(x) = n^2x^2 - 1$$

Given that n is a positive constant, which function is which?

- 25.** Find a value for the constant j making

$$y = (jx - 4)(x - j) - (2x - 1)(3x - 5)$$

a linear function, and state its slope and y -intercept.

Practice Final Solutions

Math E-8: College Algebra

December 10, 2009

1. We have

$$\begin{aligned}\frac{r-s-rs}{s-r-r(s-r)} &= \frac{-2-5-(-2)5}{5-(-2)-(-2)(5-(-2))} \\ &= \frac{-7+10}{7+2(7)} \\ &= \frac{3}{21} = \frac{1}{7}.\end{aligned}$$

2. We have

$$\begin{aligned}f(-3) &= \frac{3(-3)^2+1}{(-3)+2} \\ &= \frac{27+1}{-1} \\ &= -28.\end{aligned}$$

3. We have

$$\begin{aligned}\frac{3t^2+1}{t+2} &= 5 \\ 3t^2+1 &= 5(t+2) && \text{Clearing denominator} \\ 3t^2+1 &= 5t+10 \\ 3t^2-5t-9 &= 0 && \text{Collecting like terms}\end{aligned}$$

We can solve this using the quadratic formula with $a = 3, b = -5, c = -9$:

$$\begin{aligned}t &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 3(-9)}}{2 \cdot 3} \\ &= \frac{5 \pm \sqrt{133}}{6}.\end{aligned}$$

Approximate values for t are $t = -1.0888, 2.755$.

4. We have

$$\begin{aligned}p\sqrt{q} + np &= k \\ p\sqrt{q} &= k - np && \text{subtract} \\ \sqrt{q} &= \frac{k - np}{p} && \text{divide} \\ q &= \left(\frac{k - np}{p}\right)^2 && \text{square both sides}\end{aligned}$$

5. We have

$$\begin{aligned}p\sqrt{q} + np &= k \\ p(\sqrt{q} + n) &= k && \text{factor} \\ p &= \frac{k}{\sqrt{q} + n} && \text{divide}\end{aligned}$$

6. We have

$$\begin{aligned} g(t) &= 4t - 2 \\ \text{so } g(2t + 1) &= 4(2t + 1) - 2 && \text{substitute } 2t + 1 \text{ for } t \\ &= 8t + 2 && \text{simplify.} \end{aligned}$$

7. We have

$$\begin{aligned} w(2v + 7) &= 3(2v + 7) + 2 && \text{because } w(v) = 3v + 2 \\ &= 6v + 23 \\ w(v - 1) &= 3(v - 1) + 2 && \text{because } w(v) = 3v + 2 \\ &= 3v - 1 \\ \text{so } 3w(v - 1) &= 3(3v - 1) \\ &= 9v - 3. \end{aligned}$$

Solving gives

$$\begin{aligned} w(2v + 7) &= 3w(v - 1) \\ 6v + 23 &= 9v - 3 \\ -3v &= -26 \\ v &= \frac{-26}{-3} = \frac{26}{3}. \end{aligned}$$

Checking our answer, we see that

$$\begin{aligned} 2v + 7 &= 2 \cdot \frac{26}{3} + 7 \\ &= \frac{52}{3} + \frac{21}{3} && \text{find common denominator} \\ &= \frac{73}{3} \\ \text{so } w(2v + 7) &= w\left(\frac{73}{3}\right) \\ &= 3 \cdot \frac{73}{3} + 2 && \text{because } w(v) = 3v + 2 \\ &= 75, \\ \text{and } v - 1 &= \frac{26}{3} - \frac{3}{3} && \text{find common denominator} \\ &= \frac{23}{3} \\ \text{so } 3w(v - 1) &= 3w\left(\frac{23}{3}\right) \\ &= 3 \left(3 \cdot \frac{23}{3} + 2 \right) && \text{because } w(v) = 3v + 2 \\ &= 3 \cdot 25 \\ &= 75 && \text{as required.} \end{aligned}$$

8. We have

$$\begin{aligned} f(t) &= 4^{t+1} \cdot 5^{2t-1} \\ &= 4^t \cdot 4^1 \cdot 5^{2t} \cdot 5^{-1} \end{aligned}$$

$$= \frac{4}{5} \cdot 4^t \cdot 25^t$$

$$= \frac{4}{5} \cdot 100^t.$$

This is an exponential function in standard form $f(t) = ab^t$ with $a = 4/5, b = 100$.

9. We have

$$\begin{aligned} g(t) &= \frac{5}{3\sqrt[3]{t^2}} \\ &= \frac{5}{3} \cdot \frac{1}{(t^2)^{1/3}} \\ &= \frac{5}{3} \cdot \frac{1}{t^{2/3}} \\ &= \frac{5}{3} \cdot t^{-2/3}. \end{aligned}$$

This is a power function $g(t) = kx^p$ with $k = 5/3$ and $p = -2/3$.

10. We have:

$$\begin{aligned} \underbrace{\text{Number of miles}}_r &= 0.62 \times \underbrace{\text{Number of kilometers}}_R \\ \text{so } \text{Number of people} &= f\left(\frac{\text{Number of miles}}{\text{in } R \text{ km}}\right) \\ \text{within } R \text{ km} &= f(0.62R) \\ \text{so } \text{Number of households} &= \frac{1}{4} \times \text{Number of people} \\ &= 0.25f(0.62R). \end{aligned}$$

11. We have $w(x) = b + mx$ where

$$\begin{aligned} m &= \frac{w(0.7) - w(0.3)}{0.7 - 0.3} \\ &= \frac{0.01 - 0.07}{0.4} \\ &= -0.15. \end{aligned}$$

We can use the point $(0.3, 0.07)$ to solve for b :

$$\begin{aligned} w(0.3) &= b + m(0.3) \\ 0.07 &= b + (-0.15)(0.3) \\ b &= 0.115. \end{aligned}$$

Thus, $w(x) = 0.115 - 0.15x$.

12. We have

$$\begin{aligned} f(19) &= ab^{19} = 47.2 \\ f(42) &= ab^{42} = 23.3 \\ \frac{f(42)}{f(19)} &= \frac{ab^{42}}{ab^{19}} = \frac{23.3}{47.2} && \text{take ratios} \\ \text{so } b^{23} &= \frac{23.3}{47.2} \\ b &= \left(\frac{23.3}{47.2}\right)^{1/23} \\ &= 0.9698. \end{aligned}$$

Solving for a , we have

$$\begin{aligned} a(0.9698)^{19} &= 47.2 && \text{because } f(19) = 47.2 \\ a &= \frac{47.2}{(0.9698)^{19}} \\ &= 84.569, \\ \text{so } f(t) &= 84.569(0.9698)^t. \end{aligned}$$

Checking our answer, we see that

$$\begin{aligned} f(42) &= 84.569(0.9698)^{42} \\ &= 23, && \text{as required.} \end{aligned}$$

13. Since the half-life is 5 days, we have $v(t) = a(0.5)^{t/5}$. We know that

$$\begin{aligned} a(0.5)^{10/5} &= 200 && \text{Because } v(10) = 200 \\ a(0.5)^2 &= 200 \\ 0.25a &= 200 \\ a &= 800, \end{aligned}$$

$$\text{so } v(t) = 800(0.5)^{t/5}.$$

14. We have

$$\begin{aligned} 2550(1.087)^t &= 3900 \\ 1.087^t &= \frac{3900}{2550} \\ t \log 1.087 &= \log \frac{3900}{2550} \\ t &= \frac{\log \frac{3900}{2550}}{\log 1.087}, \end{aligned}$$

or about 5.0932 years.

15. We have

$$\begin{aligned} \text{First population} &= 6200(1.031)^t && \text{growth rate is 3.1\%} \\ \text{Second population} &= 4000 \cdot 2^{t/12} && \text{doubling time is 12.} \end{aligned}$$

Solving gives

$$\begin{aligned} 4000 \cdot 2^{t/12} &= 6200(1.031)^t \\ 4000 \left(2^{1/12}\right)^t &= 6200(1.031)^t && \text{exponent rule} \\ \frac{\left(2^{1/12}\right)^t}{1.031^t} &= \frac{6200}{4000} && \text{divide} \\ \left(\frac{2^{1/12}}{1.031}\right)^t &= 1.55 && \text{exponent rule} \\ \log \left(\frac{2^{1/12}}{1.031}\right)^t &= \log 1.55 && \text{take logs} \\ t \log \left(\frac{2^{1/12}}{1.031}\right) &= \log 1.55 && \text{log rule} \\ t &= \frac{\log 1.55}{\log \left(\frac{2^{1/12}}{1.031}\right)} && \text{divide} \\ &= 16.093. \end{aligned}$$

16. Taking the hint, we add the first two equations to obtain

$$\begin{aligned} 3x + 4y + z + (2x - 3y - z) &= -1 + 8 \\ 5x + y &= 7. \end{aligned}$$

Together with the third equation, we now have the system

$$\begin{cases} 5x + y = 7 \\ 5x - 2y = 16. \end{cases}$$

Doubling the first of these two equations, then adding them, gives

$$\begin{aligned} 2(5x + y) + 5x - 2y &= 2 \cdot 7 + 16 \\ 10x + 2y + 5x - 2y &= 30 \\ 15x &= 30 \\ x &= 2 \\ \text{so } 5 \cdot 2 + y &= 7 && \text{use } 5x + y = 7 \\ y &= -3 \\ \text{so } 3 \cdot 2 + 4(-3) + z &= -1 && \text{use } 3x + 4y + z = -1 \\ 6 - 12 + z &= -1 \\ z &= 5. \end{aligned}$$

Thus, $x = 2$, $y = -3$, $z = 5$. Checking our answer, we have

$$\begin{aligned} 2x - 3y - z &= 2 \cdot 2 - 3(-3) - 5 \\ &= 4 + 9 - 5 \\ &= 8, && \text{as required.} \end{aligned}$$

17. We have

$$\begin{aligned} y &= 12x - 2x^2 - 8 \\ y + 8 &= 12x - 2x^2 \\ -\frac{y+8}{2} &= x^2 - 6x && \text{Divide by } -2 \\ -\frac{y+8}{2} + 9 &= x^2 - 6x + (-3)^2 && \text{Complete the square} \\ -\frac{y+8}{2} &= (x-3)^2 - 9 \\ y + 8 &= -2(x-3)^2 + 18 \\ y &= -2(x-3)^2 + 10. \end{aligned}$$

This is a quadratic equation in vertex form with $a = -2$ and vertex $(h, k) = (3, 10)$. We can find the y -intercept by evaluating y at $x = 0$:

$$\begin{aligned} y &= 12 \cdot 0 - 2 \cdot 0^2 - 8 \\ &= -8. \end{aligned}$$

We can find the x -intercepts by solving for $y = 0$:

$$\begin{aligned} -2(x-3)^2 + 10 &= 0 \\ -2(x-3)^2 &= -10 \\ (x-3)^2 &= 5 \\ x-3 &= \pm\sqrt{5} \\ x &= 3 \pm \sqrt{5}. \end{aligned}$$

18. Since $v(t) = \frac{t-1}{2t-3}$, we see that

$$v(w(t)) = \frac{w(t)-1}{2w(t)-3}.$$

Since we already know that $v(w(t)) = \frac{2t^2}{4t^2-1}$, we can compare numerators to infer

$$\begin{aligned} w(t) - 1 &= 2t^2 \\ w(t) &= 2t^2 + 1. \end{aligned}$$

Checking our answer, we have

$$\begin{aligned} v(w(t)) &= \frac{w(t)-1}{2w(t)-3} \\ &= \frac{2t^2}{2(2t^2+1)-3} \\ &= \frac{2t^2}{4t^2-1}, \end{aligned}$$

as required.

19. Taking the transformations one at a time, we have:

$$\begin{aligned} y &= p(x-2) & y &= p(x) \text{ shifted right} \\ y &= -p(x-2) & y &= p(x-2) \text{ flipped vertically} \\ y &= -3p(x-2) & y &= -p(x-2) \text{ stretched vertically} \\ y &= -3p(x-2) + 4 & y &= -3p(x-2) \text{ shifted up,} \\ \text{so } q(x) &= -3p(x-2) + 4. \end{aligned}$$

20. No. The second equation is quadratic, and we can solve it by factoring:

$$\begin{aligned} x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0, \end{aligned}$$

so the solutions are $x = 3$ and $x = -1$. However, $x = -1$ is *not* a solution to the original first equation:

$$\sqrt{2(-1)+3} = \sqrt{1} \neq -1.$$

21. We have

$$\begin{aligned} 2 \log P - 3 \log Q + \log(2R) &= \log(P^2) - \log(Q^3) + \log(2R) \\ &= \log \frac{P^2}{Q^3} + \log(2R) \\ &= \log \frac{2RP^2}{Q^3}. \end{aligned}$$

22. The function g is defined only if the input to the square root operation, $(x-4)(x+6)$, is non-negative. The graph of $y = (x-4)(x+6)$ is an upward-opening parabola with x -intercepts (zeros) at $x = -6$ and $x = 4$. Thus, the graph lies on or above the x -axis for $x \leq -6$ and $x \geq 4$. This means the domain of f is all x such that $x \leq -6$ or $x \geq 4$.
23. The graph of $y = 25 + (x-3)^2$ is an upward-opening parabola with vertex $(x, y) = (3, 25)$. This means the least value of y on the parabola is $y = 25$, which occurs at $x = 3$. Thus, the least value of h is $h(3) = \sqrt{25 + (3-3)^2} = \sqrt{25} = 5$, so the range of h is $y \geq 5$.

24. • To find the zeros of f (if any), we solve $x^2 + nx + n^2 = 0$. Letting $a = 1, b = n, c = n^2$, we see that the discriminant of this equation is:

$$\begin{aligned} b^2 - 4ac &= n^2 - 4(1)(n^2) \\ &= -3n^2. \end{aligned}$$

Thus, since n positive, the discriminant is negative, which tells us this equation has no solutions, so f has no zeros.

- Writing g in vertex form gives

$$\begin{aligned} g(x) &= x^2 - 2nx + n^2 \\ &= (x - n)^2. \end{aligned}$$

Thus, the vertex of g is the point $(n, 0)$, so its graph touches the x -axis at only one point, namely $x = n$. We conclude that g has only one zero.

- Writing h in factored form gives

$$\begin{aligned} h(x) &= n^2 x^2 - 1 \\ &= (nx - 1)(nx + 1) \\ &= n \cdot \left(x - \frac{1}{n}\right) \cdot n \cdot \left(x + \frac{1}{n}\right) \\ &= n^2 \left(x - \frac{1}{n}\right) \left(x + \frac{1}{n}\right). \end{aligned}$$

Thus, h has two zeros, at $x = 1/n$ and $x = -1/n$.

25. We don't need to multiply out this expression to see that the terms involving x^2 are:

$$\begin{aligned} (jx - 4)(x - j) &= jx^2 + \text{other terms} \\ -(2x - 1)(3x - 5) &= -6x^2 + \text{other terms}. \end{aligned}$$

By choosing $j = 6$, the terms involving x^2 vanish, as required for a linear function. This means we have

$$\begin{aligned} y &= (6x - 4)(x - 6) - (2x - 1)(3x - 5) && \text{because we choose } j = 6 \\ &= 6x^2 - 36x - 4x + 24 - (6x^2 - 10x - 3x + 5) && \text{multiply out} \\ &= 6x^2 - 6x^2 - 40x + 13x + 24 - 5 && \text{collect like terms} \\ &= -27x + 19, && \text{simplify} \end{aligned}$$

so $m = -27$
 $b = 19$.

Final

Math E-8: College Algebra
December 17, 2009

1. Find the domain of

$$y = \frac{1}{\sqrt{x-3} - \sqrt{11-x}}.$$

2. The range of f is $3 < y < 8$. Find the range of $g(x) = 2f(x-1) + 3$.

3. Say what must be true about k in order for neither of the following equations to have a solution:

$$\begin{aligned} 6x^2 + 21x + k &= 0 \\ kx^2 + 91x + 3k &= 0. \end{aligned}$$

4. Evaluate $g(-2)$ if

$$g(x) = 2\sqrt{25 - 4\sqrt{10 - 3x}}.$$

5. Evaluate $h(4t^2)$ if $h(t) = \sqrt{9t - 4}$.

6. Find $q^{-1}(x)$ if $q(x) = \frac{4-3x}{7-9x}$.

7. Solve $w(0.2x + 1) = 0.2w(1 - x)$ if $w(x) = 0.5 - 0.25x$.

8. Two fourth-degree polynomials f and g have coefficients given by:

$$\text{Coefficients of } f: a_i = (-1)^{i+1} \cdot \frac{2^i}{3^{i+1}}$$

$$\text{Coefficients of } g: b_i = (-1)^i \cdot \frac{i+1}{2i+1}.$$

Find the degree, the leading coefficient, and the constant term of the polynomial $y = f(x)g(x)$.

9. Rewrite the equation $1.088^t = 12.5$ in the form $10^{At} = 10^B$ and give possible values for the constants A and B .
10. Given $f(x) = (x-3)^2 + 2$, find the vertex of the graph of $g(x) = 2f(x-4) + 4$.

11. Without bothering to express the polynomial h in standard form, find a_n :

$$h(t) = 3((2t+1)^3 + 3)^2 + 5.$$

12. A population starts at 11,500 and decreases by 2.8% per year. Another population starts at 23,400 and has a half-life of 14 years. When are the two populations equal in size?

13. Write in vertex form, and state the values of a , h , and k : $y = 3x^2 - 18x + 8$.

14. Write $y = 3(0.5x - 4)(4 - 20x)$ in the form $y = k(x-r)(x-s)$, and state the values of k , r , s .

15. Given that $v = x^{\log A}$ and $w = x^{\log B}$, write in terms of v and w without using logs:

$$x^{\log(AB^2)}.$$

16. What must be true about p and q if the vertices of both of the following quadratic functions lie in Quadrant II (that is, to the left of the y -axis and above the x -axis)?

$$y = p(x-3)(x-q)$$

$$y = (x+4+p)^2 + q + 5.$$

17. Solve the system
$$\begin{cases} 10x + 4y = -3 \\ 6x - 5y = 13 \end{cases}.$$

18. Use what you know about the quadratic formula to find a quadratic equation having

$$x = \frac{-2 \pm \sqrt{8}}{2}$$

as solutions. Your equation should be in standard form with integer (whole number) coefficients.

19. Find a possible formula for $q(x)$ given

$$p(x) = x^2 + 1 \quad \text{and} \quad q(p(x)) = \frac{x^2 + 2}{2x^2 + 3}.$$

Answer Problems 20–21 exactly provided
 $g(t) = 7 + 36 \cdot 3^{t/5}$.

20. Evaluate $g(-10)$. **21.** Solve $g(t) = 50$.

The graph of $y = f(x)$ includes the points $(0.4, 4.2)$ and $(1.9, 0.9)$. In Problems 22–23, find a possible formula for this function assuming it is:

22. linear

23. exponential

24. Write in the form kx^p and state the values of k and p :

$$\frac{\sqrt{x} \sqrt[3]{x^5}}{5(2x^3)^4}.$$

25. For each of the following functions, say whether the graph has exactly three different x -intercepts, all to the right of the y -axis. Briefly explain your reasoning. **Note:** An x -intercept is a point where the graph touches *or* crosses the x -axis.

(a) $y = (x^2 - 4)(x^2 - 5x + 6)$

(b) $y = (x - 4)^2(x^2 - 5x + 6)$

(c) $y = (x^2 - 9x + 20)(x^2 - 7x + 12)$

(d) $y = (x - 2)^2(x - 3)^3(x^2 - 5x + 6)$

Final Solutions

Math E-8: College Algebra
December 17, 2009

1. • Since we can't take the square root of a negative number, we require:

$$\begin{aligned}x - 3 &\geq 0 \\x &\geq 3 \\ \text{and } 11 - x &\geq 0 \\11 &\geq x \\x &\leq 11.\end{aligned}$$

Thus, x must lie between 3 and 11, inclusive.

- The denominator will be zero if:

$$\begin{aligned}\sqrt{x-3} &= \sqrt{11-x} \\x-3 &= 11-x && \text{square both sides} \\2x &= 14 \\x &= 7.\end{aligned}$$

Note that in the second step, both sides are nonnegative, so it is OK to square them. We conclude that $x \neq 7$, for otherwise we would divide by a negative. To check this, we see that:

$$\begin{aligned}\text{At } x = 7: \quad y &= \frac{1}{\sqrt{7-3} - \sqrt{11-7}} \\&= \frac{1}{\sqrt{4} - \sqrt{4}} \\&= \frac{1}{0}.\end{aligned}$$

In conclusion, the domain is all values of x between (and including) 3 and 11, except $x = 7$. Another way to say this is all values of x such that $3 \leq x < 7$ or $7 < x \leq 11$.

2. Taking the transformations one step at a time:

- First, the graph of f is shifted to the right by 1 unit. Since this does not affect the vertical position of the graph, the range of the resulting function is the same as the range of f , that is, $3 < y < 8$.
- Next, the graph is stretched vertically by a factor of 2. The original minimum y -value of 3 is stretched to a new minimum y -value of 6, and the original maximum y -value of 8 is stretched to a new maximum y -value of 16, resulting in an intermediate function whose range is $6 < y < 16$.
- Finally, the graph is shifted up 3 units. The intermediate minimum y -value of 6 is shifted to a new minimum of 9, and the intermediate maximum of 16 is shifted a maximum of 19.

Thus, the range of g is $9 < y < 19$.

3. In order for these two quadratic equations not to have solutions, their discriminants must be negative.

- For the discriminant of the first equation to be negative, we have

$$\begin{aligned}21^2 - 4 \cdot 6 \cdot k &< 0 \\441 - 24k &< 0 \\441 &< 24k \\k &> 18.375.\end{aligned}$$

- For the discriminant of the second equation to be negative, we have

$$\begin{aligned}
 91^2 - 4 \cdot k \cdot 3k &< 0 \\
 8281 - 12k^2 &< 0 \\
 8281 &< 12k^2 \\
 k^2 &> 690.083 \\
 k &> 26.269 \\
 \text{or } k &< -26.269.
 \end{aligned}$$

Since we already know $k > 18.375$, we can rule out $k < -26.269$ and $k > 18.375$. Thus, $k > 26.269$.

4. We have

$$\begin{aligned}
 g(-2) &= 2\sqrt{25 - 4\sqrt{10 - 3(-2)}} \\
 &= 2\sqrt{25 - 4\sqrt{16}} \\
 &= 2\sqrt{25 - 4 \cdot 4} \\
 &= 2\sqrt{9} \\
 &= 6.
 \end{aligned}$$

- 5.

$$\begin{aligned}
 h(4t^2) &= \sqrt{9 \cdot 4t^2 - 4} \\
 &= \sqrt{36t^2 - 4}.
 \end{aligned}$$

We can rewrite this as

$$\begin{aligned}
 \sqrt{36t^2 - 4} &= \sqrt{4(9t^2 - 1)} \\
 &= 2\sqrt{9t^2 - 1}.
 \end{aligned}$$

6. If $y = q(x)$, then $x = q^{-1}(y)$, so we solve:

$$\begin{aligned}
 y &= \frac{4 - 3x}{7 - 9x} \\
 y(7 - 9x) &= 4 - 3x && \text{multiply by denominator} \\
 7y - 9xy &= 4 - 3x && \text{distribute} \\
 3x - 9xy &= 4 - 7y && \text{collect } x\text{-terms at left} \\
 x(3 - 9y) &= 4 - 7y && \text{factor} \\
 x &= \frac{4 - 7y}{3 - 9y} && \text{divide} \\
 \text{so } f^{-1}(x) &= \frac{4 - 7x}{3 - 9x}.
 \end{aligned}$$

7. We have

$$\begin{aligned}
 w(0.2x + 1) &= 0.5 - 0.25(0.2x + 1) \\
 &= 0.5 - 0.05x - 0.25 \\
 &= 0.25 - 0.05x \\
 0.2w(1 - x) &= 0.2(0.5 - 0.25(1 - x)) \\
 &= 0.2(0.5 - 0.25 + 0.25x) \\
 &= 0.2(0.25 + 0.25x)
 \end{aligned}$$

$$= 0.05 + 0.05x.$$

Thus, solving gives

$$\begin{aligned} w(0.2x + 1) &= 0.2w(1 - x) \\ 0.25 - 0.05x &= 0.05 + 0.05x \\ -0.1x &= -0.2 \\ x &= 2. \end{aligned}$$

Checking our answer, we see that

$$\begin{aligned} w(0.2 \cdot 2 + 1) &= w(1.4) = 0.5 - 0.25(1.4) = 0.15 \\ 0.2w(1 - 2) &= 0.2(-1) = 0.2(0.5 - 0.25(-1)) = 0.2(0.75) = 0.15, \end{aligned}$$

as required.

8. The constant terms are given by:

$$\begin{aligned} \text{Constant term of } f : \quad a_0 &= (-1)^{0+1} \cdot \frac{2^0}{3^{0+1}} = -\frac{1}{3} \\ \text{Constant term of } g : \quad b_0 &= (-1)^0 \cdot \frac{0+1}{2 \cdot 0 + 1} = 1 \end{aligned}$$

Likewise, since both polynomials are of degree $n = 4$, the leading coefficients are given by:

$$\begin{aligned} \text{Leading coefficient of } f : \quad a_4 &= (-1)^{4+1} \cdot \frac{2^4}{3^{4+1}} = -\frac{16}{243} \\ \text{Leading coefficient of } g : \quad b_4 &= (-1)^4 \cdot \frac{4+1}{2 \cdot 4 + 1} = \frac{5}{9} \end{aligned}$$

. Focusing only on the constant and leading terms, this means:

$$\begin{aligned} f(x) &= -\frac{1}{3} + \cdots - \frac{16}{243} \cdot x^4 \\ g(x) &= 1 + \cdots + \frac{5}{9} \cdot x^4 \end{aligned}$$

$$\begin{aligned} \text{so } f(x)g(x) &= \left(-\frac{1}{3} + \cdots - \frac{16}{243} \cdot x^4 \right) \left(1 + \cdots + \frac{5}{9} \cdot x^4 \right) \\ &= -\frac{1}{3} \cdot 1 + \cdots - \frac{16}{243} \cdot \frac{5}{9} \cdot x^4 \cdot x^4 \\ &= -\frac{1}{3} + \cdots - \frac{80}{2187} \cdot x^4. \end{aligned}$$

We conclude that the degree of $f(x)g(x)$ is $n = 8$, the leading coefficient is $-80/2187$, and the constant term is $-1/3$.

9. We have

$$\begin{aligned} 1.088^t &= 12.5 \\ (10^{\log 1.088})^t &= 10^{\log 12.5} && \text{definition of log} \\ 10^{t \log 1.088} &= 10^{\log 12.5} && \text{exponent rule,} \\ \text{so } A &= \log 1.088 \\ B &= \log 12.5. \end{aligned}$$

10. We see that f is a quadratic function in vertex form where $(h, k) = (3, 2)$. Taking the transformations of f one step at a time, we have:

- The graph of f is shifted right by 4, moving the vertex to $(7, 2)$.
- The graph is then stretched vertically by a factor of 2, moving the vertex to $(7, 4)$.
- The graph is then shifted up by 4, moving the vertex to $(7, 8)$.

Thus, the vertex of g is $(7, 8)$.

11. We see that

$$\begin{aligned}
 h(t) &= 3 \left((2t+1)^3 + 3 \right)^2 + 5 \\
 &= 3 \left((2t)^3 + \text{other terms} \right)^2 + 5 \\
 &= 3 \left(8t^3 + \text{other terms} \right)^2 + 5 \\
 &= 3 \left((8t^3)^2 + \text{other terms} \right) + 5 \\
 &= 3 \left(64t^6 + \text{other terms} \right) + 5 \\
 &= 192t^6 + \text{other terms}, \\
 \text{so } n &= 6, a_n = 192.
 \end{aligned}$$

12. We have

$$\begin{aligned}
 \text{First population} &= 11,500(0.972)^t && \text{growth rate is } -2.8\% \\
 \text{Second population} &= 23,400(0.5)^{t/14}. && \text{half-life is 14.}
 \end{aligned}$$

Solving gives

$$\begin{aligned}
 11,500(0.972)^t &= 23,400(0.5)^{t/14} \\
 \frac{(0.972)^t}{(0.5)^{t/14}} &= \frac{23,400}{11,500} && \text{divide} \\
 \frac{(0.972)^t}{((0.5)^{1/14})^t} &= 2.0348 && \text{exponent rule} \\
 \left(\frac{0.972}{(0.5)^{1/14}} \right)^t &= 2.0348 && \text{exponent rule} \\
 1.0213^t &= 2.0348 && \text{evaluate left-hand side} \\
 \log(1.0213^t) &= \log 2.0348 && \text{take logs} \\
 t \log 1.0213 &= \log 2.0348 && \text{log property} \\
 t &= \frac{\log 2.0348}{\log 1.0213} && \text{divide} \\
 &= 33.706 \text{ years} && \text{evaluate right-hand side.}
 \end{aligned}$$

13. We have

$$\begin{aligned}
 y &= 3x^2 - 18x + 8 \\
 y - 8 &= 3x^2 - 18x \\
 \frac{y-8}{3} &= x^2 - 6x \\
 \frac{y-8}{3} + 9 &= x^2 - 6x + 9 && \text{complete the square} \\
 \frac{y-8}{3} + 9 &= (x-3)^2 \\
 \frac{y-8}{3} &= (x-3)^2 - 9 \\
 y - 8 &= 3(x-3)^2 - 27 \\
 y &= 3(x-3)^2 - 19 \\
 \text{so } a &= 3, \\
 h &= 3, \\
 k &= -19.
 \end{aligned}$$

14. We have

$$\begin{aligned} y &= 3(0.5x - 4)(4 - 20x) \\ &= 3(0.5)(x - 8)(-20)(x - 4/20) \\ &= -30(x - 8)(x - 0.2), \end{aligned}$$

so $k = -30$, $r = 8$, $s = 0.2$. Note that the order of r and s doesn't matter.

15. We have:

$$\begin{aligned} x^{\log(AB^2)} &= x^{\log(AB^2)} \\ &= x^{\log A + \log B^2} \\ &= x^{\log A} x^{2 \log B} \\ &= x^{\log A} (x^{\log B})^2 \\ &= vw^2. \end{aligned}$$

16. • The first function is in factored form. We see that its graph has x -intercepts at $x = 3$ and $x = q$. We know from the symmetry of the graph (a parabola) that the vertex lies midway between these zeros. Thus, in order for the vertex to be to the left of the y -axis, q must be farther to the left than $x = -3$; that is, $q < -3$. Moreover, if the parabola opens up, its vertex will lie *below* the x -axis. Therefore, the parabola must open down, so $p < 0$.
- Writing the second function in vertex form, we have

$$y = (x + 4 + p)^2 + q + 5 = \left(x - \underbrace{(-4 - p)}_h \right)^2 + \underbrace{(q + 5)}_k.$$

Thus, its vertex is $(-4 - p, q + 5)$. In order for the vertex to lie to the left of the y -axis, we require

$$\begin{aligned} -4 - p &< 0 \\ p &> -4. \end{aligned}$$

We already know $p < 0$, so we see that p must be strictly between -4 and 0 . In order for the vertex to lie above the x -axis, we require

$$\begin{aligned} q + 5 &> 0 \\ q &> -5. \end{aligned}$$

We already know $q < -3$, so we see that q must be strictly between -5 and -3 .

In conclusion, $-4 < p < 0$ and $-5 < q < -3$.

17. We have

$$\begin{array}{ll} 3(10x + 4y) = 3(-3) & \text{Multiplying 1st equation by 3} \\ 30x + 12y = -9 & \\ -5(6x - 5y) = -5 \cdot 13 & \text{Multiplying 2nd equation by } -5 \\ -30x + 25y = -65 & \\ 30x + 12y + (-30x + 25y) = -9 + (-65) & \text{Adding these two equations} \\ 37y = -74, & \end{array}$$

so $y = -2$. Substituting this into the first equation gives

$$\begin{aligned} 10x + 4(-2) &= -3 \\ 10x - 8 &= -3 \\ 10x &= 5 \\ x &= \frac{1}{2}. \end{aligned}$$

Thus, $(x, y) = (0.5, -2)$.

18. According to the quadratic formula, one solution to the equation $ax^2 + bx + c = 0$ is given by

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}.$$

Since

$$x = \frac{-2 \pm \sqrt{8}}{2},$$

we can try letting $-b = -2$, so $b = 2$, and $a = 1$. This means the discriminant, $b^2 - 4ac$, must equal 8:

$$\begin{aligned} b^2 - 4ac &= 8 \\ 2^2 - 4 \cdot 1 \cdot c &= 8 \\ 4 - 4c &= 8 \\ -4c &= 4 \\ c &= -1. \end{aligned}$$

Thus, one possible equation is

$$x^2 + 2x - 1 = 0.$$

19. We have

$$\begin{aligned} q(p(x)) &= \frac{x^2 + 2}{2x^2 + 3} \\ &= \frac{(x^2 + 1) + 1}{2(x^2 + 1) + 1} && \text{rearrange numerator and denominator} \\ &= \frac{p(x) + 1}{2p(x) + 1}. && \text{since } p(x) = x^2 + 1 \end{aligned}$$

We infer that $q(x) = \frac{x + 1}{2x + 1}$.

20.

$$\begin{aligned} g(-10) &= 7 + 36 \cdot 3^{-10/5} \\ &= 7 + 36 \cdot 3^{-2} \\ &= 7 + 36 \cdot \frac{1}{4} \\ &= 7 + 4 \\ &= 11 \end{aligned}$$

21. We have

$$\begin{aligned} 7 + 36 \cdot 3^{t/5} &= 50 \\ 36 \cdot 3^{t/5} &= 43 \\ 3^{t/5} &= \frac{43}{36} \\ \log(3^{t/5}) &= \log \frac{43}{36} \\ \frac{t}{5} \cdot \log 3 &= \log \frac{43}{36} \\ t &= \frac{5 \log \frac{43}{36}}{\log 3}. \end{aligned}$$

An approximate value for t is $t = 0.8087$.

22. We have $f(x) = b + mx$ where

$$\begin{aligned} m &= \frac{f(1.9) - f(0.4)}{1.9 - 0.4} \\ &= \frac{0.9 - 4.2}{1.5} \\ &= -2.2. \end{aligned}$$

Solving for b , we have

$$\begin{aligned} f(0.4) &= b + m(0.4) \\ 4.2 &= b - 2.2(0.4) \\ b &= 5.08, \end{aligned}$$

so $f(x) = 5.08 - 2.2x$. Checking our answer, we see that

$$\begin{aligned} f(0.4) &= 5.08 - 2.2(0.4) = 4.2 \\ f(1.9) &= 5.08 - 2.2(1.9) = 0.9. \end{aligned}$$

23. We have $f(x) = ab^x$ where

$$\begin{aligned} \frac{f(1.9)}{f(0.4)} &= \frac{ab^{1.9}}{ab^{0.4}} = \frac{0.9}{4.2} \\ b^{1.9-0.4} &= \frac{0.9}{4.2} \\ b^{1.5} &= \frac{0.9}{4.2} \\ b &= \left(\frac{0.9}{4.2}\right)^{1/1.5} \\ &= 0.3581. \end{aligned}$$

Solving for a , we have

$$\begin{aligned} f(0.4) &= ab^{0.4} = 4.2 \\ a &= \frac{4.2}{b^{0.4}} \\ &= 6.334, \end{aligned}$$

so $f(x) = 6.334(0.3581)^x$. Checking our answer, we see that

$$\begin{aligned} f(0.4) &= 6.334(0.3581)^{0.4} = 4.2 \\ f(1.9) &= 6.334(0.3581)^{1.9} = 0.9. \end{aligned}$$

24. We have

$$\begin{aligned} \frac{\sqrt{x} \sqrt[3]{x^5}}{5(2x^3)^4} &= \frac{1}{5} \cdot \frac{x^{1/2} (x^5)^{1/3}}{2^4 (x^3)^4} \\ &= \frac{1}{5} \cdot \frac{1}{16} \cdot \frac{x^{1/2} \cdot x^{5/3}}{x^{3 \cdot 4}} \\ &= \frac{1}{80} \cdot \frac{x^{\frac{1}{2} + \frac{5}{3}}}{x^{12}} \\ &= \frac{1}{80} \cdot \frac{x^{\frac{13}{6}}}{x^{12}} \\ &= \frac{1}{80} \cdot \frac{x^{\frac{13}{6}}}{x^{12}} \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{80} \cdot x^{\frac{13}{6}-12} \\
&= \frac{1}{80} \cdot x^{\frac{13}{6}-\frac{72}{6}} \\
&= \frac{1}{80} \cdot x^{-\frac{59}{6}},
\end{aligned}$$

so $k = 1/80$ and $p = -59/6$.

25. (a) No. Factoring, we see that

$$y = (x^2 - 4)(x^2 - 5x + 6) = (x - 2)(x + 2)(x - 2)(x - 3),$$

so there are 3 x -intercepts, with one to the left of the y -axis.

(b) Yes. Factoring, we see that

$$y = (x - 4)^2(x^2 - 5x + 6) = (x - 4)^2(x - 2)(x - 3),$$

so there are 3 x -intercepts, all to the right of the y -axis.

(c) Yes. Factoring, we see that

$$y = (x^2 - 9x + 20)(x^2 - 7x + 12) = (x - 4)(x - 5)(x - 4)(x - 3) = (x - 4)^2(x - 5)(x - 3),$$

so there are 3 x -intercepts, all to the right of the y -axis.

(d) No. Factoring, we see that $y = (x - 2)^2(x - 3)^3(x^2 - 5x + 6) = (x - 2)^2(x - 3)^2(x - 2)(x - 3) = (x - 2)^3(x - 3)^3$, so there are only 2 x -intercepts.