Math 168 Weekly assignment #8 Due Thursday, Nov 8, 10 AM

About polyhedra from nets

There are three main steps to this assignment, covered in greater detail below: create the net of a polyhedron, convert it into one or more files for cutting, and then assemble the cut pieces into the actual model. Within that framework, you have a fair amount of freedom. You can use different tools for the two software steps, and there are different strategies for building your polyhedron from the cut pieces. Because it's the building that drives the design, note that the steps are described with step 3 ***first****, then step one, then step two, even though you will execute them in the usual 1-2-3 order. Choose the construction approach that suits you, or a variation on any of the below suggestions, and then design toward that approach.*

3. In class on Friday, Nov 9, you will assemble your cut pieces into the final polyhedron, and then check out other polyhedra and identify their 3D point groups.

A. Obtain all your piece or piecess from the front of the classroom.

B. Pre-fold all of the score lines in the direction that they will ultimately bend when the polyhedron is assembled.

C. Start to make the edge-to-edge connections of your polyhedron in the style that you have selected. Here are some popular styles:

i. No flaps, tabs, or slits at all, just the faces of your polyhedron. In this style, you simply line up the edges of adjacent faces, and then tape them on the inside or on the outside. Pros: you can get very crisp edges. Requires the least complexity in the design/layout phase. Cons: lining up edges can be difficult. It is difficult to get tape into the inside of a corner. Tape on the outside can detract from the appearance of the model. If you are taping on the inside, it is extremely difficult to tape the edges of the last face to be closed on the inside. So likely you end up with some tape on the outside. Slight misalignments early on can compound into gaps between faces later in the construction. The resulting model can be prone to having faces cave in if the model is subjected to pressure, as the tape joint is the only thing holding edges together.

ii. Single flaps at each edge-to-edge joint. In this style, you slip each flap inside and then secure it to the face it connects to with tape or glue. (I am not certain what sort of glue will bond the material being used, so I will bring a couple of options, including "crazy glue," which you should only use with rubber gloves as it can bond skin.) Pros: relatively easy to make all connections. The flaps of the final face can often just be slipped inside the remaining opening like closing an ordinary box. Cons: Edges can be somewhat more rounded than without flaps. The extra material of the flaps can distort the shape of the polyhedron slightly. A face or faces where you were not able to tape or glue the flaps can sometimes slip out, or conversely, may droop into the interior of the model, possibly necessitating exterior tape.

iii. Dual flaps (i.e., a flap on *every* "exposed" edge of the net). In this style, you fold every flap back to half of the dihedral angle between the two faces being joined, and then you connect the flaps with tape, glue, or paper-binding clips. Pros: very easy to make all connections except for the last face. Cons: The sharp creases needed for the dual flaps can be tricky to make with narrow flaps. The quality of those creases determines how crisp the edges of your polyhedron are. The last face poses a definite problem; there is no way to clip the final junctions, since you can't reach inside to work the clips, and taping on the inside is if anything more difficult than with no flaps. Glue may work if you have the dual flaps folded accurately.

Note there is an alternate style when constructing with dual flaps: fold both flaps *outside* the polyhedron. This creates very emphasized edges with a sort of strut along them. Clips are not recommended for this construction style, because of the visual clutter they create. Glue or double-sided adhesive are recommended. You need to understand that this method leaves a very noticeable amount of material outside of the boundaries of the polyhedron being modeled, and be comfortable with that feature, before choosing this style.

iv. Flaps and tabs. On one side of every edge-to-edge junction, place a flap. On the other side, generally relatively centered, place a small tab (a little rectangular projection, sometimes with rounded or mitered corners). Cut a slit in the flap at the correct position for the tab to slide into. Then, you construct by slipping the tabs into the slots on the corresponding flaps (which then go under the adjacent faces of the polyhedron from which the tab is emanating). Early in the construction, you generally need to tape the tab in place inside the polyhedron. As its curvature develops, you may find that your tabs are holding well in their slots with no additional adhesive. Pros: This produces the most secure and stable models of any of these styles. The final face closes up just the same as any of the other faces. Cons: More complexity in the earlier stage of laying out the cut design.

D. In any case, whatever your style of connections are, you generally work from one point on the net around the polyhedron. Incorporate your additional pieces as needed, if you divided your net up into multiple pieces for the purposes of making it larger or controlling the colors as desired. Proceed in this way until there is just one opening with one face left to close up. As mentioned in the above styles, securing that last face often requires some change in the procedure as compared to the other faces, since you can no longer reach inside the model.

E. When you are done with your model, label it in some way that does not obviously refer to you, but which will be different from other students. (For example, pick your favorite three-digit number and label it with that, or your favorite three-letter word, etc.) Then examine the other models. For three of them other than yours, write down their label and their 3D point group and hand that in (put your name on what you hand in). Also on what you hand in, let me know the label you used on your own polyhedron.

F. When all of the models are built, we will discuss the results.

Now, going back to the beginning:

1. Choose a polyhedron and construct its net. You may choose any polyhedron that does not have a trivial symmetry group, *except* for one of the Platonic solids (too standard/simple). Also, you may *not* simply use a published image of a polyhedron net, even of an otherwise allowed polyhedron; you must model the polyhedron itself and lay out its net. Below are two main possibilities and a couple of alternate approaches for creating the net.
   (a) Use the polyhedron net constructor at https://polytemplate.com. This will guide you through a few questions to initialize your model – don't be afraid of the "advanced" options – and then you can apply "truncate" and "dual" operations to your model as desired. You can select a couple of different flap styles (what this site calls "tabs" we call "flaps"). An SVG file of the net of your polyhedron, which you can download by right-clicking or control-clicking and selecting an option like "Save this link" or "Download this link", appears on the lower right of the display. Some significant caveats: this site uses a fairly stupid dual algorithm (my Open Neighborhood talk next Monday, Oct 29, will address this sort of issue, you may want to attend) and so it can produce a "polyhedron" with non-planar "faces." Such a net will likely not build well! Also, its layout algorithm allows tabs to overlap, so you will either have to clean these tabs up, or generate a net without tabs and add tabs in the next step. Sometimes the layout algorithm generates a "Template Error". When that

happens, one or more faces have been left out of the net. If it is just a few, you can add them back in by hand in the following step if it's a model you would otherwise like to build.

(b) Use a dynamic geometry program like Geogebra and in 2D mode, just lay out all of the faces connected to each other in a net. This is particularly plausible when the polyhedron you want has only regular-polygon faces, because regular polygons are generally easy to draw. You just need to keep track of each face that you have laid out, and make sure they are all connected correctly without missing any faces or laying out any face twice. You just connect the faces edge to edge as they are connected on the polyhedron, but keeping everything flat. Of course, some of the connections will open up into gaps that will go away when you fold the model up. Note that Geogebra can export an SVG file.

(c) You can use a hybrid method of (a) and (b): polytemplate will give you the "polyhedron data" listing the edge lengths and internal angles of each face. You can then lay these shapes out in Geogebra, or in a CAD program such as the freely-avaliable QCAD (http://qcad.org). Use the "label faces" option in polytemplate to help you keep track of the faces you've laid out and how they should be connected.

(d) There is a very old polyhedron modeling program called Poly (http://www.peda.com/poly/) that you can try. I do not know if it will export in SVG or PDF format, as required. If you decide to try it, I will be interested to hear if it works for you.

(e) You can get one of the Stella family of programs (https://www.software3d.com/Stella.php). They produce nets. They are somewhat pricey commercial programs, and I am not advocating your purchasing any of them, just making you aware of their existence. I am not certain if they will export in SVG or PDF, but I think they will. I am not guaranteeing it, but it is likely. Certainly, these programs have a wide library of polyhedra and produce attractive images.

(f) You may know of another application that can produce polyhedron nets. You are welcome to use it, just let me know what program you used.

(g) Whatever program you used, submit the program's internal file format of your polyhedron model (if applicable; polytemplate just produces an svg file) and the raw svg or pdf that you use as input to the next step. Be sure to label or name that file so that I know not to cut it.

2. Edit/clean up the net to produce a cuttable design.

   (a) For this, you will need to use something that will import the net you produced in step 1, probably as an svg or pdf file, edit it, and then export it back as an svg or pdf file for cutting. (svg is preferred when possible). Some options include: Inkscape (http://inkscape.org), a free vector-drawing program analogous to Illustrator; Vectr (http://vectr.com), a less-powerful/simplified web-based app along the same lines; LibreOffice (http://libreoffice.org), a free office suite, the drawing program of which can handle svgs and pdfs; Primal Draw (http://primaldraw.com), a similar on-line drawing program, Gravit Designer (https://designer.gravit.io/), another on-line vector-drawing program; or QCAD (mentioned above). There may be other options; use what is comfortable to you. The key is that your program should be able to import the svg or pdf you produced in step 1, modify it, and export the results as svg or pdf (preferably the former).

   (b) Import the net you produced in step 1. If it does not have the flap/tab style you want, add flaps or tabs to edges as required to provide all of the material you will need for building. You may need to track which edge will connect up with which as it folds up. This is easy for pairs of edges near the beginnings of the gaps, but can be trickier for edges farther away. It may be helpful to make a rough temporary paper model of your polyhedron as a guide. If you are unsure what the shape of a flap should look like, check out nets of polyhedra published at places like Wolfram Mathworld or other websites as inspiration. Remember your flaps cannot overlap! (There is only one layer of material in a sheet, of course.)

(c) Decide what color polyhedron you want. This will determine the sheet size. (See the list of colors and sheet sizes on assignment 7.) Your design must be scaled to fit within a 12 mm border around the edges of the sheet. If that will make the edges/faces of your polyhedron too small, split your net into two pieces along one edge of the net and lay the pieces out on separate pages. (For a complicated polyhedron, or if you want to get certain colors on certain regions of your finished model, you may want to split it into more than two pieces. That's fine; the rule is simply that your final design must fit on no more than four sheets of material.) Remember to add flaps/tabs of your chosen style on the appropriate new edges you created by splitting the net.

(d) Once you have your final scaled net laid out on one or more pages, ready it for cutting. Make sure that all of the outer boundaries are black, and if possible, join them into a single continuous path. (If you cannot figure out how to do that joining, it is not absolutely necessary.) All of the edges of faces of your polyhedron which are not on the boundary need to be scored (rather than cut). Indicate this by making those lines red in your layout. Finally, if you added tabs, make sure there are black line segments indicating cuts along the proper portion of the edges into which they will be inserted. (But do *not* slit the entire edge. Just make a large enough slit for the tab to insert into. You don't need slits if you didn't choose to use tabs. Flaps don't require slits.) Remove any extraneous lines that should neither be cut nor scored, or make them a color other than red or black. Remember: everything that is black will be cut, everything that is red will be scored. Don't worry about the fills of regions, they are irrelevant and unnecessary; only the lines will be cut/scored.

(e) I recommend you print out your final design (you may have to reduce it to fit on your paper, but turn in the full size file), cut it out on the black lines, fold on the red lines, and make sure it seems as though it will go together properly. Make any necessary adjustments. Add in any cuts or scores you forgot.

(f) Submit your final svg or pdf file or files (svg is preferred if you have a choice; up to 4 cut files will be accepted). If there is an internal file format for the application you used, submit that as well. Clearly indicate what color each submitted svg or pdf file should be cut out of. Note that I will not be scaling these files for you; make sure to submit them scaled to the size you desire. Your files must be submitted by 10AM on Thursday Nov 8.

(g) What is the 3D point group of the polyhedron your pieces will make?

(h) I will start cutting these in my office at 10 AM on Thursday Nov 8. You are encouraged to stop by if possible to see the cutting and scoring process. I will cut them in the order that people arrive.

Challenge problems

1. **No mirrors?** You will receive challenge problem credit if your net produces a polyhedron model with a non-trivial point group correctly identified, such that the point group contains no mirror symmetries.