Math 223a : Algebraic Number Theory

Instructor: Alison Miller	**e-mail:** abmiller@math.harvard.edu
Office: Science Center 527	**Office Hours:** W 2-3, Th 3-4
Meeting Time: MF 3-4:15	**Location:** Science Center 310
Course Assistant: Ben Gunby	**e-mail:** bgunby@math.harvard.edu

Course Website: https://canvas.harvard.edu/courses/44564

Course Description

Math 223 is a graduate-level course in algebraic number theory. Math 223a will cover local class field theory and Math 223b global class field theory.

Tentative List of Topics

Brief overview of local and global class field theory. Theory of local fields, extensions and ramification; Galois cohomology; local class field theory; Lubin-Tate theory. Time permitting: Brauer groups and/or applications to global class field theory.

Prerequisites

There are two main prerequisites for this class.

The first is a previous class on number fields, e.g. Math 129 at Harvard. Chapter I of Neukirch's *Algebraic Number Theory* (see below) is a good reference for the relevant material. This semester we'll mostly be developing the theory of local fields, so this background won't be strictly necessary much of the time, but will provide important context.

The second is exposure to homology/cohomology in some other context (simplicial/singular, de Rham, etc). We'll be developing the theory of Galois cohomology as an important tool, and I'll expect that you've previously seen chain complexes, the snake lemma, and such.

References

There is no textbook for Math 223. However, I will post PDF lecture notes on the Canvas website immediately after each class. You can also see last year's lecture notes at.

There are many good references for the course material: my recommendations are below. (Hyperlinks go to the SpringerLink website, which allows users on the Harvard network to download free PDFs of the books for free. Harvard affiliates can also download the book off-campus using EZProxy.)

Other recommended references:

- *Class Field Theory course notes* by James Milne,
  `http://www.jmilne.org/math/CourseNotes/CFT310.pdf`
  These notes come the closest in terms of exposition to the way I teach this class.

- *Algebraic Number Theory* by Jürgen Neukirch.
  `https://link.springer.com/book/10.1007/978-3-662-03983-0`
  This is a really good overview of algebraic number theory from the basics through class field theory.

- *Class Field Theory: the Bonn Lectures* by Jürgen Neurkich.
  `https://link.springer.com/book/10.1007/978-3-642-35437-3`
  More focused on the proofs of class field theory than Neukirch's other book with a more systematic treatment of Galois cohomology.

- *Local Fields* by Jean-Pierre Serre.
  `https://link.springer.com/book/10.1007/978-1-4757-5673-9`
  The classic text for local fields. Excellent but a little terse and dry.

- *Algebraic Number Theory* edited by Cassels and Frohlich. This is also a classic: the author list is a who's who of 20th century algebraic number theorists. A little uneven, and watch out for typos (there are well-documented online errata), but many chapters are very good.

- *Primes of the form* $x^2 + ny^2$ by David Cox.
  Not so relevant to the material we'll be covering in the fall, but this is one of the best places to go for statements of the basic theorems of global class theory and some of their major applications.

Software and online resources

Online computer algebra systems can be helpful for working out examples in algebraic number theory. You are encouraged to use them on problem sets, and I will occasionally post online demonstrations on the course webpage to supplement course material.

For this class, we recommend use of Sage and MAGMA.

Sage (`http://www.sagemath.org`) is an open-source computer algebra system based on the computer language Python. You can download it for use on your computer or use it online through the web notebook interface at `https://www.cocalc.com`.

MAGMA is a proprietary computer algebra system that has more features than Sage. For our purposes, the online MAGMA calculator at `http://magma.maths.usyd.edu.au/calc/` should be sufficient for any calculations you need.

The LMFDB `http://www.lmfdb.org/` is an excellent database of number theoretic objects, including, but not limited to, local and global fields. You can use its "show commands" feature to look up how to compute the things it does in Sage/MAGMA.

Homework and Grading Policies

If you are taking the class for a grade, your final class grade will be based 80% on weekly homework and 20% on the final paper.

Homeworks will be assigned weekly, and will be due Fridays at midnight. You may submit them online before the start of class or bring a hard copy to class. All homeworks will be weighted equally and the lowest two homework grades will be dropped.

The final assessment for this class will be a 5-10 page paper on a topic related to the course material.

You are encouraged to discuss the homework problems with your classmates, but you must write them up independently. You should acknowledge everyone you worked with in your homework writeups, as well as any external sources you consulted.