Rossby waves

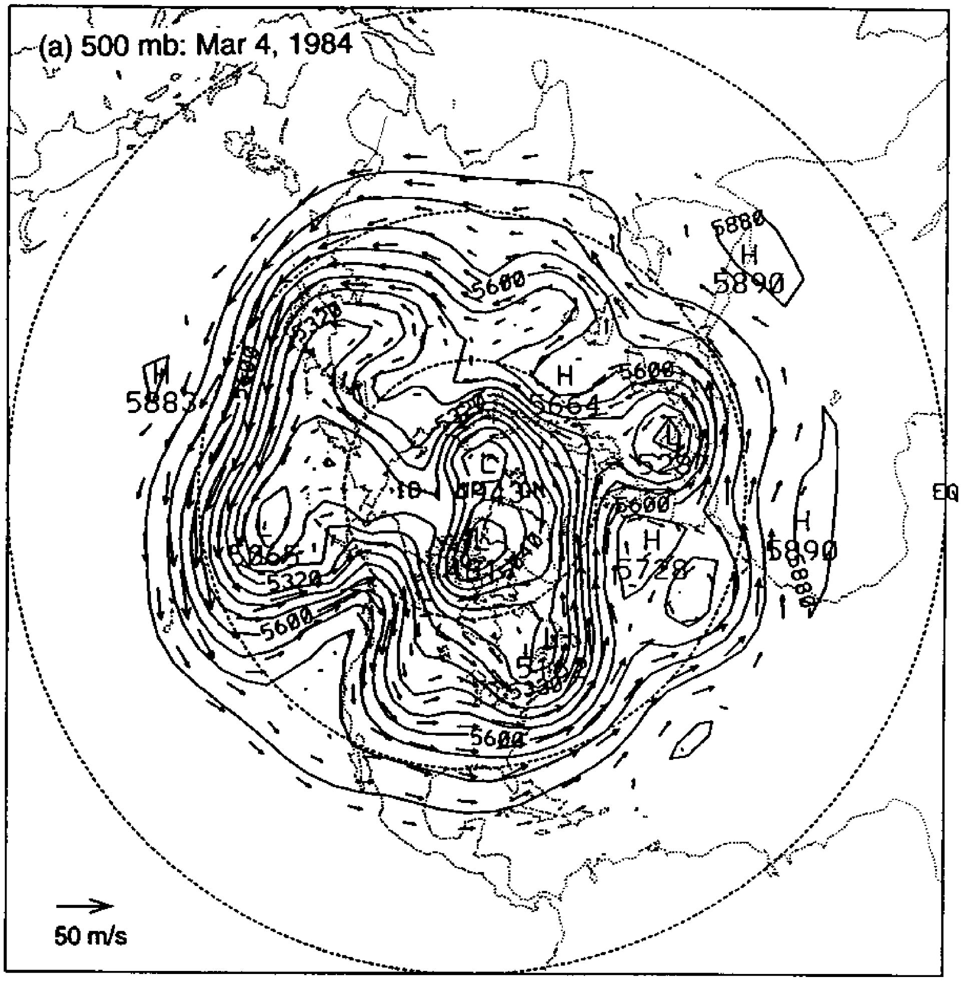
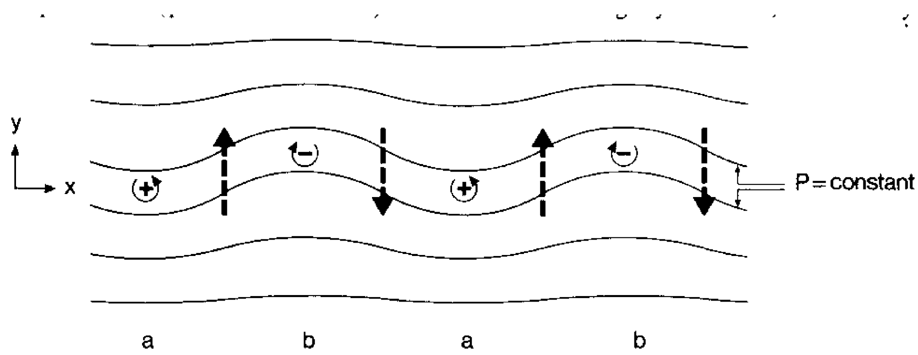


Figure 1.9 (a) Height (contours) of and horizontal velocity (vectors) on the 500-mb isobaric surface for March 4, 1984. (*continues*)

Away from the equator, gravity wave adjustment is trapped horizontally within the deformation radius. However, there are other waves that can communicate over large distances.

The variation of the Coriolis parameter with latitude (or more generally variations in PV) gives rise to a new type of winds called Rossby waves. Rossby waves are a key mechanism by which different parts of the atmosphere/ocean communicate. How it works can be qualitatively understood in terms of vorticity conservation.



latitude	f ($\times 10^{-4}\,\mathrm{s}^{-1}$)	β ($\times 10^{-11}\,\mathrm{s}^{-1}\,\mathrm{m}^{-1}$)
90°	1.46	0
60°	1.26	1.14
45°	1.03	1.61
30°	0.73	1.98
10°	0.25	2.25
0°	0	2.28

Table 6.1: Values of the Coriolis parameter, $f = 2\Omega \sin\varphi$ — Eq.(6.42) — and its meridional gradient, $\beta = \frac{df}{dy} = \frac{2\Omega}{a}\cos\varphi$ — Eq.(10.10) — tabulated as a function of latitude. Here Ω is the rotation rate of the Earth and a is the radius of the Earth.

Start with PV conservation in a shallow water system:

$$\frac{D}{Dt}\left(\frac{\varsigma + f}{H}\right) = 0$$

Now simplify it by demanding that H is a constant. Physically, one could place two rigid places to bound the fluid:

$$\frac{D}{Dt}(\varsigma + f) = 0$$

This is known as the nondivergent barotropic vorticity equation. Being barotropic means that density (temperature) is constant on constant pressure surfaces. Linearizing it around a mean state with a zonal velocity of U, we have:

$$\frac{\partial}{\partial t}\varsigma + U\frac{\partial}{\partial x}\varsigma + v\frac{df}{dy} = 0$$

Define β=df/dy and assume it's a constant, i.e. $f=f_0+\beta y$. This is known as the β-plane approximation. Recall that

$$\varsigma = \nabla^2\psi$$

$$v = \frac{\partial\psi}{\partial x}$$

we have

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\nabla^2\psi+\beta\frac{\partial\psi}{\partial x}=0 \tag{1.1}$$

Assume a wave solution of the form exp(i(kx+ly-ωt)), we have the dispersion relation

$$\omega = Uk-\beta\frac{k}{k^2+l^2}$$

The zonal phase speed relative to the mean wind is:

$$c_x-U=\frac{-\beta}{k^2+l^2}$$

and is always westward. The phase speeds depend on the wavenumber so Rossby waves are dispersive. For a typical midlatitude synoptic-scale disturbance, with l=k and with wavelength of ~6000km, we estimate c_x-U~-8m/s. This is smaller than typical mean zonal wind, so synoptic-scale Rossby waves usually move eastward (in terms of phase). Very long Rossby waves can however be stationary or even propagate westward (relative to the surface).

One can also compute the group velocity of the Rossby waves. The group velocity is

$$c_{gx}=U+\frac{\beta\left(k^2-l^2\right)}{\left(k^2+l^2\right)^2}$$

$$c_{gy}=\frac{2\beta kl}{\left(k^2+l^2\right)^2}$$

The group velocity can be either eastward or westward (relative to the mean flow), depending on the ratio of the zonal and meridional wavenumbers.

Now consider Rossby waves that are forced in a particular region (by baroclinic instability, heating or orography, e.g.) To the north, it should have positive c_{gy} while to the south, it should have negative c_{gy}. This implies that the phase lines are from northwest to southeast to the north, and from southwest to northeast to the south, like the banana-shapes pattern that we saw before. This converges zonal momentum to the region of Rossby wave generation.

An example of the propagation of Rossby waves on a sphere:

Fig. 10.15 The vorticity pattern generated on a sphere when a constant angular velocity westerly flow impinges on a circular forcing centered at 30°N and 45°W of the central point. Left to right, the response at 2, 4, and 6 days after switch on of the forcing. Five contour intervals correspond to the maximum vorticity response that would occur in 1 day if there were no wave propagation. Heavy lines correspond to zero contours. The pattern is drawn on a projection in which the sphere is viewed from infinity. (After Hoskins, 1983.)

Rossby wave can be used to explain the zonal asymmetry in the mean circulation that we saw:

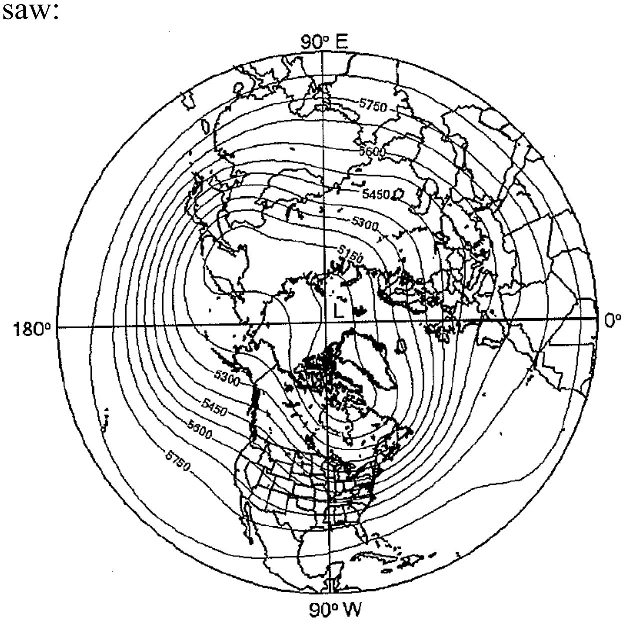


Fig. Jan. mean 500hPa geopotential height.

Now consider flow over planetary scale orography. As the flow moves towards the orography, it's compressed in the vertical. By PV conservation, it will gain negative relative vorticity. The reverse will happen when the flow extends in the vertical. One may include these as sources of vorticity in Eq. (1.1):

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\nabla^2\psi+\beta\frac{\partial\psi}{\partial x}=-\frac{f_0}{H}U\frac{\partial h_T}{\partial x} \qquad (1.2)$$

where U is the mean zonal wind, and the mean meridional wind is assumed to be zero. H is the depth of the fluid, f_0 is the Coriolis parameter at the latitude of interest and $\beta=df/dy$. ψ is the streamfunction so that $v=\partial\psi/\partial x$ and vorticity $\zeta=\nabla^2\psi$. h_T is the height of the topography. We should assume h_T is uniform in y (the meridional direction) and has zonal wavenumber of k and amplitude of h_0.

Since this is a linear equation with constant coefficients, the response ψ has the same frequencies and wavenumbers as the forcing h_T. Decompose h_T into the different Fourier components and we can treat them separately. Denote the component of h_T with zonal wavenumber k and frequency ω as

$$\hat{h}_0 \exp\left[i(kx-\omega t)\right]$$

and its response as

$$\hat{\psi}\exp\left[i(kx-\omega t)\right]$$

Equation (1.2) for this Fourier component then becomes:

$$\left[i\left(\omega-Uk\right)k^2+ik\beta\right]\hat{\psi}=-\frac{f_0U\hat{h}_0}{H}ik$$

or:

$$\hat{\psi}=-\frac{f_0U\hat{h}_0}{Hk^2\left[\frac{\omega}{k}-\left(U-\frac{\beta}{k^2}\right)\right]}$$

which is the solution. From this expression, we can see whenever the phase speed of the forcing, which is given by ω/k, matches that of a free Rossby wave mode with a zonal wavenumber k, we have resonance. For orographic forcing, the frequency is zero and resonance happens when $U=\beta/k^2$.

The response to steady forcing such as orography is therefore:

$$\hat{\psi}=\frac{f_0\hat{h}_0/H}{k^2-k_s^2}$$

where $k_s=\mathrm{sqrt}(\beta/U)$ is the wavenumber at which the free Rossby wave is stationary. Therefore there is a resonance effect when the scale of the orography matches that of the stationary Rossby wave. Charney and Eliassen (1949) did this calculation with some simple damping and simple y dependence, and were able to produce the observed geopotential height distribution quite well given the orography from the Himalayas and the Rockies (see below). In reality, the strong diabatic heating as the cold continental air moves over the ocean also contribute significantly to the stationary wave pattern. Less pronounced orography in the southern hemisphere explains why it’s more zonally symmetric in its mean circulation.

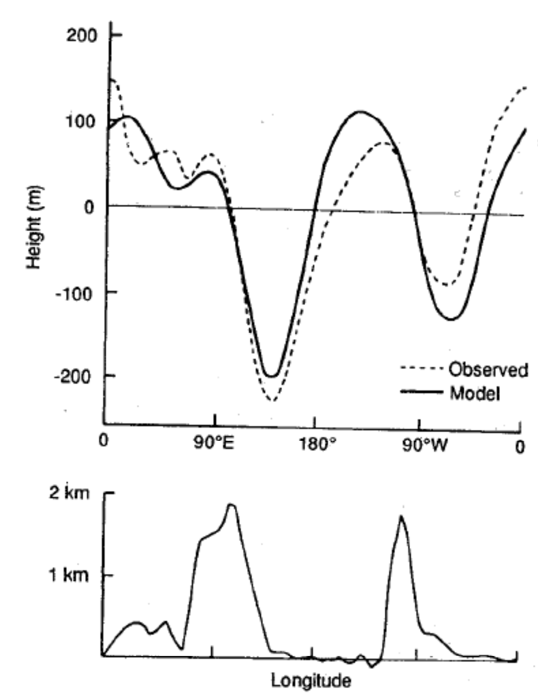


Fig. 7.15 (Top) Longitudinal variation of the disturbance geopotential height ($\equiv f_0\Psi/g$) in the Charney–Eliassen model for the parameters given in the text (solid line) compared with the observed 500-hPa height perturbations at 45°N in January (dashed line). (Bottom) Smoothed profile of topography at 45°N used in the computation. (After Held, 1983.)

The stationary wave pattern gives rise to the Asian and North American jet streams. Can you locate them on the two dimensional plot of the observed stationary wave pattern?

The answer is below:

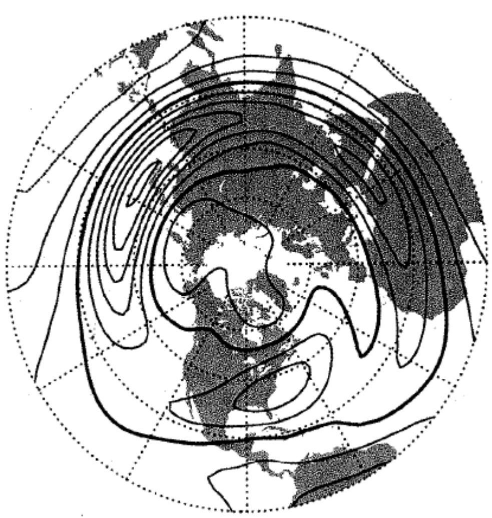


Fig. 6.2 Mean zonal wind at the 200-hPa level for December–February averaged for years 1958–1997. Contour interval 10 m s^{-1} (heavy contour, 20 m s^{-1}). (Based on NCEP/NCAR reanalyses; after Wallace, 2003.)

The jet streams are regions of strong thermal wind, which is what's behind baroclinic instability. So these regions are also favorable for storm development. Moreover, the strong jets provide a strong PV gradient and a good wave guide for the Rossby waves so these regions are also known as the storm tracks. The storm tracks move seasonally, being more equatorward in the winter. This affects the precipitation in many regions.

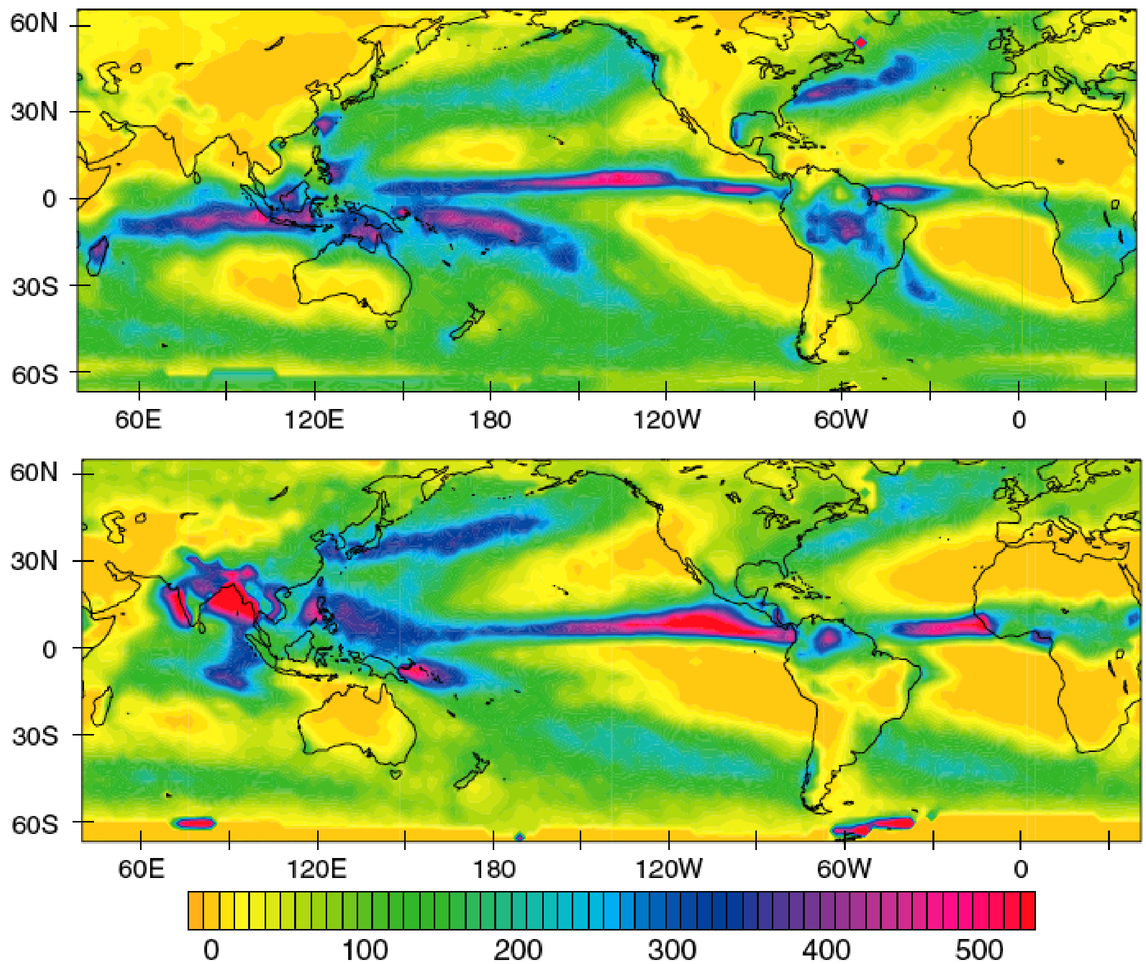


Fig. Rainfall pattern for January (top) and July (bottom) in mm/month. (The high precipitations near the Antarctic ice shelf are spurious due to problems with the algorithm)

Like gravity waves, Rossby waves can also propagate vertically. For a mean state that is homogeneous horizontally but varies with height, with some approximations (Boussinesq and quasi-geostrophic), the dispersion relation can be written as

$$\frac{\omega}{k} = U - \frac{\beta}{k^2 + l^2 + m^2 f_0^2 / N^2}$$

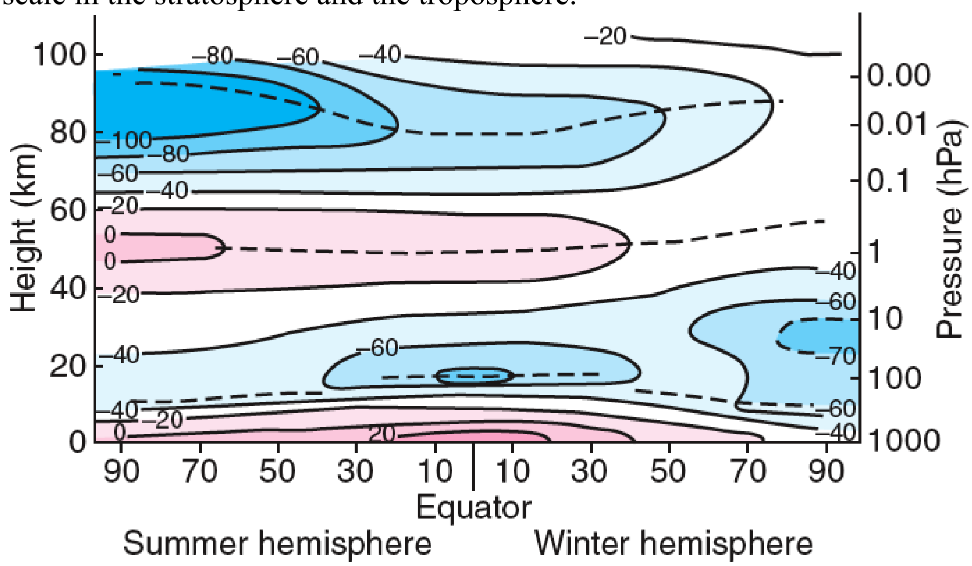
where m is the vertical wavenumber (see Salby's book Chapter 14 for a derivation). We see that m becomes imaginary if

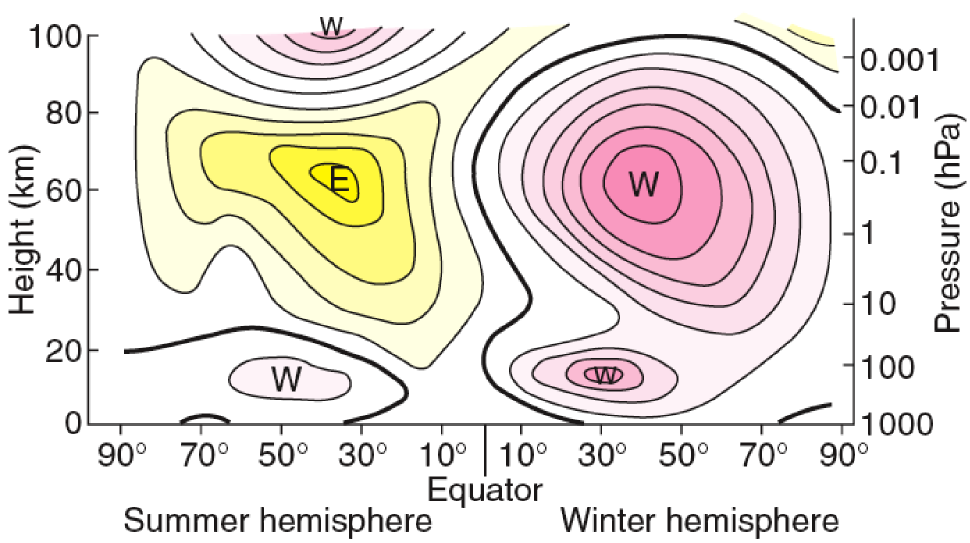
$$U < \frac{\omega}{k}$$

or

$$U > \frac{\beta}{k^2 + l^2} + \frac{\omega}{k}$$

Thus, Rossby waves will not propagate if the mean westerly winds become too weak or too strong. In the summer hemisphere in the stratosphere, winds are easterly so no Rossby waves can propagate into the stratosphere. In the winter hemisphere, for small scale waves such as the synoptic scale (a couple thousand km) waves, the second criterion can be satisfied shortly above the tropopause so only the very largest planetary waves can propagate into the stratosphere, explaining the difference in the observed wave scale in the stratosphere and the troposphere.





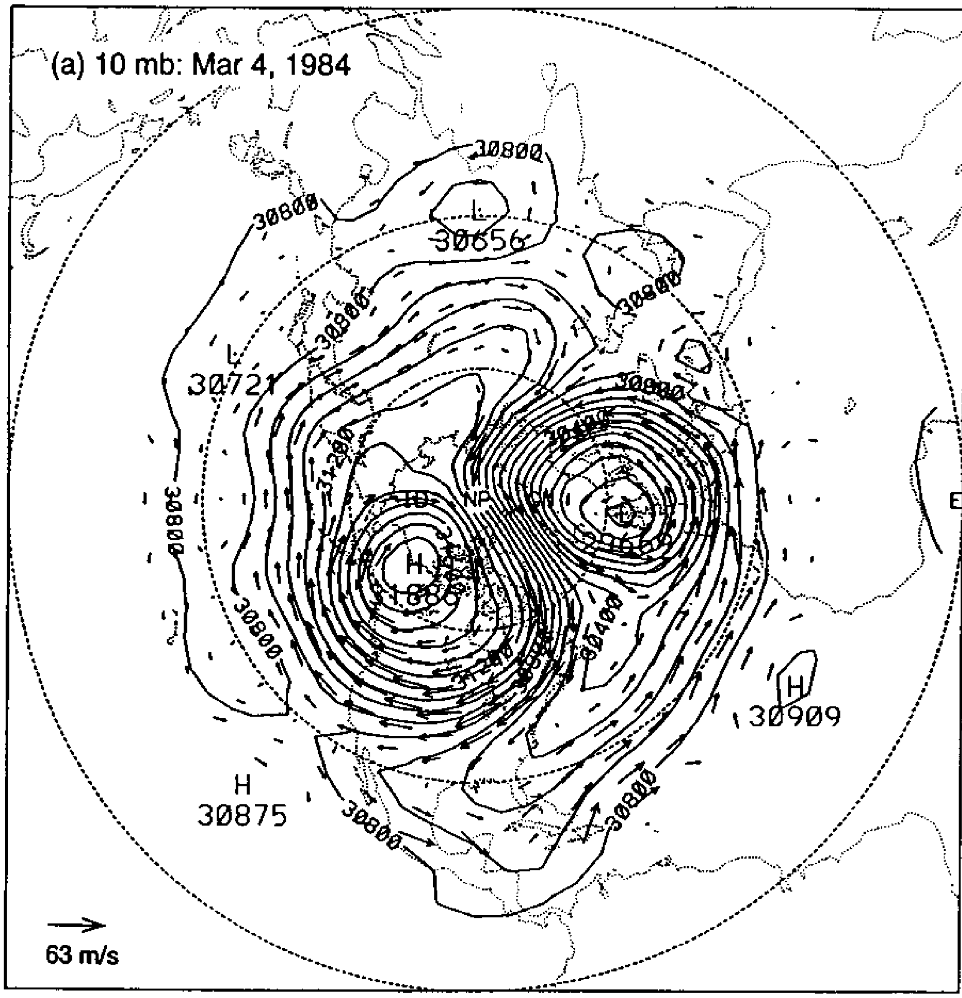


Figure 1.10 As in Fig. 1.9, but for the 10-mb isobaric surface. (*continues*)

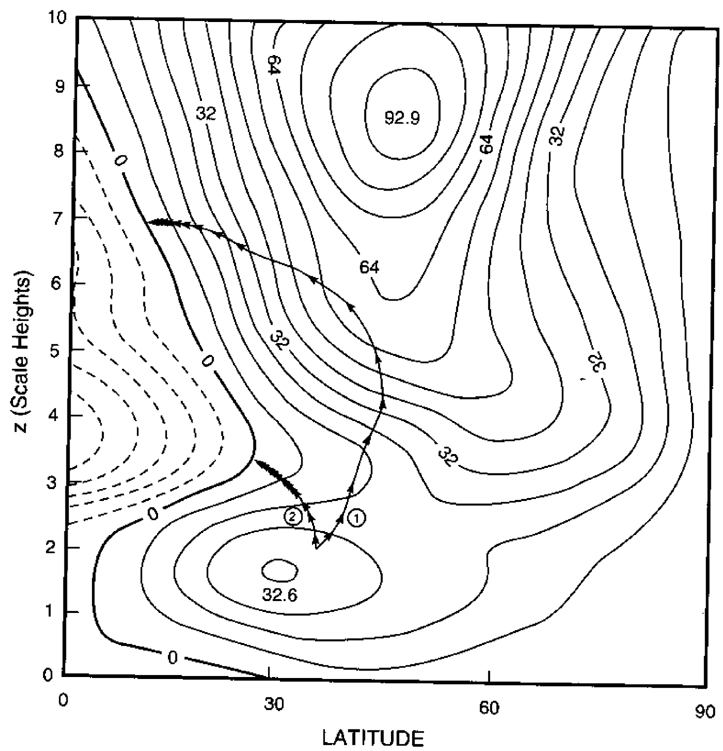


Figure 14.19 Rays of two stationary planetary wave components introduced into the lower stratosphere in zonal-mean winds (contoured in $m\,s^{-1}$) representative of northern winter on a midlatitude beta plane. Arrowheads mark uniform increments of time moving along a ray at the group velocity c_g. The ray for component 1, which initially propagates upward and poleward, encounters a turning line in strong westerlies of the polar-night jet, where wave activity is refracted equatorward. As the ray approaches easterlies, it next encounters a critical line ($\bar{u} = 0$), where c_g vanishes. Propagation then stalls, leaving wave activity to be absorbed, so the ray terminates. The ray for component 2 is initially directed upward and equatorward, so wave activity encounters the critical line and is absorbed even sooner. Thus, wave activity introduced between tropical easterlies and strong polar westerlies can propagate vertically only a limited distance before being absorbed. (Frequency of arrowheads near the critical line reduced for clarity.)

Upward propagation of Rossby waves, in particular the stationary waves forced by orography, and their dissipation have an important role in the dynamics of the stratosphere. As the planetary waves propagate up in the winter stratosphere, because of dissipation (both thermal and mechanical), they decelerate the polar night jet and force air to move poleward. This wave pumping is what drives the circulation in the stratosphere, known as the Brewer-Dobson circulation. This wave pumping adiabatically warms the winter polar stratosphere. Otherwise, the winter polar stratosphere would be a lot colder (and the polar night jet would be a lot stronger, in fact over 300m/s).

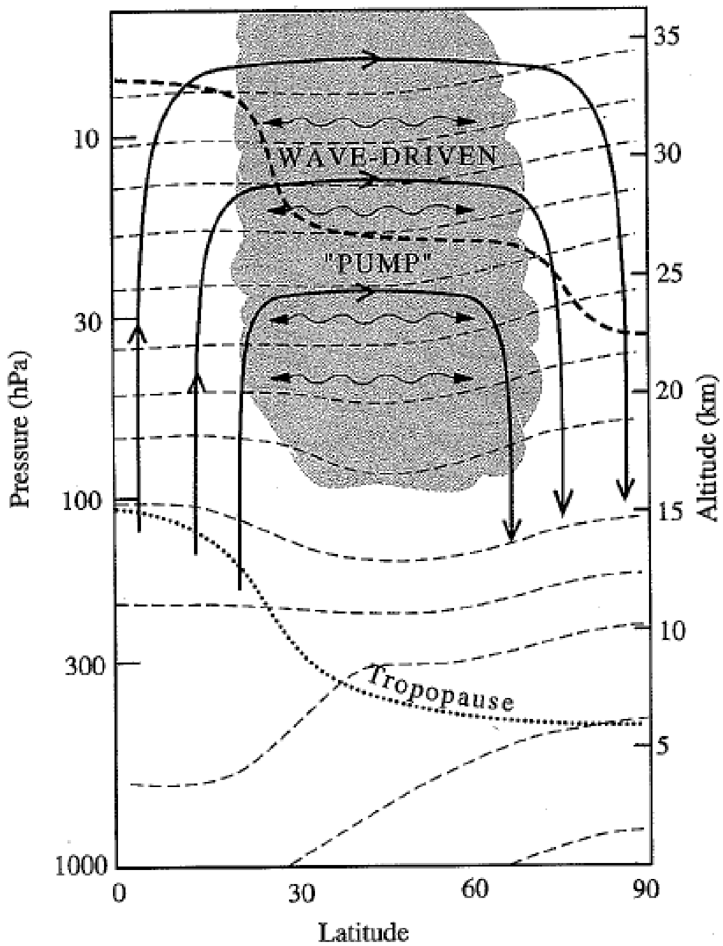
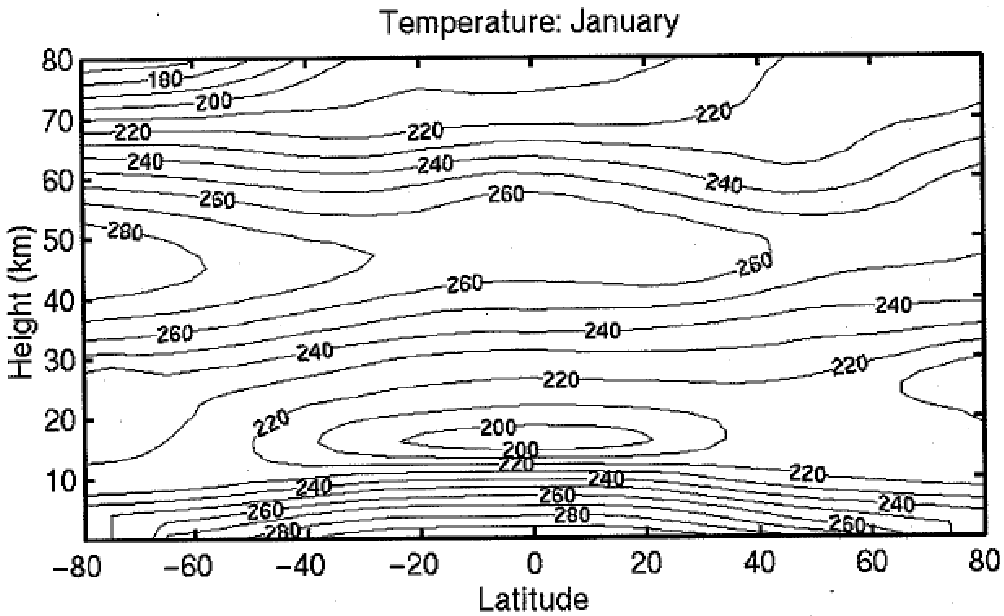


Fig. 12.8 Schematic cross section of the wave-driven circulation in the middle atmosphere and its role in transport. Thin dashed lines denote potential temperature surfaces. Dotted line is the tropopause. Solid lines are contours of the TEM meridional circulation driven by the wave-induced forcing (shaded region). Wavy double-headed arrows denote meridional transport and mixing by eddy motions. Heavy dashed line shows an isopleth of mixing ratio for a long-lived tracer.



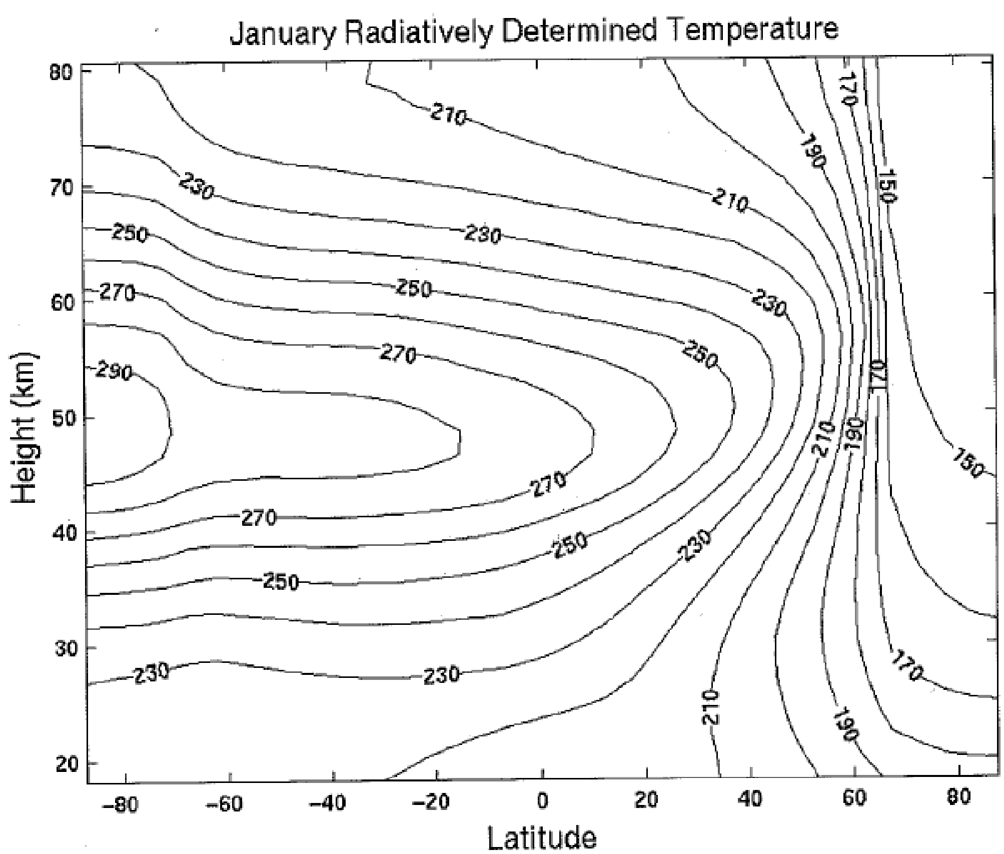


Fig. 12.4 Radiatively determined middle atmosphere temperature distribution (K) for Northern winter soltice from a radiative model that is time marched through an annual cycle. Realistic tropospheric temperatures and cloudiness are used to determine the upward radiative flux at the tropopause. (Based on Shine, 1987.)

The Brewer-Dobson circulation is well confirmed by the observed distributions of chemical tracers.

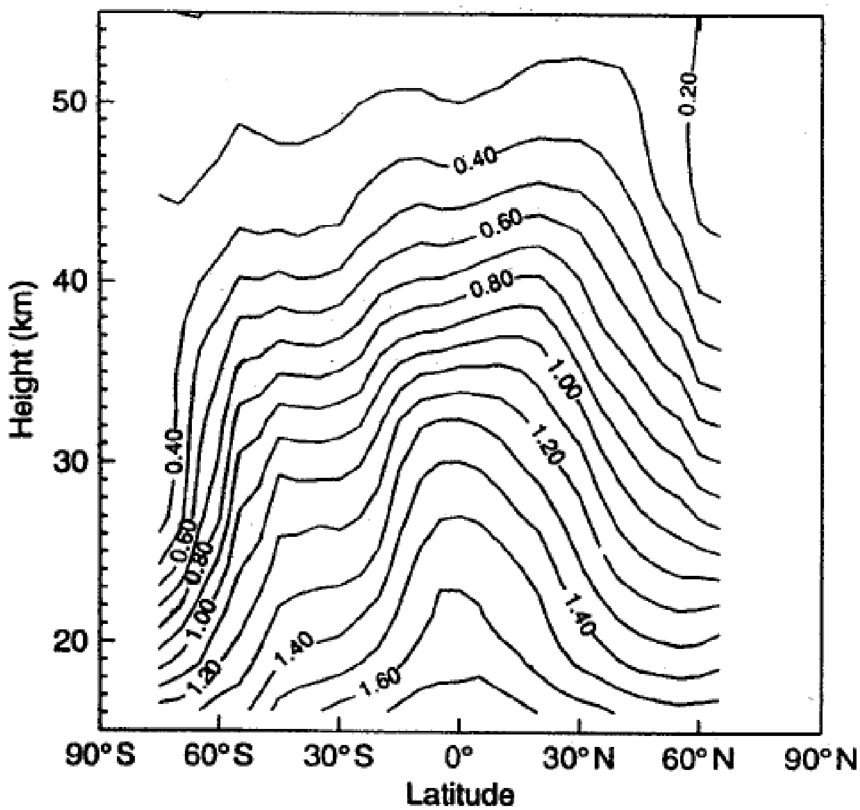


Fig. 12.9 October zonal mean cross section of methane (ppmv) from observations by the Halogen Occultation Experiment (HALOE) on the Upper Atmosphere Research Satellite (UARS). Note the strong vertical stratification due to photochemical destruction in the stratosphere. The upward bulging mixing ratio isopleths in the equatorial region are evidence of upward mass flow in the equatorial region, whereas the downward sloping isopleths in high latitudes are evidence of subsidence in the polar regions. The region of flattened isopleths in the midlatitudes of the Southern Hemisphere is evidence for quasi-adiabatic wave transport due to wintertime planetary wave activity. (After Norton, 2003.)

Another good example is ozone.

Horizontal propagation of Rossby waves is also very important. It is a mechanism by which tropical sea surface temperature changes affect the global circulation.

(a)

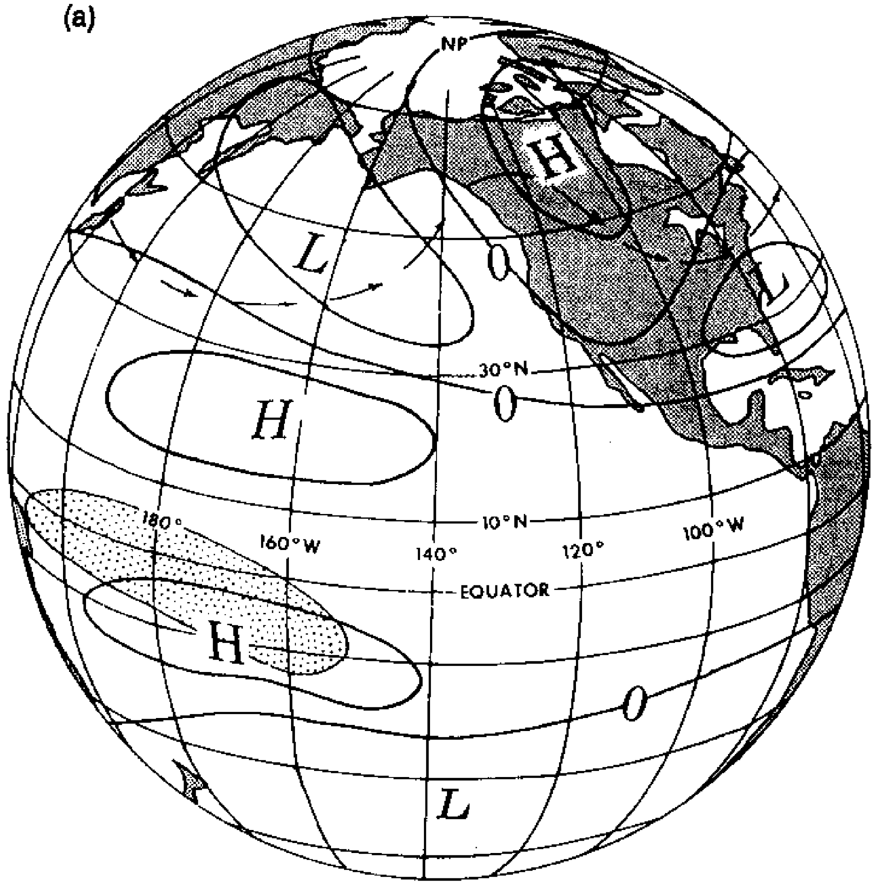


Figure 14.20 (a) Anomalous planetary wave field for northern winters during El Niño, when anomalous convection (stippled) is positioned in the tropical central Pacific. The anomalous ridge over western Canada and trough over the eastern United States characterize the so-called *Pacific North America (PNA) pattern* that upsets the normal track of the jet stream (wavy trajectory) and cyclone activity during El Niño winters. After Horel and Wallace (1981); see Wallace and Gutzler (1981) for a detailed plot. (*continues*)