

Radiative transfer and radiative equilibrium

Before we start, let's summarize the take-home messages from the last topic:

We looked at how the atmosphere interacts with radiation through absorption, scattering and emission. Gas absorptions occur in discrete frequencies, corresponding to allowed transitions in energy states. The position and strength of these lines are measured and/or calculated based on quantum mechanics and archived. The absorption by gas molecules depends on their structures. Certain species (such as H₂O, CO₂, O₃) are active in infrared radiation while others such as N₂ and O₂ (fortunately for us...) are not, because of the lack of electric dipole moments in the latter¹. The absorption lines are broadened by collision and Doppler effects to affect a broader range of frequencies.

Air molecules and particles in the atmosphere also scatter light. For spherical particles, Mie theory gives the complete solution of the scattering given the size of the sphere, the index of refraction, and the wavelength of the light. Scattering by nonspherical ice particles remains a research topic and can be treated with numerical techniques. For visible light, Rayleigh scattering by air molecules are important, but it becomes unimportant in the infrared as it is proportional to frequency to the fourth power. Cloud particles are big enough to be in the Mie scattering regime and the scattering is less wavelength-dependent.

Our current goal is to determine the vertical temperature structure that arises if radiative processes act alone. Earlier, we looked at this problem with our simple greenhouse models and assumed the atmosphere is a homogeneous layer. Now we relax this assumption to look at the continuous case. To do so, we need the equation of radiative transfer.

Changes of intensity over an infinitesimal path dl is described by the Lambert-Bouguer-Beer's law:

$$dI_v = -e_v n I_v dl \quad (1)$$

where dI_v is the gain in intensity (i.e. negative means loss in intensity), n is the number density, e_v is, by definition, the extinction coefficient per molecule and has the dimension of area. We can also define the absorption coefficient, k_v and the scattering coefficient s_v . Since extinction is the combination of absorption and emission, $e_v = k_v + s_v$. It is often useful to define the single-scattering albedo $a_v = s_v / e_v$. If we view the extinction process as a collision between a photon of negligible size and a molecule of a collision cross section, e_v is this cross section. The physical content of this law is in the proportional dependence of dI_v on I_v and n , which can be understood intuitively from the collision view.

¹ People have also sought to come up with molecules that are super efficient so that we can warm up Mars in case we need to migrate over there (Keeping Mars warm with new supergreenhouse gases, Gerstell, M.F.; Francisco, J.S.; Yung, Y.L.; Boxe, C.; Aaltonee, E.T.; Published; Proceedings of the National Academy of Sciences of the United States of America, v. 98, p. 2154-2157, 2001)

Over this infinitesimal path, the pencil of radiation can also gain intensity from emission and scattering of light coming from other directions. An example of the latter is the light beam we see when we added milk into water in the aquarium. Including these in the source function, we can write:

$$\frac{dI_v}{ne_v dl} = -I_v + J_v \quad (2)$$

J_v is the source function. This is the radiative transfer equation or the Schwartzchild's equation of transfer.

If you find the equation somewhat abstract, we have seen an analog of this already in our simple leaky greenhouse model.

Radiative transfer in a plane parallel atmosphere

The atmosphere is a thin layer and tends to vary more slowly horizontally than vertically. This makes a plane parallel atmosphere a good approximation in dealing with radiative transfer, where we regard the atmosphere as horizontally homogeneous and the radiation field horizontally uniform as well (still functions of height and directions).

For convenience, define optical depth τ

$$\tau_v(z) = \int_z^\infty ne_v dz' \quad (3)$$

We have

$$\frac{\mu dI_v}{d\tau_v} = I_v - J_v \quad (4)$$

where $\mu = \cos\theta$, θ being the zenith angle. τ/μ is the optical path and is greater than the optical depth when light comes in at a slant angle. The sign change is because τ is defined to be an integral from infinity to a certain height and increases with decreasing height. If the absorption and scattering properties and the temperature profiles of the atmosphere are known, together with boundary conditions, this equation can be solved for the radiation fields at every point and every direction. The solution in general requires numerical techniques.

In the following, we will look at some limiting cases.

We shall first consider the case without scattering. Under clear sky (no aerosol or clouds) conditions, this is an excellent approximation for infrared radiation (why?). In this case, we only need to consider the emission source function. The amount of emission into the pencil of radiation is $\epsilon_v B_v$. By Kirchhoff's law, the emissivity of the material along dl is the same as its absorptivity $k_v ndl$. Therefore, $dI_v = -ne_v dl + nk_v B_v dl$. Without scattering, $k_v = e_v$ we have

$$\frac{\mu dI_v}{d\tau_v} = I_v - B_v(T) \quad (5)$$

With the proper boundary condition, Eq. (5) can be solved when T is known as a function of τ . Multiply both sides of Eq. (4) by $\exp(-\tau/\mu)$ and use

$$d(e^{-\tau/\mu} I) = -\frac{1}{\mu} e^{-\tau/\mu} I d\tau + e^{-\tau/\mu} dI$$

we have:

$$\mu \frac{d(e^{-\tau/\mu} I)}{d\tau} = -e^{-\tau/\mu} B$$

which can be integrated along a path. In the presence of scattering, the source function J would have depended on I itself. But without scattering, the source function J is replaced by $B(T)$, and this can be integrated to give the solution.

For instance, for an upward direction, the solution can be written as

$$I_v^+(\tau_v, \mu) = I_v^+(\tau_{vs}, \mu) \exp\left[\frac{\tau_v(z) - \tau_{vs}}{\mu}\right] + \int_{\tau_v(z)}^{\tau_{vs}} B_v(\tau_v') \exp\left[\frac{\tau_v(z) - \tau_v'}{\mu}\right] \frac{d\tau_v'}{\mu} \quad (6)$$

where $0 < \mu \leq 1$. For a downward direction, the solution is:

$$I_v^-(\tau_v, \mu) = I_v^-(0, \mu) \exp\left[\frac{\tau_v(z)}{\mu}\right] + \int_0^{\tau_v(z)} B_v(\tau_v') \exp\left[\frac{\tau_v(z) - \tau_v'}{\mu}\right] \frac{d\tau_v'}{-\mu} \quad (7)$$

where $-1 \leq \mu < 0$. τ_{vs} is the optical depth at the surface.

The term $\exp\left(-\left|\frac{\Delta\tau_v}{\mu}\right|\right)$ where $\Delta\tau_v$ is a layer's contribution to the optical depth (often averaged over frequency/wavelength for a given spectral band) is called the transmission function of that layer. One minus the transmission in our absorption only case is called the absorptance.

For infrared radiation, the boundary conditions are $I_v^+(\tau_v, \mu) = B(T_s)$; $I_v^-(0, \mu) = 0$

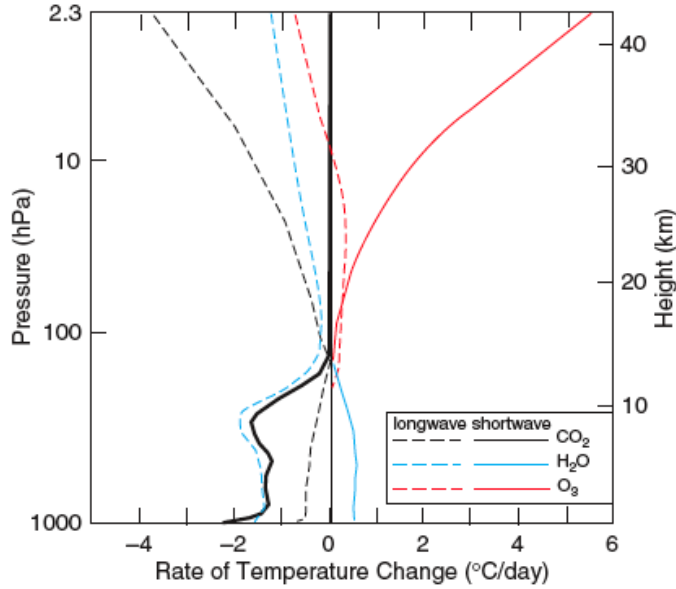
For solar radiation, emission by the atmosphere is negligible. When we also neglect scattering, which is an acceptable approximation for calculating radiative heating, the source function is 0. The solution is then simply the exponential decay of the incoming solar beam and the reflected sun light. The latter can be calculated from the reflectivity of the surface, which can be a function of the incident and reflecting angles. A surface with reflectivity independent of the reflecting angle is called a Lambertian surface.

Every part of the atmosphere absorbs and emits energy. The net value gives us the heating rate. Consider a thin layer in a plane parallel atmosphere. Define the net flux $F = F^+ - F^-$, where F^+ and F^- are the upward and downward fluxes, respectively. The energy loss/gain is the difference between the net flux at the upper interface and that at the lower interface. The rate that air is heated by radiation is:

$$\frac{\partial T}{\partial t} = \frac{-1}{c_p \rho} \frac{\partial F}{\partial z}$$

To calculate the fluxes, we need to integrate I_v over all angles and all frequencies. This is straightforward but computationally expensive. This is true particularly for the integration over frequency because of the complicated and rapid changes of absorption cross sections. Approximate methods have been developed. One method is to use an

effective absorption cross section for a whole spectral band. Integration over all angles can also be simplified by assuming some simple shapes for the radiation field. A popular one is to assume a semi-isotropic field of radiation. This gives the two-stream approximation. Such calculations can be done numerically. This is an example:



[Adapted from S. Manabe and R. F. Strickler, *J. Atmos. Sci.*, 21, p. 373 (1964).]

Here we shall make some simplifications and try to gain some understanding of the qualitative behavior.

At frequencies of solar radiation, emission by the atmosphere is negligible, so $B_v \sim 0$. We now consider that light beams only go up and down, so that $F=I$. (For light beams at a slant angle, simply replace the optical depth with the optical path below).

We have

$$\begin{aligned} \frac{dI_v^\uparrow}{d\tau_v} &= I_v^\uparrow \\ -\frac{dI_v^\downarrow}{d\tau_v} &= I_v^\downarrow \end{aligned} \quad (8)$$

where optical depth τ is again defined as

$$\tau_v(z) = \int_z^\infty n e_v dz' \quad (9)$$

As $F=I$, integrate over all (relevant) wavelengths and use an effective optical depth τ , we have

$$\begin{aligned} -\frac{dF_v^\uparrow}{d\tau_v} &= -F_v^\uparrow \\ \frac{dF_v^\downarrow}{d\tau_v} &= -F_v^\downarrow \end{aligned} \quad (10)$$

We shall consider absorptions by O_2 and O_3 in the UV. These are very large, so we only need to consider the downward solar beam (nothing reaches the surface so there is no reflection), and $F_v^\downarrow(z) = F_v^\downarrow(\infty)\exp(-\tau_v(z))$ where $F(\infty)$ is the incoming solar flux at $z=\infty$.

For a small frequency interval, the rate of energy absorption is $\frac{dF_v^\downarrow(z)}{dz}d\nu$, and the heat rate due to absorption in this frequency range is:

$$\frac{1}{c_p\rho} \frac{dF_v^\downarrow(z)}{dz} d\nu = \frac{-F_v^\downarrow d\nu}{c_p\rho} \frac{d\tau}{dz} = \frac{F_v^\downarrow d\nu}{c_p\rho} nk_v$$

where n is the number density of the absorption molecules and k is the effective absorption cross section. Write $\rho = n_{air}M_{air}$, the product of the number density and the molecular weight of air, we have

$$\frac{F_v^\downarrow d\nu}{c_p\rho} nk_v = \frac{F_v^\downarrow d\nu}{c_p M_{air}} \frac{n}{n_{air}} k_v = \frac{F_v^\downarrow(\infty) d\nu}{c_p M_{air}} C k_v \exp(-\tau_v(z))$$

We have defined the volume mixing ratio $C = n/n_{air}$.

The heat rate is obtained by integrate the above expression over all frequencies. Note that for frequencies with stronger absorption, the solar beam gets attenuated more quickly as it moves downward, so the transmission function ($\exp(-\tau_v)$) is small, and the heating is the produce of k_v and the transmission function. At height z , the absorption is the strongest at frequencies where the optical depth is close to 1. For frequencies with greater optical depth, the transmission function is too small (no enough photons reach there). For frequencies with smaller optical depth, which also implies a smaller absorption cross section, the molecules are not as good at absorbing the photons. Alternatively, for each of the frequencies, the same argument also gives a height range with the strongest absorption, which is known as the Chapman layer. At lower and lower heights, regions of zero/small transmission becomes wider and wider, frequencies with optical depth of one get pushed to regions with smaller and smaller k_v and the product of k_v and ($\exp(-\tau_v)$) becomes smaller and smaller.

Now let's see what the atmospheric temperature profile should look like. As the sunlight goes through the atmosphere, higher energy UV light (shorter than 0.2micron) is strongly attenuated by O_2 , so $|F|$ decreases, but the mixing ratio of O_2 is constant. We should see large SW heating rates at higher altitudes. Assuming the infrared cooling capability is roughly the same with height, we should see temperature to be higher at high altitudes. This is what we see in the thermosphere.

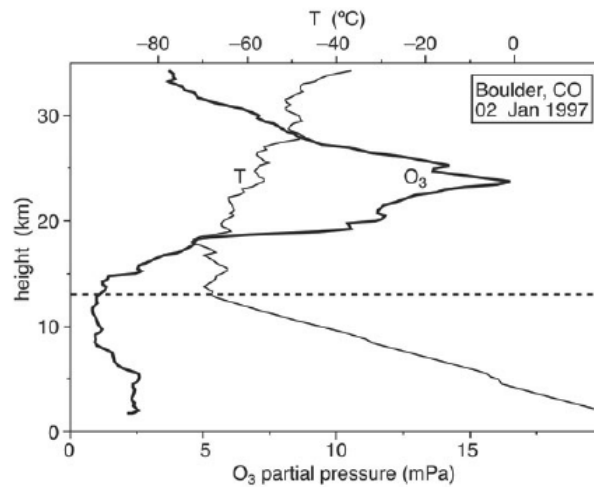
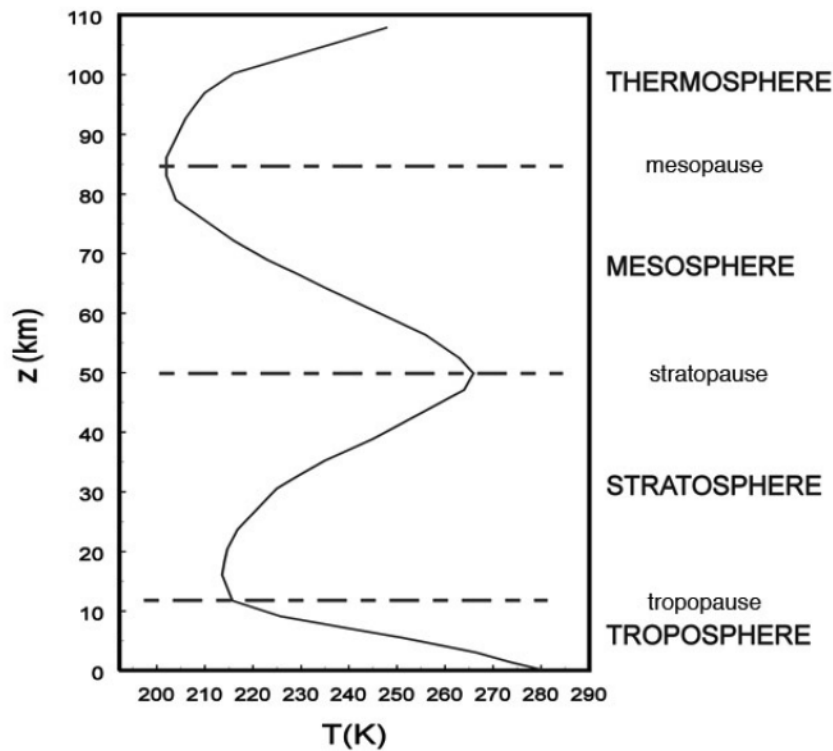


Figure 3.2: A typical winter ozone profile in middle latitudes (Boulder, CO, USA, 2 Jan 1997). The heavy curve shows the profile of ozone partial pressure (mPa), the light curve temperature ($^{\circ}\text{C}$) plotted against altitude up to about 33 km. The dashed horizontal line shows the approximate position of the tropopause. [Balloon data, courtesy of NOAA Climate Monitoring and Diagnostics Laboratory.]

The existence of an O_3 layer that peaks between 20 and 30 km modifies the picture. While O_3 mixing ratio peaks there, F decreases as light moves downward through the

atmosphere. The product of the two, and hence the SW heating rate is higher, near 50km. This gives rise to the mesosphere and the stratosphere, separated by the stratopause.

We have explained the existence of the thermosphere, mesosphere, and stratosphere! Note that the above discussion is only qualitative. The infrared cooling capability in reality is not constant, as the discussions below will illustrate.

Now let us consider the radiative transfer in the longwave and radiative-equilibrium. Here the emission source function is $B_v(T)$, and $I_v(\tau_{vs}, \mu) = B(T_s), I_v(0, \mu) = 0$

If the total radiative heating is nonzero over parts of the atmosphere, temperatures there will adjust, which then modifies the radiative heating field. In the absence of other processes, the equilibrium state is one with zero radiative heating everywhere, i.e. one in radiative-equilibrium. The adjustment of temperature mostly modifies the longwave radiation, which is tied to temperature by the Planck's law. Shortwave heating rate has some dependence on temperature through the dependence of absorption coefficients on temperature but this dependence is weaker.

So it is instructive to keep the shortwave heating fixed and consider the adjustment of temperature and longwave radiation alone. In fact, we will assume shortwave heating is zero for the atmosphere. We will also assume infrared absorption by the atmosphere is uniform at all wavelengths. This is called a gray atmosphere. The problem at hand is thus a continuous version of the leaky greenhouse problem we looked at before.

We shall again assume that radiation is either straight upward or downward so that $F=I$. We could choose to use the semi-isotropic approximation, but the results only differ by replacing B with $B^*=\pi B$. With these simplifications, we have

$$\frac{dF^\uparrow}{d\tau} = F^\uparrow - B \quad (11)$$

$$-\frac{dF^\downarrow}{d\tau} = F^\downarrow - B \quad (12)$$

Let's define the net flux $F = F^\uparrow - F^\downarrow$ and the total flux $\bar{F} = F^\uparrow + F^\downarrow$ so that we have:

$$\begin{aligned} \frac{dF}{d\tau} &= \bar{F} - 2B \\ \frac{d\bar{F}}{d\tau} &= F \end{aligned} \quad (13)$$

Derivation of Eq. (13) from Eqs. (11) and (12) is trivial. But let us pause for a moment and look at their physical content to get an intuitive feeling. In the first equation, F -bar times $d\tau$ is the amount of energy absorbed and $2Bd\tau$ is the amount of energy emitted. The second equation is more obscure physically, but says that the total flux can change with τ only when there is a net flux. If there is not, it is symmetric between up and down and the total flux gradient is zero.

Radiative equilibrium requires $F=\text{const}$, so we have

$$\bar{F} = 2B \quad (14)$$

Integrating the second Eq. in (13), we have

$$2B(\tau) = \bar{F} = F\tau + 2B(\tau = 0)$$

Consider energy balance at the top of the atmosphere (TOA), where $F^\uparrow = F_0 = (1 - \text{albedo})S/4$, where S is the solar constant. Since $F^\downarrow = 0$ at TOA,

$$F = \bar{F} = F^\uparrow = F_0$$

$$B(\tau = 0) = F_0 / 2$$

$$B(\tau) = \frac{F_0}{2}(\tau + 1)$$

We have got the temperature structure in the atmosphere as a function of τ . Now consider energy balance at the surface (looks familiar?),

$$B(T_s) = F_0 + F^\downarrow(\tau_s)$$

From their definitions, we have

$$F^\downarrow = \frac{1}{2}(\bar{F} - F)$$

As the net flux F is constant and equal to F_0 , and use Eq. (14), we have:

$$B(T_s) = B(\tau_s) + \frac{F_0}{2}$$

Note the jump at the surface.

We have solved the problem in optical depth space. To get the solution in height, we need to know τ as a function of z . To do so, we need to know how the number of absorbing gas molecules is distributed with height. The atmosphere is held close to the surface of the planet by gravity. The vertical forces acting on the atmosphere at rest are gravity and the pressure forces.

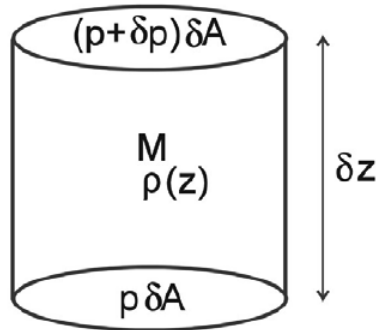


Figure 3.5: A vertical column of air of density ρ , horizontal cross-sectional area δA , height δz and mass $M = \rho \delta A \delta z$. The pressure on the lower surface is p , the pressure on the upper surface is $p + \delta p$.

These are in balance to a very good approximation as wave adjustment can very effectively remove any imbalance. This gives the hydrostatic balance.

$$\frac{dp}{dz} = -\rho g \quad (15)$$

For an ideal gas, we have

$$p = \rho RT \quad (16)$$

where R is the gas constant, which is the universal gas constant divided by the molecular weight of the gas in question. Eliminate ρ from the two equations, we have

$$\frac{dp}{p} = -\frac{g dz}{RT} = -\frac{dz}{H}$$

We have defined the scale height $H=RT/g$.

While T is not a constant with height, assuming it is yields a simple expression for p as a function of height:

$$p = p_s e^{-z/H}$$

The following figure shows it is not too far off.

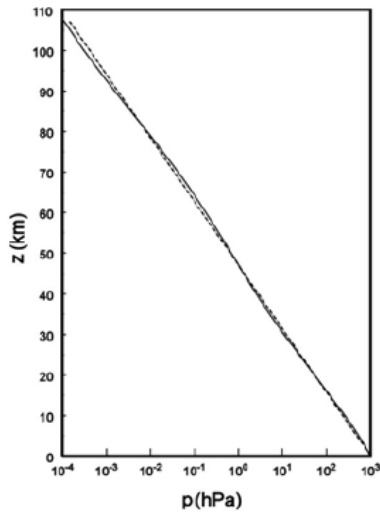


Fig 3.6 of Marshall and Plumb. Observed profile of pressure (solid) plotted against that calculated from the equation above with $H=6.8\text{km}$.

For an isothermal atmosphere, the density profile is simply: $\rho = \frac{p_s}{RT} e^{-z/H}$

The number density n of the absorbing gas is ρ times the volume mixing ratio C , then divided by the molecular weight of the air M_{air} . We will assume that C and k_v are constant. These are not accurate but will give us the qualitative behavior. This gives an exponential distribution for τ

$$\tau = \frac{p_s}{g} \frac{C}{M} k_v e^{-z/H} = \tau_s e^{-z/H}$$

For a gas with uniform mixing ratio such as CO_2 , τ has an exponential distribution with H equal to the scale height. In the troposphere, water vapor is the most important gas for infrared radiative transfer. We can also roughly fit an exponential for τ due to water vapor but H is considerably smaller ($\sim 2\text{km}$) because of the rapid decrease in water vapor mixing ratio with height. Some examples of the resulting temperature profiles are shown here:

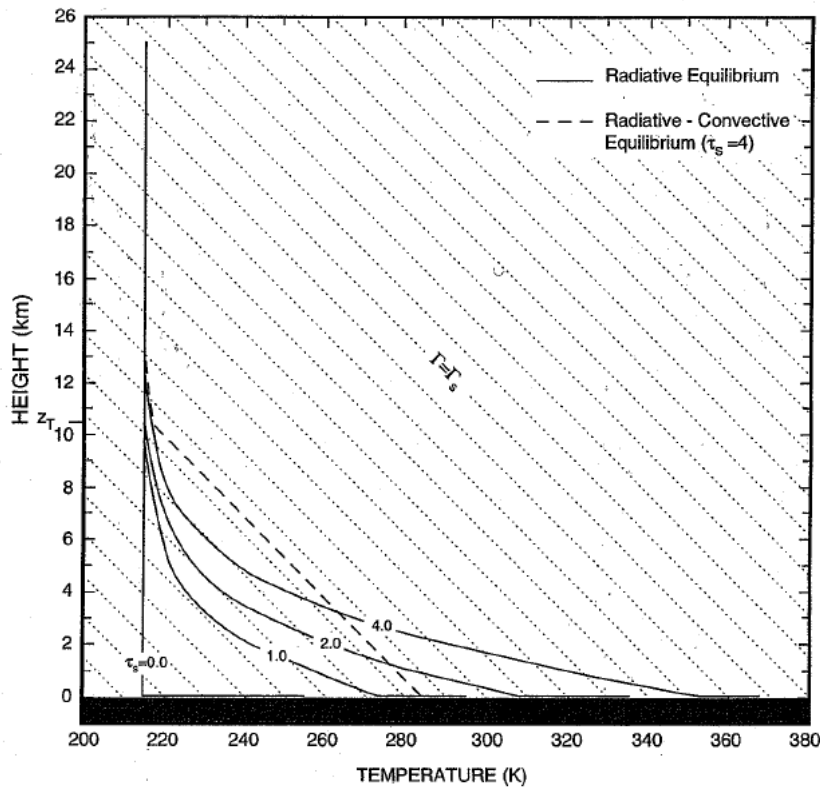


Figure 8.21 Radiative equilibrium temperature (solid lines) for the gray atmosphere in Fig. 8.20, with a profile of optical depth representative of water vapor (8.69), presented for several atmospheric optical depths τ_s . Saturated adiabatic lapse rate (dotted lines) and radiative-convective equilibrium temperature for $\tau_s = 4$ (dashed line) superposed.

The optical depth=4 case gives roughly the behavior of the radiative equilibrium of a realistic radiative transfer model.

The above gives the basic feature, but the gray approximation exaggerates the discontinuity at the surface. The real atmosphere is not gray and absorption happens in spectral lines/bands. The radiative-equilibrium profiles can be calculated more accurately using a radiative transfer model, where band models have been developed for different gas species and applied to absorption and emission within relevant wavelength ranges. This is the approach that climate models use.

With a more realistic radiative transfer model, there is SW heating and the discontinuity at the surface is relatively small (a few K). Below is a rough energy flow diagram. Note the LW flux from the atmosphere to the surface is greater than the SW flux absorbed by the surface!

The SW absorption in the stratosphere is mainly by ozone and oxygen. That in the troposphere is mostly by water vapor (13 units out of the 17), and rest is by clouds (3 units) and CO_2 , O_3 , and O_2 (1 unit).

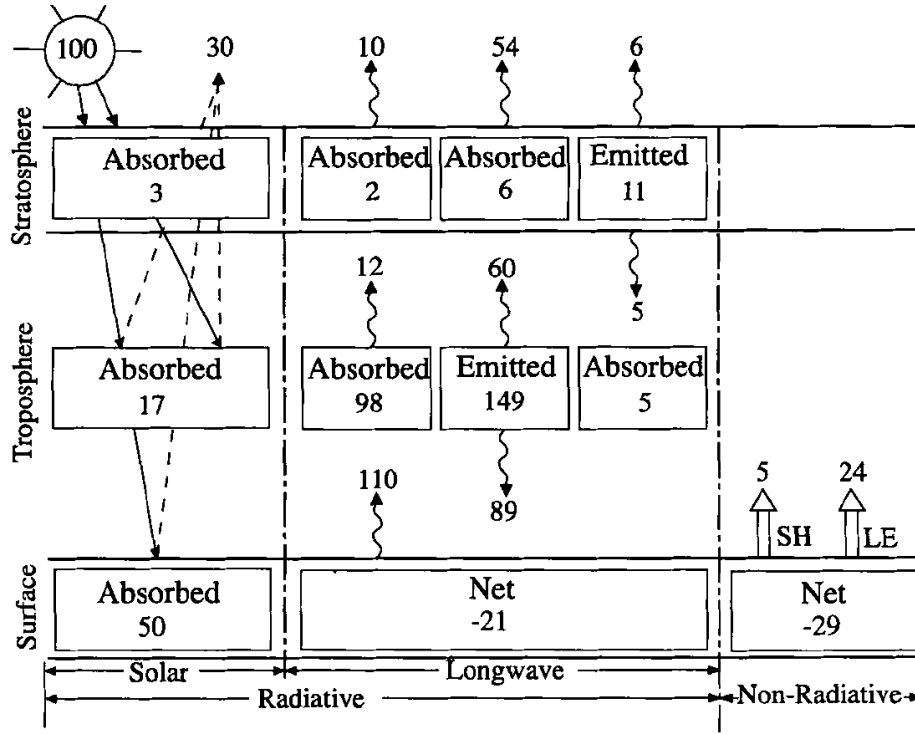


Fig. 2.4 Radiative and nonradiative energy flow diagram for Earth and its atmosphere. Units are percentages of the global-mean insolation (100 units = 342 W m^{-2}).

Compared with observed temperature profiles, the radiative equilibrium profile gives a temperature decrease with height $-dT/dz$, called the lapse rate, that is too large in the lower troposphere. To explain this, we need to consider hydrostatic instability. But before we do so, let's look at some applications of what we have learned so far, including remote temperature sensing.

Remote temperature sensing

For infrared radiation, when τ_{vs} is large, we have at the top of the atmosphere (for an upward direction)

$$I_v(\tau = 0) = \int_0^{\tau_{vs}} B_v(T) \exp[-\tau_v'] d\tau_v'$$

$$= \int_0^{\infty} -B_v(T) \exp[-\tau_v'] \frac{d\tau_v'}{dz} dz$$

When τ_{vs} is large, emission from the surface cannot penetrate to the TOA so the surface term dropped out. We are again considering the upward beam only.

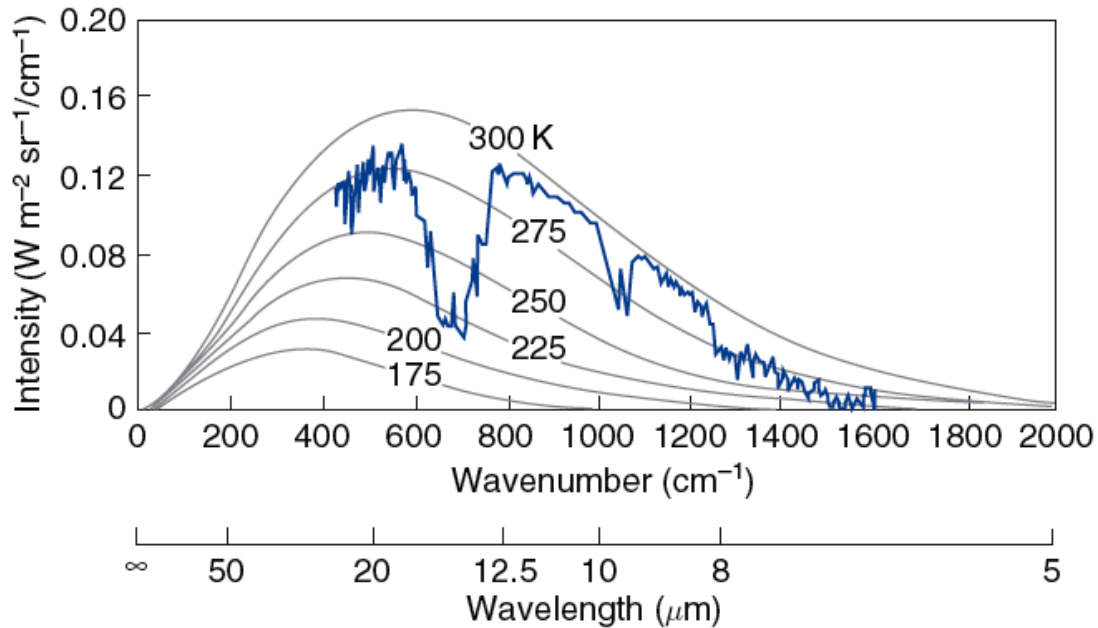
This is a weighted sum of $B_v(T)$ which is related to the temperature profile $T(z)$, and

$W_v = -\exp[-\tau_v'] \frac{d\tau_v'}{dz}$ is the kernel or weighting function, which represents the contribution efficiency from a layer of unit thickness to the satellite observed radiance.

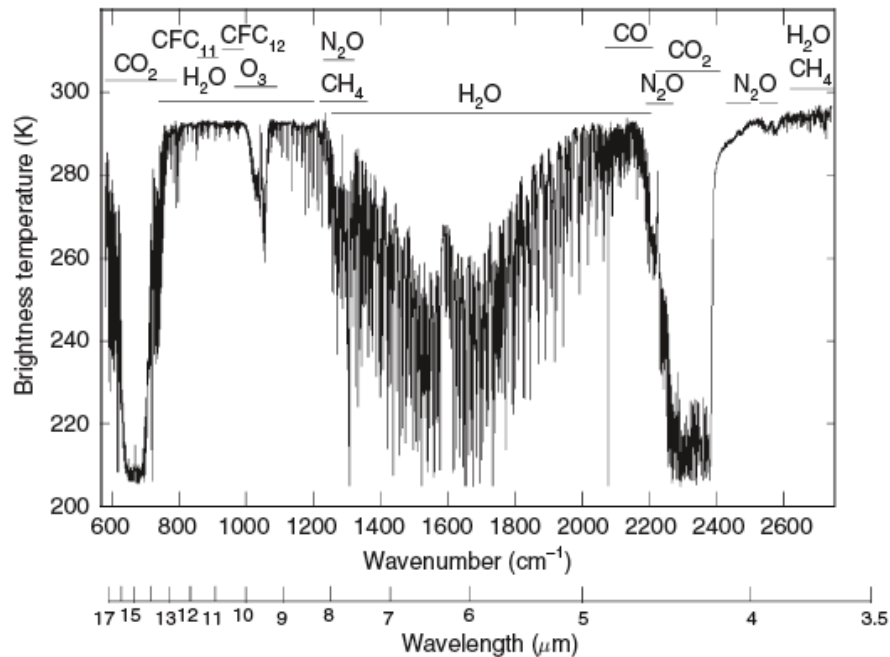
In some textbooks, the peak in the weighting function is given a specially meaning. For example, if again assuming an exponential profile for τ , we have

$$W_v = \exp[-\tau_v'] \frac{\tau_v'}{H} \quad (17)$$

and can find where the weighting function peaks by differentiating W , and requiring it to be zero. This gives $\tau=1$. Some call this the emission level. (τ/μ , the optical path or thickness, should be used if we consider a slant path), meaning that the region of unit optical thickness makes the largest contribution to the measured intensity. However, this is not correct because the peak is no longer present if a layer of unit mass (per unit area) instead of unit thickness is used. What is key to understanding remote temperature sensing is simply that these weighting functions are different. If we take measurements at frequencies that correspond to different absorption strengths (e.g. from line center to the wings), then we are measuring the weighted sum of $B_v(T)$ with different weighting functions. As the absorption strength increases, the peak of this weighting function (or more generally regions contributing to the satellite observed radiance) moves upward towards lower pressure. This tells us the temperature structure. The following figures illustrate this.

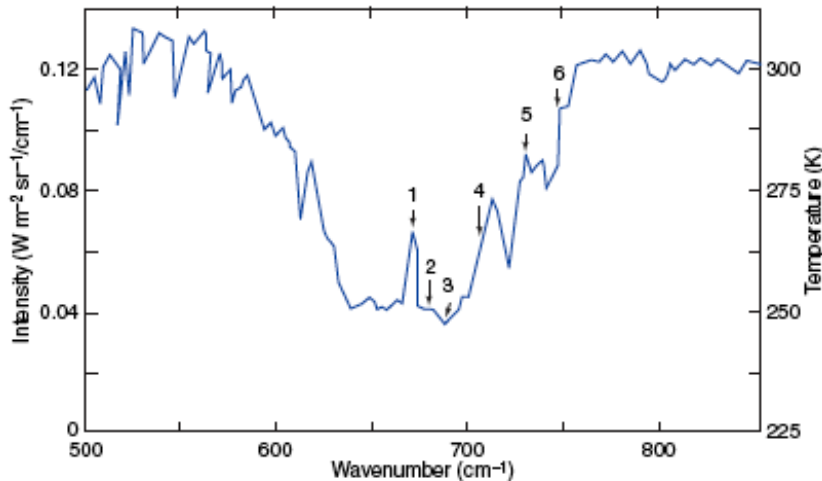


Monochromatic intensity of radiation emitted from a point on the Earth measured by an infrared spectrometer.

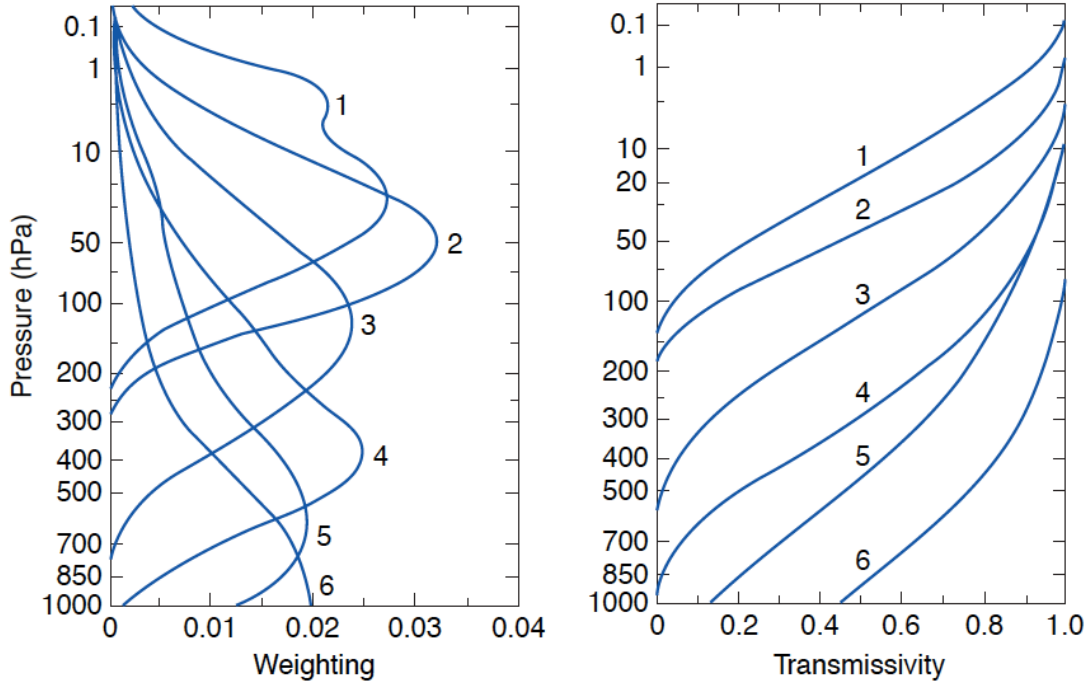


Finer resolution data from NASA ER-2 aircraft

Intensity at a particular frequency is uniquely related to a blackbody temperature. So intensity measurements are often presented in terms of this temperature, which is called brightness temperature.



Intensities and equivalent blackbody temperatures observed from the satellite. The channels are numbered in order of decreasing absorptance.



An illustration of weighting functions and transmissivity functions.

Well-mixed trace gases such as CO₂ are well suited for remote temperature sensing because it can be assured that the variations in the radiances from one sounding to another are mainly due to differences in the vertical profiles of B_v rather than to the differences in the vertical profiles of the concentrations of the absorbing constituents.

What do we learn about the temperature profile from the above figures?

With measurements over a number of channels, we can approximate the atmosphere as n isothermal layers, each with its own temperature T_n, and write

$$I_i = W_{i,s} B_i(T_s) + \sum_n W_{i,n} B_i(T_n) \quad (18)$$

We have included the contribution from surface temperature here as well. This can be solved for T_n and T_s, and is called an inverse problem, the simplest of which is a least-square fit.

The effect of doubling CO₂

We have learned most of what is needed to understand the radiative effects of doubling CO₂. Let's first consider the effect of doubling CO₂ on the stratosphere. As we mentioned earlier, SW heating in this region is mostly by O₃ and the absorption is largely independent of temperature. Therefore we may consider it as fixed. The LW heating rate of a layer of air is determined by its emission to space and its radiative energy exchange with air above and below. CO₂ is the main emitter and absorber in this region in LW.

Picture this as layers of the atmosphere exchanging photons. Consider two layers, A and B, separated by some parts of the atmosphere. At a particular frequency, the number of

photons from layer A that are absorbed by layer B is the product of three things: the number of photons emitted by A, which is itself the product of the emissivity of layer A and its blackbody emission or Planck function, the transmission function across the atmosphere between A and B, and the absorptivity of B. The number of photons emitted by layer B that are absorbed by layer A is the product of the emissivity of layer B and its blackbody emission or Planck function, the same transmission function across the atmosphere between A and B, and the absorptivity of A. From the Kirchoff's law, the exchange of photons between A and B is thus proportional to the difference in A and B's Planck functions. This exchange of photons is also proportional to the transmission function of the layer between A and B; layers A and B and only exchange photons effectively when they are close in optical depth space. Because the contrast between the temperature of a layer and those of the layers above and below is much smaller compared to the difference between the temperature of this layer and that of space. This allows us to neglect the radiation energy exchange between this layer and layers above and below and consider only the heat loss through radiation to space. This is called the cool-to-space approximation, and is surprisingly good in many cases, as long as temperature variations have large vertical wavelengths so that temperature varies slowly in optical depth space. This does not work so well for ozone absorption at 9.8 microns; because there is little ozone in the troposphere, temperature changes substantially between the tropopause and the surface even though optical depth changes very little.

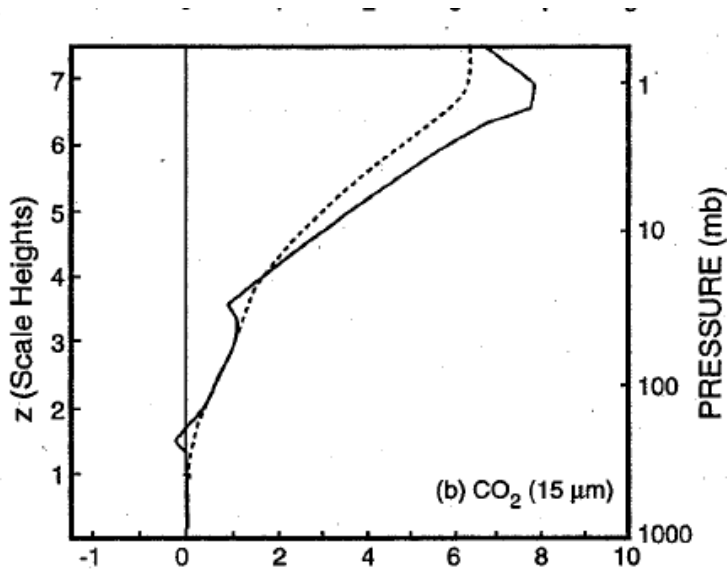


Figure: Total LW cooling by the CO₂ 15 micron band (solid) and that from the cooling to space approximation (dashed).

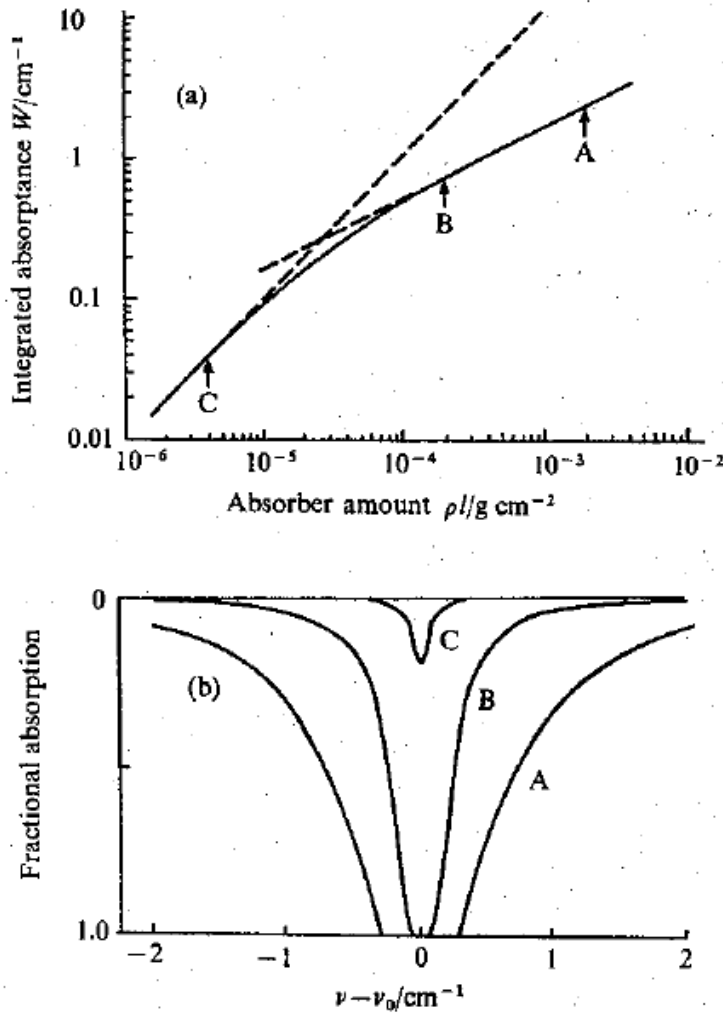
Now we have a situation where the SW heating is fixed, and we increase the number of emitters by doubling CO₂. The stratosphere should cool because of doubling CO₂. We can in fact estimate how much cooling should occur. Consider the radiative balance in the stratosphere:

$$\epsilon B = \text{SW heating.}$$

where ϵ is the emissivity and is proportional to the square root of the number of CO₂ molecules as CO₂ absorption line centers are saturated in the 15 micron band. This

behavior is illustrated in the following figure, and can be proven given the pressure broadening line shape and provided the lines are strong.
(Houghton Fig.4.2)

Fig. 4.2. (a) Curve of growth of a typical spectral line with $s = 10^4 \text{ cm}^{-1} (\text{g cm})$ and $\gamma_0 = 0.06 \text{ cm}^{-1}$ showing the linear and square root regions of growth.
(b) Actual shapes of the spectral line for different values of ρl corresponding to the values shown by the arrows in (a).



For weak lines, increasing the number of absorbers increases the absorption linearly. For strong lines, at the line center, there is already complete absorption, increasing the number of absorbers thus increases the total absorption by widening the spectral region where absorption is strong, which goes as the square root of the number concentration as seen from the pressure broadening line shape $\frac{s}{(\nu - \nu_0)^2 + \alpha^2}$, where s is proportional to the number concentration, and for strong lines, we are interested in the wings and α can be neglected.

Doubling CO₂ will then cause B to decrease to $(1/2)^{(1/2)}$ of its current value, and the temperature to decrease to $(1/2)^{(1/8)}$ of its current value (say 280K). The expected temperature decrease is ~20K. This is not far from the results from more detailed models.

The effect of doubling CO₂ on the troposphere.

I'm sure everybody has heard about global warming due to CO₂ increase. Some of you also know the perturbation of doubling CO₂ on the radiation is ~4W/m², and the effect of CO₂ increase is not linear but about logarithmic. We have learned a number of reasons why the radiative effects should scale sublinearly with the concentration of the gas (saturation at line center, line overlapping). Below sketches another argument. From our discussion earlier about the CO₂ 15 micron band, we see that much of the emission comes from the upper troposphere. As we increase CO₂, the emission level ($\tau=1$) will move upwards. As temperature decreases with height in the troposphere, CO₂ emission is at a lower temperature, giving rise to a greater greenhouse effect. The emission here is from wings of individual lines, for which one can show that $\tau \propto p^2$ (based on the same argument presented earlier)². So pressure at the emission level will decrease as $[\text{CO}_2]^{1/2}$. And the altitude of the emission level will increase as $(\ln[\text{CO}_2])/2$ because $z \propto \ln p$, so does the emission temperature, and hence the greenhouse effect, because the lapse rate dT/dz is roughly constant. For more discussions of these, go to Chapters 4 and 14 of John Houghton's *The physics of atmospheres*, third edition, Cambridge University Press, 2002). What we have discussed is an extension of the toy greenhouse models that we talked about. In the CO₂ 15 micron band, the absorption is strong so we can view it as opaque. In the toy greenhouse model, we assume the atmosphere to be homogeneous so that when it's opaque, making it more opaque does not change things. The discussion above allows us to extend this to the case of an inhomogeneous atmosphere.

Radiative timescale

If we perturb the temperature field that is in radiative equilibrium, how long does it take to relax back to its original (equilibrium) state? The precise answer depends on the spatial scale of the perturbation. But for a very rough estimate, let's consider the atmosphere as a blackbody and its temperature is perturbed by ΔT , we have

$$-\frac{d\Delta T}{dt} = \frac{1}{c_p \rho H} 2\Delta F = \frac{8\sigma T^3}{c_p \rho H} \Delta T$$

where H is the scale height. This gives a radiative relaxation timescale of ~6 days. The cooling-to-space approximation can be used to think about this more precisely. The stratosphere will adjust more efficiently as its transmission function to space is closer to one, while the troposphere is slower to adjust as the transmission function to space in the bulk of the troposphere is reduced by the presence of water vapor and clouds. Taking

² If we consider the overlapping of different lines, then the dependence is further weakened. A limiting case is where there is enough overlap that all wavelengths are blocked and transmission is zero and remains zero no matter how much one increases the amount of absorbers. To read more on this, check out Chapter 4 of Goody and Yung, 1989, *Atmospheric Radiation*, theoretical basis, second edition.

representative numbers, the radiative timescale for the stratosphere is $\sim 3\text{-}5$ days and that for the troposphere is about 10 days. Thus radiative processes act slowly compared to other processes such as convection. Also over a diurnal cycle, we see temperature at 50 km varies by $\sim 4\text{K}$ as solar heating varies over the course of a day. This variation would be greater if the radiative timescale were shorter.

A few words on clouds

Clouds consist of liquid water droplets or ice particles suspended in the atmosphere. Our everyday experience indicates that clouds can have complicated 3-dimensional shape. In climate models, it is often simplified as uniform and infinite in the horizontal within a grid box.

Given the size distribution of cloud droplets/particles, the albedo of a cloud depends on the liquid/ice water content and the solar zenith angle. Clouds reflect more when there are more droplets/particles and when sunlight comes in at a slant angle (figure below). The albedo saturates at some point: it no longer increases when you increase the number of droplets. This situation is called optically thick, is the same reason why a large glass of milk looks no different from a small glass of milk.

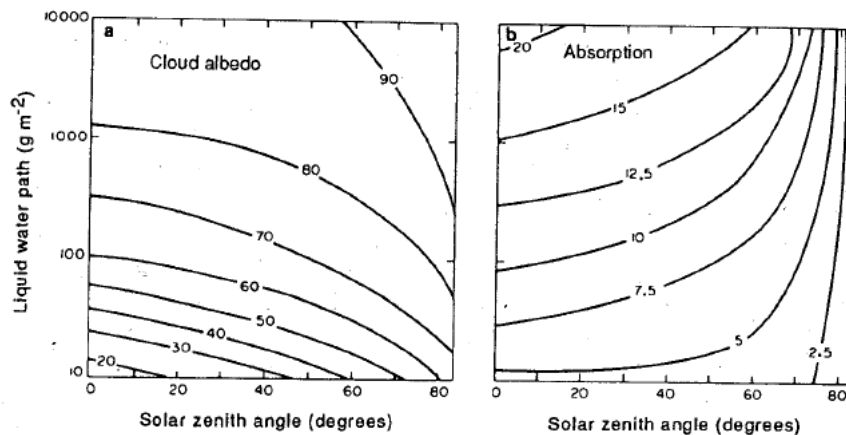


Fig. 3.13 The dependence of (a) cloud albedo and (b) cloud absorption on cloud liquid water path and solar zenith angle. Values are given in percent. [From Stephens (1978). Reprinted with permission from the American Meteorological Society.]

For the same amount of cloud water/ice, smaller particles tend to increase the albedo (figure below) as the total cross section is greater. Below is based on radiative transfer calculations for realistic atmospheric conditions.

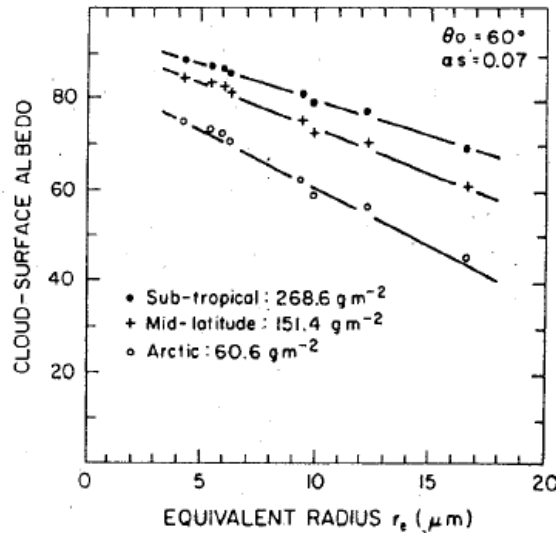


Fig. 3.14 The dependence of planetary albedo on the size of cloud droplets. [From Slingo and Schrecker (1982). Reprinted with permission from the Royal Meteorological Society.]

The absorption by clouds decreases with solar zenith angle. This is because with a larger zenith angle, there is greater chance, because of the stronger forward scattering by cloud particles, for a photon to be scattered away from the clouds versus into the clouds. The latter may eventually lead to absorption. The same idea also explains why wet sand is darker than dry sand: in wet sand, the scattering is more in a forward direction so it takes a larger number of scattering events for light to come back out, hence more absorption.

Even relatively thin clouds are opaque to terrestrial radiation (figure below). For example, a 1km thick status cloud with liquid water content of say 0.3g/m^3 (for cirrus, this number can be 10 or 100 times smaller) will have an emissivity close to 1. So one may consider clouds as blackbodies.

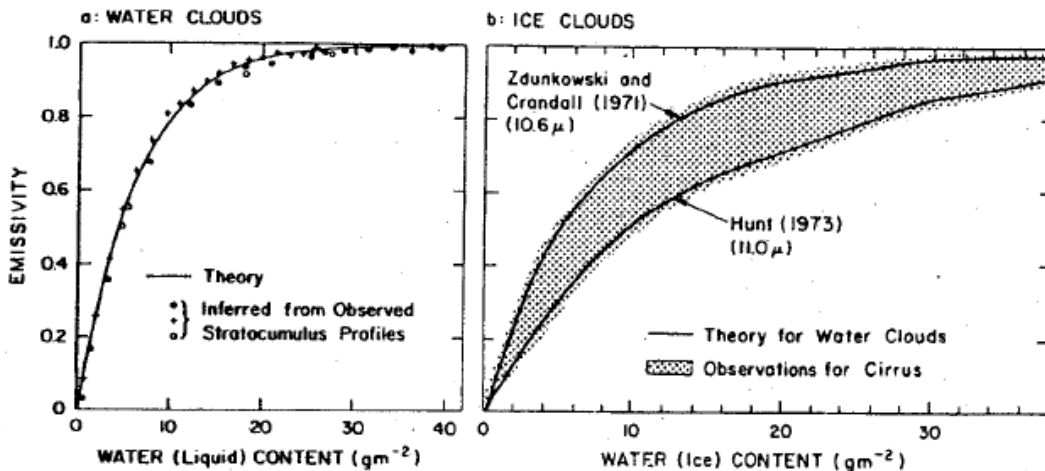


Fig. 3.15 The dependence of the longwave emissivity on (a) liquid water content [from Slingo *et al.* (1982); reprinted with permission from the Royal Meteorological Society] and (b) ice content [from Griffith *et al.* (1980); reprinted with permission from the American Meteorological Society].

This is seen clearly from observations.

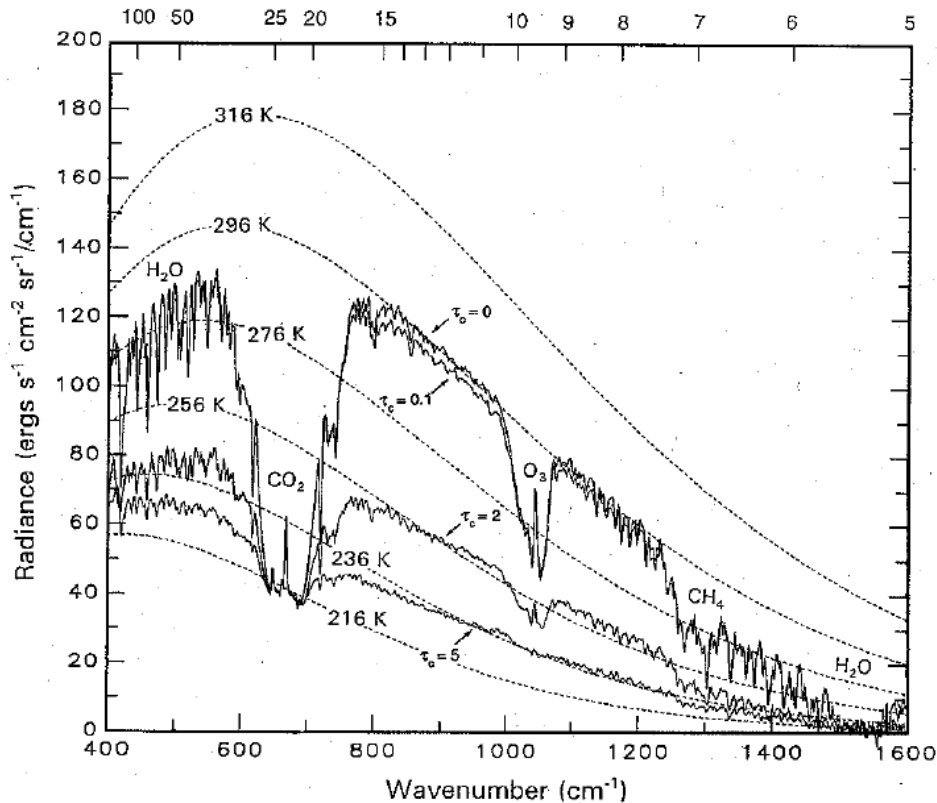


Figure 9.35 Spectrum of outgoing LW radiation observed by Nimbus-4 IRIS for a clear-sky region ($\tau_c = 0$) at (134°E, 12°N) and neighboring cirrus-covered regions ($\tau_c > 0$). Courtesy of B. Carlson (NASA/Goddard Institute for Space Studies).

Radiative effects of clouds

Clouds, being bright, reduce the amount of solar radiation that Earth absorbs. This is a cooling effect. It also reduces the amount of thermal emission, which is a warming (greenhouse) effect. The net effect is the difference between the two. The greenhouse is strongest for clouds at high altitudes (i.e. lower temperatures), and the weakest when clouds are close to the surface. The net radiative forcing of clouds is close to zero for clouds in tropical deep convection, which are white and cold, and is the strongest (on the negative side) for boundary layer clouds. Tenuous cirrus clouds, however, are cold and opaque in the infrared but not very reflective in the visible so that they have a net warming effect. The following is based on radiative transfer calculations (from Hartmann, Moy, Fu, J. Climate, 2001) and optical depth is for visible wavelength:

