Math 168 Weekly assignment #3 Due Monday, Oct 1

About building

This is the first "building" assignment in this class. Of course, you will need to do some software modeling to prepare your building plan. Here are the ground rules for all such assignments: Some of the questions will require saved files from the modeling software. Some of the questions may require written answers, which should be narrative/descriptive in nature. These should be typed into any generally readable text document format (even just plain text ".txt" or Markdown ".md" is fine, or any commonly readable word processing format, or PDF). You should collect up a single written file, and all of the saved files, and submit them electronically all at once. Not for this assignment, but for future building assignments, you may need to hand in physical objects you have created as well; those you should bring to class.

1. On Monday, Oct 1, the class will have access to the following building materials: 2400 2cm cube foam blocks (720 each of orange and green, 480 each of red and blue), and ample adhesives for attaching the cubes face to face. Keeping in mind that six constructions will be built that day, limiting each project to a maximum of 400 cubes, your first goal is to choose a mathematically interesting structure that can be built by attaching cubes face to face. We will be talking about some mathematically interesting aspects of the three-dimensional integer lattice in class on Friday, Sep 28, but you likely will not want to, and ought not to, wait until then to begin this assignment. So you will need to independently choose an appropriate structure. Here are several suggestions, but you are not limited to these options; however, you do need to describe the mathematical significance of the structure you choose.
   (a) You could create loops of blocks which are knotted structures – see the wikipedia page on knot theory. Given the amount of time and number of blocks available, you would likely want to plan on creating more than one knot. For example, you might want to create both right- and left-handed versions of an asymmetric knot, as well as an example of a symmetric knot. Or you might want to construct the knotted structure that uses the fewest possible cubes, and a model of a different knot that requires more cubes to create. Or you might want to create a single model of a more complicated knot that has some special property – the wikipedia page includes links for further reading.
   (b) You could create an approximation to a 3D fractal. There are some likely-looking candidates on the wikipedia page "List of fractals by Hausdorff Dimension." You have to scroll down quite a ways because the early fractals on the list all lie in the plane. You can search other places for 3D fractals as well.
   (c) You could create an illustration of a theorem connected with three dimensions. For example, there is a beautiful block-based "proof" of the formula for the sum of the first n squares, see https://i.pinimg.com/originals/7b/2a/57/7b2a57f67c79d595a739328cd04fc75a.png
   There are a few more in the book "Proofs Without Words" that you can find online, but you will have to comb through it: most of the proofs are two dimensional, and some of the three-dimensional ones use relatively uninteresting shapes to build. But "Sums of Triangular Numbers II" is very nice, for example.
   (d) Can you make a 3D version of the "dragon curve"? You can do an internet search to find out what the usual 2D dragon curve is. You'll find that it can be created by taking a long strip of paper and folding it in half many times, then unfolding each fold in turn but only to 90 degrees, not unfolding it all the way. So imagine instead that you take a wire and fold it in half many times, and then unfold each fold to 90 degrees, but also at right angles to the

previous unfolding. You get a very interesting wandering path in three dimensions this way which you can model.

(e) You could model a 3-dimensional random walk, using a die to determine which face of each successive cube you will leave by.

(f) You could make a model of a 3-D Hilbert Curve or other space-filling curve.

(g) You could make a set of polycubes and show how they pack together. See the wikipedia page on polycubes and http://puzzler.sourceforge.net/docs/polycubes.html. There are many other possibilities along these lines. For example, there are 12 chiral (asymmetric in 3D) pentacubes, that come in 6 mirror-image pairs. They have a combined volume of 60, but they are probably too “bumpy” to make a 3x4x5 solid. On the other hand, it might be that if you add the 12 “solid pentominoes” – which are just the pentacubes of height 1 – you could perhaps fit them all into a 4x5x6 solid. That would be a very elegant construction, if it is possible (I do not know the answer).

(h) Take the infinite ternary square-free sequence constructed by this paper: https://arxiv.org/pdf/0712.0139v1.pdf and turn it into a path in three dimensions by interpreting “1” as “move right”, “2” as “move forward”, and “3” as “move up”. Model the resulting path. Similarly, you can take any interesting sequence that has at most six different values (since there are also left, back, and down) and model it as a path in three dimensions and build that from cubes.

There are many more possibilities; you are not limited to the list above, but you are welcome to choose something from that list. In any case, for this question, describe the mathematical structure you will be modeling using cubes connected face-to-face, what its mathematical significance is, and why you chose it.

2. Use VoxelBuilder (voxelbuilder.com) to model the structure you want to build. It has a very simple point and click interface that makes it quick to connect lots of virtual cubes. It’s easiest with VoxelBuilder to get the colors of your component cubes correct from the beginning – it doesn’t really have a way to change the color of an existing cube, you just have to delete the cube and put in a new one. So think about how you will use the four colors available to highlight the structure of what you’re building ahead of time. That doesn’t mean that you can’t or shouldn’t change midstream or after the fact – if you get a better idea of how to use the color, you should. Just delete and re-fill your cubes in one at a time to change color. It just may save you some time to think about color first.
   When you have completed a model of the structure you want to build, under the “Actions” menu on the top right select “Export PNG” and save the model to a file. (It may look like only a two-dimensional snapshot of the model, but VoxelBuilder actually annotates the .png file so that it is possible to restore the full 3D model.) Submit the .png file with your assignment.
   One final note: if you create a large enough structure that you find you cannot zoom out far enough in VoxelBuilder to see the whole thing, and that lack of perspective is getting annoying, you can find directions at http://studioinfinity.org/blog/2018/04/05/boxtahedral-trusses/ for increasing the limit on zooming out.

3. How does your use of color bring out the properties of your model?

4. When you’re happy with your goal structure, it is time to make a build plan. Start with a build inventory: list exactly the components you will need for your build, with quantities. Don’t forget to include adhesive to attach the cubes, even though I am supplying it. If your structure has natural subcomponents, you may want to give the inventory per component, and then show the total. (If there isn’t a natural way to break down your structure into parts, it is fine to simply

give the overall totals.) In addition to the raw table, you should describe how you obtained the totals, including any calculations; if you found the numbers simply by counting, describe the system by which you counted to avoid error. (In practice in actual builds, we both double and triple-check to ensure that we have exactly the right quantity of everything, *and* we bring extras of every item in case of counting error, loss en route, or breakage/loss on site.)

5. Next, write a structured narrative build plan, indicating how you will build your structure. Determine the order of construction: where in your structure will building start and how will it proceed? Indicate important milestones in the construction. Will you construct portions of the final structure and then connect them, or build from one point and proceed sequentially? Assume you will have two people working on the build. What tasks can they do in parallel, and at what points will it be particularly helpful for them to work together? (Sometimes you just need four hands to get certain components in place properly.) Keep in mind that the cubes are small – just two centimeters on a side – and you need to be able to reach the area where you're attaching things, so you may need to order your construction from the "inside out." Or it might be helpful to build from the "bottom up" or the "top down" – it really depends on the nature of what you're building. Keep in mind efficiency: we have 75 minutes to build the structure, so you want to construct in parallel as much as possible. Finally, although it may come as a shock, the foam cubes supplied will **not** be mathematically precise cubes, so devote a portion of your plan to discussing how you will tell if building errors are accumulating and what strategies you will use to keep the final construction as close to a cubic grid as possible given irregularities in the materials.
   Write out the build instructions as clearly as possible, without being overly verbose. Remember that someone will likely be helping to build this structure based on what you write, and that you don't want to confuse yourself – you will definitely be referring to your own build instructions during the process.
   If it is helpful, you may want to create VoxelBuilder models showing some of the components or intermediate stages of the construction as well. If you do, submit the exported .png files for those models as well.
   Submit your build instructions with this assignment. If it's convenient, you may want to bring a printed copy of your inventory and build instructions, and any images you created, with you to class – it can be easier to refer to paper than a computer screen while building. But it is not required to bring a printout.

6. Looking ahead, as soon as you arrive in class on Monday Oct 1, the materials will be laid out for you on the table in the front of class. Before you start building, find a partner in class, and compare your structure with the one your partner proposed to build. Come to a **consensus** as to which of the two models you will build – there will only be time and materials to build one of them. (Note two things: nobody's grade will be at all affected by which structure you choose to build, and if you really want to build the structure you designed but don't get a chance to in class, I will be happy to supply the materials needed for you to do so on your own time.) As soon as you have a partner and the two of you have agreed which structure you will build, feel free to get started, even if it happens to be slightly before 1:30 – you can take advantage of that time if you happen to arrive early, but we cannot go past 2:45 out of respect for Algebraic Number Theory after us. [This item does not require a response.]

Challenge problems

Building challenge problems will involve planning and executing more substantial, intricate, or sophisticated designs. As with the other challenge problems, you can do any, all, or none of the *challenge problems; each successful effort on a problem will reduce the weight of all of the other elements of the course.*

1. **High-precision cubic lattice build**. As alluded to above, EVA foam cubes (the material we will be using) tend to have low quality tolerances: the edges may vary by a significant percentage from the nominal 2 cm, faces may not be parallel, etc. Also, because the material is lightweight and not particularly sturdy, structures built with EVA foam tend to be fairly transitory. These characteristics are adequate for the projects contemplated above, but problematic for certain types of construction, or if you seek a permanent result. In particular, mathematical structures that include multiple paths of cubes that loop or reconnect with themselves, but only after independent paths spanning eight or more cubes, tend not to render well in EVA foam. (By the time one path has meandered around to reconnect to itself or another path, the accumulated errors have become large enough that the connections are awkward.) The difficulties are especially acute if the loops are interlinked and/or if independent portions of the model should fit together or interlock to appreciate the underlying structure.
   For this problem, propose and model a mathematically significant structure that exhibits one or more of the above characteristics, or for some other reason that you can clearly articulate, would not render well in EVA foam cubes glued face to face. Research another type of commercially available cubical stock (wood, plastic, etc.) that would provide the required tolerance, and determine a suitable adhesive for that material. Submit a description of your proposed structure, its mathematical significance, your software model, and the exact proposed inventory of materials. (Note a detailed construction plan along the lines of (4) above is not required.)
2. **Execute your build**. High-quality submissions to Challenge 1 above will be eligible for grants to cover the cost of the materials. If you submit such a design, indicate whether you would also execute your build if you received such a grant. (If so, and if you receive such a grant, then you are under some moral obligation to actually complete the build.) If you execute a design submitted for Challenge 1 (it is fine if others assist you in the actual construction) and submit the physical results, you will receive challenge problem credit in this class, and the structure will be yours to keep at the conclusion of the Spring 2019 semester (or immediately after grading if you supplied the materials without a grant.)