Math 168 Weekly assignment #11

About generalizing from 2D to 3D

In class we encountered the *Lissajous curves*, which can be given as *parametric curves* in the plane with the equations

$$x = a\cos(j\theta)$$
$$y = b\sin(k\theta),$$

where a, b are arbitrary positive real numbers and j, k are arbitrary positive integers, and θ is the "parameter" which varies from 0 to τ. In some sense, these curves generalize the circle, which is the special case when $a = b$ and $j = k$.

If you are not familiar with these curves from class or from some other context, you should use any graphing software you are comfortable with to experiment, trying different values of j and k especially, and seeing what the effect of varying a and $b$is. Geogebra will do; set up a slider named θ (or just t is easier to type), which should run from 0 to τ (you can just type in 6.2831853), define a point (say A) whose coordinates are given by the expressions you want, and then tell GeoGebra to construct the locus of A for the slider t.

Now that you have a clear picture what Lissajous curves are like, your assignment, briefly stated, is to generalize Lissajous curves to three dimensions, and then 3D print a model of one 3D Lissajous curve that you particularly like. That's all there is to it; the remainder of this document consists of some steps for accomplishing this (and gives specific items that you should submit for the assignment).

1. Write down a description in words of what you want the 3D Lissajous curves to be like. What characteristics should they have? (submit this)

2. Decide what sort of mathematical object will best model the concept you described in question 1. Then determine what, for you, will consist of the "general form" of a 3D Lissajous curve; it may have some constants in it which will produce different examples depending on their values, like a, b, j, k above. Submit a concise mathematical description of this general form.

3. If you decided on something other than a parametric curve in three dimensions $(x(t), y(t), z(t))$ in question 2 above, that is totally fine, but then it will be up to you to find software which will help you turn that concept into STL files suitable for 3D printing. In this case, submit the following material: what the program is you used, the native file modeling your 3D Lissajous curve, and the STL file it produced, and pick back up at step 9 below.

4. If you did decide on a parametric curve for your 3D Lissajous curve, then download OpenSCAD from openscad.org, and copy the files from the openscad directory on Canvas to a directory on your computer. Start up OpenSCAD and tell it to open the file "mathgrrl_sweeper_168.scad". You should see a 2D Lissajous curve. (Well, actually, it's a 3D object created by sweeping a circular cross-section along a 2D Lissajous curve. But that doesn't count as generalizing 2D Lissajous curves to three dimensions, so don't just hand in this default curve...) (Nothing to submit for this item.)

5. At the very top of the file, you should see the spot where the three parametric expressions for the curve are defined. Note that by simply setting z to a constant 0, we can cover the case of 2D parametric curves as well. You should put your parametric expressions for your 3D generalization in place of the default expressions that are already there, including replacing this 0 for z. When you have entered your expressions, hit "F5" or click on the icon just to the right of the text display of the .scad file that looks like a cube with two ">" symbols over it. You should see your version of 3D Lissajous jump into life on the screen! Does it look the way you expected/hoped? (Nothing to submit for this item.)

6. Try different instances of your version of 3D Lissajous curves until you find one you like, that you want to have a physical model of. Remember to hit F5 or that icon any time you make a change to the file that you want to see the effects of.(Nothing to submit for this item.)

7. Just below the parametric equations in the .scad file are a few more settings. When you have the curve you like, try varying these to see the effect. The values you should use for the first two in your final file for 3D printing are specified in the comments, but the third parameter is up to you; it just depends on how thick you would like the "tube" or "rope" of your curve to be compared to the overall size of the model. (Nothing to submit for this item.)

8. When you have finalized the settings, and are happy with the shape, here's how to convert it to an STL file for 3D printing. First, render the curve by clicking on the icon just to the right of the icon mentioned above. Then under the File menu, subheading "Export," select "Export as STL" and save that file under whatever name you would like. Submit your edited .scad file, and the resulting STL file.

9. Submit all of the above items via Canvas by Tuesday, Dec 4. Then you need to turn your STL file into a physical object to turn in at the prototype session on Wednesday Dec 5 at 1:30 PM (you will get these back at the final class open house). The below step tells you how to do that, but WARNING: the SEAS lab is being very generous to help our class with this. They will not take kindly to all of you showing up en masse Wednesday morning to do the 3D printing, plus they have only two 3D printers to use, so it just won't work, plus you have your final project prototypes. So I **strongly** recommend that you do not leave this to get your STL file on Canvas late on Tuesday and try to squeeze in printing Wednesday morning. The best is if you all spread out when you show up at the lab. There is no reason you can't start on this assignment right now and have it ready to go Thursday or Friday, say, and go by the lab then. There is no attendant at the lab during the weekend, so you can't go then unless you are already checked out on the facilities. If you didn't get to it by Friday, be sure to be ready to go there Monday so that if there are any glitches, you can still complete by the due date. If you leave this one until the last minute, you won't like the results. You have been warned.

10. Take your STL file (maybe put it on a USB stick, not sure how they are going to want it) over to the SEAS 3D printing lab in Pierce Hall G11 any time 9AM to 5PM (well, don't arrive right at 5!) on a weekday. Tell the folks there that you are from Math 168, and ask for Steve Cortesa, Maddie, or Elaine. They will show you how to put your STL file into the software that controls the 3D printer, and how to print it. When you're done, you'll be able to take your physical 3D Lissajous; hand it in at the prototype session on Wednesday. (If you have other access to a 3D printer, or if you are already familiar with 3D printing at the SEAS labs, you are welcome to 3D print your file yourself and submit the resulting model.)