

# The Mechanics of Endogenous Innovation and Growth: Evidence from Historical U.S. Patents\*

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## Abstract

How does technological progress occur? Is the nature of innovation stable over time? We shed new light on these questions through a mixture of empirics and theory. We begin with an empirical analysis of patents granted by the United States Patent and Trademark Office. This analysis reveals several striking facts that emphasize the increasing importance of novel combinations of technologies for U.S. patents, compared to either new technology development or the reuse/refinement of older technology combinations, and the localized nature of these recombinations. We build an endogenous growth model that can match these facts and illustrate the underlying mechanics of the technological development process.

**JEL Classification:** O31, O33, O41, L16.

**Keywords:** Endogenous Growth, Innovation, Research and Development, Patents, Citations, Scientists, Technological Development; Recombinations.

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# 1 Introduction

Innovation is a key determinant of long-term growth, and thus the mechanics of technological progress have long captured the attention of economists and historians. Researchers have made substantial progress towards the development of our theoretical toolkit for modeling innovation, towards the empirical analysis of recent patent grants utilizing the NBER patent database (Hall et al. 2001), and towards the construction of detailed historical accounts of innovation and its impact. Despite this overall progress, our ability to connect these various strands together has been severely limited by data constraints. Such connections would be very valuable, however, as a healthy exchange of theory and empirics provides the best foundation for research. To date, our theory models related to long-run technological progress have raced far ahead of our long-run empirics of innovation and its evolution over time.

This study helps close this gap. We first develop a dataset with all patents granted by the United States Patent and Trademark Office (USPTO) since 1836. A crucial feature for our work is the inclusion in this dataset of all technologies listed on USPTO patent grants. Granted patents typically list several technological components, and the current count of these detailed technologies is over 150 thousand. We use this dataset to document several facts about the changing nature of patenting in the United States since the 1830s. We particularly describe how novel combinations of past technological components represents a growing share of U.S. invention compared to the development of new technologies or the reuse/refinement of previously observed combinations. We then embed these insights into a fully-specified growth model. This growth model is particularly striking for its depiction of a growth path with changing forms of innovation, and the model can match regularities in the data that we do not target in its construction. We analyze the novel spillover properties of our model, and we use a calibrated version of the model to quantify the impact of subsidies to encourage various innovation forms.

Our paper makes several important contributions in terms of data development, empirical analysis, and theoretical modelling. Our most important contribution is, in fact, the connection of these elements together. This unified framework helps us understand why U.S. technological progress is shifting towards novel combinations, including factors such as the very long useful horizons for technological components, the consequential deepening pools of technologies available for recombination, and the exponential power of combinatorial processes. Many of these themes are present in Weitzman’s (1998) seminal introduction of combinatorial processes to growth models, but we also articulate several key differences necessary to link the model to U.S. historical data. First, we measure and model the breaks on the combinatorial growth process that come through the localized nature of combinatorial processes—most all novel combinations employ technologies from a narrow age range or field set—and the spreading out of innovative effort over an ever-longer technological horizon. Second, while Weitzman (1998) focuses solely on the combinatorial power of an existing technology base, we describe the different forms of innovation (i.e., novel technologies, novel combinations, and reuse/refinement) and how profit-maximizing agents shift across them over a time in a manner that closely matches the U.S. data. As discussed later, our depiction of the origin of novel technological components on patents using frontier building blocks is important for describing the perpetuation and evolving balance of the multiple forms of technological progress.

Section 2 provides a brief introduction to discussions of combinatorial processes for technological innovation from economic history. Depictions of the combinatorial process date back to

at least Schumpeter (1911), and many accounts of major innovations (e.g., light bulb, printing press) emphasize these features.

Section 3 describes our data construction and provides the six key empirical observations that form the foundation for our model:

- 1 The composition of innovation is shifting towards novel combinations of existing technologies. The share of patents with a novel technology combination is less than 50% in the 1800s, and the share is greater than 75% for patents granted after 1970. By contrast, novel technology development declines in absolute and relative terms since the 1800s. The reuse/refinement of older technology combinations shows an important non-monotonic pattern with initial increases and subsequent declines.
- 2 The number of distinct technologies combined together in each patent is rising with time. The average count is less than two technologies per patent for much of the 1800s, and the average count is above four technologies for patents granted after 1970.
- 3 These forms of technological progress differ in their economic impact as measured by patent citations and firm-level employment growth regressions. The economic impact is highest for patents developing novel technologies, second for patents developing novel combinations, and weakest for reuse/refinement patents.
- 4 Technology use is maintained for a long time after original invention. More than 80% of technologies ever developed are still used during the course of a decade today. As a consequence, patents granted today still build extensively on technologies first observed in the 1800s.
- 5 Technological combinations are localized in nature. For example, about 70% of the technologies combined with a focal technology are born within 20 years of the focal technology’s birth year. As a consequence, combinatorial possibilities do not extend over the full range of inventions.
- 6 Novel technologies are born on patents using the most recent technologies. Novel technology patents typically have more technologies listed on the patent than contemporaneous novel combination or reuse/refinement patents.

Strumsky et al. (2011, 2012) and Youn et al. (2015) introduce the approach of using the lists of technologies on historical USPTO patent grant publications to distinguish forms of technological development over the past two centuries. Our classifications differ somewhat from Strumsky et al. (2012), but we follow their work overall in developing the first two facts. We develop Facts 3-6 to provide the foundation for our model, and Section 3 includes extensive supporting evidence around these facts. This section also describes complementary observations from the data that are less critical than these six emphasized features.

Section 3 develops our model. The model incorporates directly Facts 3-6 into its design, with Facts 1 and 2 being untargeted moments to compare against. In the model, a line of technological building blocks is continually expanded upon and used in novel combinations or reuse/refinement roles. Inventors are spread out over the line, which is always weakly increasing in length due to the sustained usefulness of building blocks (Fact 4). Each point on the line is characterized by its focal technology, the pool of technologies that it has been

shown to be useful to combine with, and whether this technology pool currently allows for novel combinations. If the technology pool is useful for novel combinations, inventors pursue them from the current pool given their greater profit contribution (Fact 3). Technology pools transition into stale states probabilistically to reflect that novel combinations exhaust the pool and that not every combination will be useful. If the technology pool has transitioned into a stale state, inventors can either reuse/refine an existing combination or seek to add another technology to the pool. The latter is governed by a random draw of another point on the technology line for an inventor, and the cost/success of incorporating this point depends upon the distance from the focal technology (Fact 5). Very distant ideas are costly to pursue, so inventors focus on reuse/refinements. Closer points are more feasible, and thus inventors seek to refresh the technology pool by adding another technology. For inventors near the leading edge of the technology line, this search process will occasionally identify a point beyond the end of the current line, opening up a novel technology that was not previously known (Fact 6).

This fairly simple model structure has strong predictive power for matching the first two facts. Fact 1 is more challenging fact, with Fact 2 being trivially implied. Consider first the rising role of novel combinations relative to reuse/refinement. We show below that even very old cohorts of technologies display this property, despite their technologies having been present for many years. In our framework this comes from the reduced likelihood over time that technology pools transition out of the novel combination state. Adding a tenth technology to a pool has a much larger impact for the potential combinations in the pool than adding a fifth technology, thereby exponentially lowering the rate at which the technology pool transitions to the stale state. With continued deepening of the pools all along the technology line, the whole innovation process shifts increasing towards novel combinations through a localized combinatorial logic. At the leading edge, the pace of novel technology development is decreasing for two reasons. First, the ever lengthening of the technology line lowers the share of inventors considering this leading edge; second, even those inventors at the leading edge experience the impact of ever-deepening technology pools focusing their efforts on novel combinations. As such, the simple framework can deliver through profit-maximizing agents the broad trends in types of innovations evident in the data, including the non-monotonic share for reuse/refinement.

[Section 4 is still to come. This section will consider a calibrated version of the model and counterfactual exercises. The presentation will show some preliminary results.]

The last section concludes. Our work connects several literatures that have largely remained separate. First, we clearly build upon a literature from economic history about the nature and processes of technical change. This literature is discussed in Section 2, and Mokyr (2011) provides a broader review. We likewise build upon a lengthy theoretical growth literature. Our framework offers an intuitive, tractable way of bringing more of the long-run empirical evidence on U.S. innovation into these growth models. The most important predecessor for the combinatorial aspects of this work is Weitzman (1996, 1998), and we earlier noted the distinctions between our approaches. This paper is also part of a broader emergence of work that utilizes empirical findings to provide a more appropriate foundation for theoretical models. Examples include Klette and Kortum (2004), Lentz and Mortensen (2008), Akcigit and Kerr (2010), and Acemoglu et al. (2013). Our work differs from this recent line of research in its historical focus and emphasis on very long-run economic changes.<sup>1</sup>

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<sup>1</sup>Prominent theoretical work in this literature includes Romer (1986, 1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995), Barro and Sala-i-Martin (1995), Kortum (1997), Aghion et al. (1997,

Finally, our work connects to an empirical literature that uses patents to study technological change and economic growth. Cohen (2011) provides a comprehensive review of the micro-economics literature on technical change, and Dosi and Nelson (2011) survey parallel work in evolutionary economics. Particularly relevant for our study is the work on combining technologies in patents by Fleming (2001), Fleming and Sorenson (2001, 2004), Furman and Stern (2011), and Gruber et al. (2013). This work emphasizes issues such as the uncertainty involved in new combinations and the role of science and scientific training in directing this process. Our work is also very closely related to the growth in team sizes for scientific progress documented by Jones (2009) and Jones et al. (2007) and also search processes for innovation (e.g., March 1991, Stuart and Podolny 1996). Our work clearly differs from these papers in its focus on the historical evolution of these processes and our efforts to synthesize these features into a framework that can be embedded into a general equilibrium model of the economy and its development.

## 2 Combinatorial Processes in Economic History

The notion that innovation has combinatorial characteristics is deeply rooted in the literature on the history of technological change. Thomas Edison's oft-cited quote that "genius is one percent inspiration and ninety percent perspiration" stems from the idea that breakthroughs arise both from 'eureka moments' and the costly assimilation of knowledge that is already known. Notably, Weitzman (1998) uses Edison as an example to motivate his formalization of recombinant growth. Edison's 1880 patent on the incandescent lamp reflected a novel combination of the candle and electricity. Researchers at his New Jersey laboratory discovered that carbonized Japanese bamboo could be a particularly effective filament material. Combining this idea with the ability to harness electricity gave rise to the first commercially practical incandescent light.

Schumpeter (1911) was among the first to consider the combinatorial character of innovation, though he did not explicitly reference technological change. He elaborated on the "new combinations" that produce new goods, new methods of production, new markets, new sources of supply, or new ways of organizing an industry. Analogous to his later 1942 writings on creative destruction, he argued "new combinations mean the competitive elimination of the old" (p.67). Schmookler (1966) discussed recombinant technology formation in a context where invention represents a "novel, useful combination of pre-existing elements" (p.60). For Schmookler, however, the potential for combinatorial growth through, for example, scientific discovery, was not the cause of technical progress; rather, combinations would only be developed in response to changes in demand. Contributions emerging from the literature on technical path dependence (e.g., David 1985) emphasize the recursive path through which each new invention is made by recombining inventions that are the product of earlier recombinations (Arthur 2009). According to Nelson and Winter (1982, p.130) "innovation ... consists to a substantial extent of a recombination of conceptual and physical materials that were previously in existence." Rosenberg (1974) describes the need to articulate the separate "pools" from which combinations can emerge.

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2001), Howitt (1999), and Hopenhayn et al. (2006). Jones (2002) quantifies the importance of innovation to U.S. economic growth. Related papers on spillover benefits are Spence (1984), Griliches (1992), Caballero and Jaffe (1993), Irwin and Klenow (1994), and Eeckhout and Jovanovic (2002).

History is replete with examples of knowledge combinations (Mokyr 1992). Windmills, which boosted energy utilization in medieval Europe, arose through the combination of "wind" and "sail", and the printing press as one of the major innovations of the early modern era (Dittmar 2011) derived from its inventor—a German metalworker called Johannes Guttenberg—observing how wine press technology operated in the Rhine Valley. Combinations led to the development of key technologies during the British Industrial Revolution. For example, when Samuel Crompton invented the famous cotton spinning mule in 1789 he combined two existing inventions: the moving carriage of the spinning jenny (invented by James Hargreaves in 1764) with the rollers of the water frame (invented by Richard Arkwright in 1769). Crompton's spinning mule was considerably more versatile than its predecessors and could spin cotton thread either coarse or fine. In the late nineteenth and early twentieth centuries, knowledge acquired from the development of "lighter than air" aircraft such as dirigibles and hot air balloons helped to jump start humankind's conquest of flight (Chanute 1997). The Wright Brothers developed their mechanical abilities in aviation from their printing and bicycle manufacturing activities.

At certain times and in certain places the potential for combinations may increase. Mokyr (2002) argues that the exchange of ideas was widespread during the European enlightenment of the late seventeenth and eighteenth centuries as intellectual institutions such as scientific societies were responsible for the diffusion of "useful" knowledge. David (1998) comments further that "scientific research communities may be studied as social networks within which ideas or statements circulate, acquire validity as reliable knowledge, and are recombined to generate further new ideas" (p.115). When communications improvements lower the cost of transmission across geographic areas they increase the number and range of individuals that an idea can reach.

Sometimes the impetus for technical change can be traced back to a single sector. In his famous study of machine tools, Rosenberg (1963) explained some of the mechanisms through which inventions get recombined or reused. He showed how the machine tool industry played a pivotal role in the diffusion and convergence of new techniques across multiple industrial sectors. Between 1840 and 1910 metallurgical advances and the adoption of standardization and precision processes by machine tools manufacturers facilitated technical solutions in the firearms, sewing machines, bicycle, and automobile industries. Key twentieth century inventions such as magnetic ferrites, the oral contraceptive pill, or matrix isolation, can all be explained by the assimilation of generations of technical know-how (NSF 1968). These innovations, while ostensibly new, were really developed from a combinatorial processes of re-configuring existing knowledge.

Notwithstanding the rate of invention due to recombination is potentially faster than the exponential rate, in the long run there may still be impediments to technological progress. Jones (2009) describes limits to potential recombinations due to what he describes as the "burden of knowledge". As knowledge accumulates and the flow of ideas becomes congested, educational and coordination challenges increase significantly as teams of inventors with specialized knowledge are needed to develop new breakthroughs. While Bresnahan (2011b) agrees with the main idea that "much of technical progress is recombinant", he also cautions like Jones that there may be obstacles to growth. Spillovers associated with reuse imply that innovation is characterized by increasing returns, yet the ability to identify the complementary capabilities of recombinant and general purpose technologies in market settings (what he describes as

"entrepreneurial knowledge") is a scarce resource relative to the function of invention. Both of these ideas—the "burden of knowledge" and the scarcity of "entrepreneurial knowledge"—speak to controversial arguments that the limits to economic growth have been reached (e.g., Gordon 2012).

### 3 Trends in U.S. Innovation

This section describes empirical regularities that are the basis for our model. We first describe our data and how we group patents into various types. We then provide a list of our key facts and a detailed discussion of their foundation.

#### 3.1 Patent Dataset

Our patent data include all patents granted by the USPTO during the 1836-2004 period. Economic historians often start historical innovation studies in 1836 due to the difficulty in developing reliable patent data earlier. During this period, the USPTO granted over seven million patents. Across such a long time span, the collected information for patents is limited. We describe next the key features we have developed.

##### 3.1.1 Technology Subclasses

Our primary focus is on the very detailed technology codes listed on patents. The USPTO classifies each granted patent with one or more technology codes. These technology codes for granted patents since 1836 are publicly available in electronic format. USPTO classifies patents at two levels. The first is called the patent class, and there are 427 patent classes listed on our patents during this period. Examples of the patent class level are "Geometrical Instruments", "Stoves and Furnaces", and "Chemistry: Electrical and Wave Energy". Hall et al. (2001) further describes these patent classes and develop a taxonomy that is often used to group these classes into 36 technology sub-categories or into six primary technology categories. We use these higher levels of technology grouping at several points in our analysis.

The second level of technology depiction is the patent subclass level. This level is the focus of our analysis, and we use the simple term "technology" to describe an individual patent subclass for expositional ease. This level of classification is very detailed, with 151,208 unique technologies listed on our patents. An example is "Stoves and Furnaces/Solar heat collector". A majority of patents list multiple technologies, as described in detail below, and how these technologies are combined together provides an important foundation for studying endogenous innovation and growth. With our sample, these technologies are combined into approximately 3.3 million unique technology combinations for U.S. domestic inventors, as defined below.

The USPTO adds new technology codes as they identify the emergence of new technologies. It is often difficult for the USPTO to identify a new technology immediately upon its invention. As the USPTO accumulates information and recognizes that a new technology code should be used, it reviews past patent grants to recode them to reflect the category as appropriate. As an example, Strumsky et al. (2011) discuss the case of a nanotechnology category that was officially recognized in 2004. After examining and updating prior patents to reflect the new code, the code was first assigned to a French patent from 1986. Strumsky et al. (2011) provide extensive details on these procedures.

When describing our trends below, we mention this updating process at several points. It is important to emphasize in advance that this updating process does not mechanically produce a trend in the data for technology counts per patent. When updating, the USPTO may add a new technology code to a patent, but they may also subtract one or more earlier technology codes if the new one is a better description. The USPTO also updates the entire database to the new scheme, even very old patents. The most important consequence for our work is that we should be cautious about over-interpretation of trends in the last decade or so. While the USPTO is extremely unlikely to identify a new technology from 1872, or even 1972, that it had earlier not recognized, this will happen to some degree for 2002. Our focus is on the very broad trends since the 1830s, and so we do not spend much time on this. In a few descriptive calculations, we drop the period after 1994 to check robustness.

### 3.1.2 Types of Inventions

A crucial part of our data development process is the assignment of types of inventions. To convey the patent classifications in the simplest manner, it is helpful to consider an illustrative patent that combines technologies A, B, and C.

- *Novel Technology:* Our first category is the development of a novel technology. We classify a patent as a novel technology development if any of the three building blocks A, B, or C appear in the grant year of the patent for the first time. We do not differentiate the first patent to invent a new technology within the grant year, instead viewing patents with the same grant year as contemporaneous work.
- *Novel Combination:* Our second category is the combining of novel combinations of technologies that have been previously developed. A novel combination would be the first time the ABC combination is observed, with all three individual technology elements A, B, and C having been previously developed.
- *Reuse/Refinement:* Our final category is one where the exact sequence of technologies has been previously observed. For our illustrative patent, it will be classified as a reuse/refinement if a patent in a prior grant year used the exact sequence ABC, too.

This approach broadly follows Strumsky et al. (2012), although we calculate our metrics with some small differences.<sup>2</sup> In calculating these groups, we use all patents filed with the USPTO to determine the dates at which technologies were born. That is, if a foreign invention

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<sup>2</sup>The most important difference is that Strumsky et al. (2012) focus their novel combinations category on the first time a pairwise combination is observed. Thus, to continue our illustration, we classify the ABC patent as a novel combination if this is the first time this triplet is observed, even if the pairwise combinations of AB, AC, and BC technologies are present before. Strumsky et al. (2012) would instead classify this case with reuse/refinement because it does not have a novel pairwise combination. In unreported work, we analyze new pairwise combinations separately from these cases where a new sequence of prior pairwise combinations is observed. New pairwise combinations behave somewhat more like novel technologies, while new sequences of prior pairwise combinations behave somewhat more like reuse/refinement. Nevertheless, these two groups behaved similarly enough that we choose to group them as described.

Patents list a primary technology code and other technology codes. The order in which non-primary technology codes are listed is arbitrary. In general, we do not consider the order of technologies in this work (e.g., ABC versus ACB), instead focusing just on the group of technologies present. An exception to this is that we discuss below the extent to which novel technologies are listed as the primary technology for patents.



develops a new technology first, we use this prior invention’s existence to classify our U.S. domestic patents. Most of our graphical trends and analytical depictions then focus on just the technological development process evident in the United States.

### 3.1.3 Additional Patent Traits

Across the full sample period, we are also able to determine the location of the inventors. To match the growth literature’s conceptual framework of a single country’s growth process, we focus most of our analysis on the United States. This single-country focus is also important from an empirical perspective, as the relative rate at which foreign inventors file patents with the USPTO has been increasing over time. From an extremely small start, foreign inventors now account for more than half of USPTO grants. Thus, our trends are most appropriately calculated over just the U.S. sample, although most of the facts that we outline below look very similar if foreign inventions are included. As patent geographic data are not electronically available for the full period, we use OCR techniques to identify each inventor’s location from the patent filings. We have 4.8 million patents that include at least one inventor living in the United States as listed on the patent grant.

Figure 1a shows the annual level of USPTO patent grants for all inventors and when restricting attention to patents filed by inventors living in the United States. We focus on this solid line that represents grants to U.S. domestic inventors. The total growth rate is 2.4% per annum for 1836-2004; looking at the period after 1868, when the big initial run-up is concluded, the growth rate is 1.1%. The growth rates for domestic patents that have either novel technologies or novel combinations are 2.6% and 1.6% over 1836-2004 and 1868-2004, respectively. As noted by Griliches (1990), changes in patent counts over time combine both changes in technological progress itself and also changes in the USPTO’s procedures and resources, which influence patent grants rates even when technological progress is stable.

We have also collected the citations that granted patents make to prior patents. Due to changes in USPTO reporting formats, these data start with the 1947 cohort of granted patents. The citations made by these patents include patents granted before 1947. Thus, for a patent granted in 1964, we can observe the full set of citations that the patent makes to prior work and the year-by-year subsequent citations that the granted patent itself receives. By contrast, for a patent granted in 1864, we can only observe the year-by-year citations the patent receives starting in 1947. This is a substantial boost over earlier patent citations data, which usually start with the 1975 grant year, and allows us to provide deeper insights into long-run impact of patents.

## 3.2 Key Empirical Facts

We repeat here the six key facts that we emphasize in this paper, and the remainder of this section provides a greater discussion of them:

**Fact 1:** *The composition of innovation is shifting towards novel combinations of existing technologies. The share of patents with a novel technology combination is less than 50% in the 1800s, and the share is greater than 75% for patents granted after 1970. By contrast, novel technology development declines in absolute and relative terms since the 1800s. The reuse/refinement of older technology combinations shows an important non-monotonic pattern with initial increases and subsequent declines.*

Figure 1b shows the distribution of patents by type over time. The development of novel technologies was dominant early in U.S. history. The particular strength over the first few decades is artificial, as we define novel technologies based upon the USPTO’s collected system. By definition, many early patents must have novel technologies. But the pattern is also evident well after these initial conditions subside. As a share of patents granted, novel technology patents have fallen from 31% in the 1800s to 0.5% since 1970. Moreover, there exists a decline in absolute numbers of novel technology patents, too, even as the U.S. economy has grown much larger. The average annual count of patents containing novel technologies that are filed by U.S. domestic inventors is 977 during the 1800s. For 1900-1934, 1935-1969, and 1970+, the averages decline to 755, 659, and 209, respectively. The last number may overstate the decline to the extent the USPTO has not identified new technologies in this era, and the rate is 295 per year for 1970-1994.

By contrast, novel combination patents show substantial growth. This category increases from 29% of patents in the 1800s to 77% in the 1970+ period. With the exception of some small tapering after 1995, the increase in the share of novel combination patents is monotonic.

Reuse/refinement patents have a non-monotonic trend. This form of technological progress initially increases to account for over 50% of patents in the late 1800s. Its share then declines to 23% after 1970. This non-monotonic feature will feature prominently in our theoretical work, as it will serve as an untargeted empirical trend to compare the model’s predictions against. We also later discuss how this non-monotonicity holds within older patent cohorts.

**Fact 2:** *The number of distinct technologies combined together in each patent is rising with time. The average count is less than two technologies per patent for much of the 1800s, and the average count is above four technologies for patents granted after 1970.*

Figure 1c shows that the average number of technology codes listed on US domestic patents has risen from 1.9 in the 1800s to 4.1 since 1970. A similar increase is evident for the median. Figure 1d shows that the share of patents that have a single technology code declined from 53% in the 1800s to 11% since 1970. The share of patents with five or more technology codes has increased from 4% in the 1800s to 31% since 1970. The distinct change observed in the 1930s is reflective of the exceptional technological progress in this era noted by Field (2003).

We spend a substantial portion of the paper developing an economic model that can help explain these features through expanding combinatorial possibilities and accumulated depth of technology pools. It is important, however, to discuss a few other simple explanations at this point. These alternatives may explain some of the trend, but they are not the key factor either.

A first explanation would be that the composition of technologies is changing—for example, moving from mechanical technologies to pharmaceuticals—and this composition could explain the increase if the newer categories tend to have more technologies per patent than older categories. Figures 2a and 2b show the patent category distribution over time and the average number of technologies per patent within the six main categories from Hall et al. (2001). While it is the case that technologies like drugs with greater technology counts are becoming more important, Figure 2b shows the increase in mean patent size in every category. Thus, the rise in technology combinations per patent is a general process, not specific to a technology group, and thus not the consequence of a changing composition of technologies invented.

Second, the increased number of technologies per patent is not due to a change in how "big" the technologies being identified today by the USPTO are compared to earlier periods.

The worry here is that greater technology divisibility may have led to new technologies being naturally smaller and more interdependent than before (e.g., a drug compound or iPhone component versus a horse shoe), and that this requires more technologies be listed per patent. This theory can be partly explained away in more recent periods by noting that many other traits of patents are much more stable, suggesting this is more of a technology combination feature. For example, in the post 1975 period, the average number of claims on patents with one technology code listed is 6.2, compared to an average of 9.4 claims for patents with 5+ technologies listed.

Moreover, a regression analysis reaches the same conclusion in a more rigorous manner. We prepare a dataset of 15.3 million observations defined as a technology  $t$  listed on a patent  $p$ . We create a measure  $TechCount_{t,p}$  that is the count of technologies listed on patent  $p$ . This measure is the same for all technologies on a patent, and we weight each observation by  $1/TechCount$  so that patents receive equal weight. We then regress  $TechCount_{t,p}$  on a set of technology cohort fixed effects  $\eta_t$  that define the decade in which the technology  $t$  is first observed. We similarly define a set of patent grant cohort fixed effects  $\theta_p$  that define the decade in which the patent is granted. The full regression takes the form,

$$TechCount_{t,p} = \eta_t + \theta_p + \epsilon_{t,p}.$$

When not including  $\theta_p$ , the coefficient values on the technology cohort fixed effects  $\eta_t$  would suggest a technology developed in the 1980s is typically combined with 1.78 (0.23) more technologies per patent than a technology developed in the 1830s. However, this cohort difference falls to 0.25 (0.11) once extending the regression to include fixed effects for grant decades  $\theta_p$ . It is further reduced to 0.11 (0.07) when controlling for patent sub-category x grant decade fixed effects. Thus, a comparison of granted patents at a point in time reveals very little relationship between how many technologies are being combined per patent and how old the given technologies are.<sup>3</sup>

A third and related concern is that the rise in technology counts is the simple product of having more potential technologies in the USPTO system to draw upon (i.e., the choice set for examiners is bigger) or similar procedural differences. It is worth noting again that when the USPTO classification system changes, the whole database is reclassified for the new system. Thus, there are institutional reasons to believe this concern is not causing the trends observed. The trends also appear robust to this concern in empirical ways. Appendix Figure 1 shows very similar patterns to Figure 1 when restricting our analysis to patents where all of the underlying technology classes are established by the USPTO by 1910. This suggests that our trends are not the result of the USPTO adding new classes that have different properties to older ones. It could still be the case that these pre-1910 categories have deepening subclasses that allow

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<sup>3</sup>This pattern is primarily a macro-trend in technology use, but it is also seen more weakly at the individual technology level. The 50th+ patents to use a technology have on average 0.5 more technologies than the originating patent for the technology. For a technology developed in the 1800s, 23% of the inventions that use the technology developed in the first ten years after the technology’s birth only contained the single technology only; 17% contained four or more additional technologies in combination. After 40 years, 17% of inventions contained the single technology, and 25% contained four or more additional technologies in combination. For a technology developed from 1935-1969, 5.6% of the inventions that use the technology developed in the first ten years after the technology’s birth only contained the single technology only; 55% contained four or more additional technologies in combination. After 40 years, 2.8% of inventions contained the single technology, and 60% contained four or more additional technologies in combination.

greater counts. We show later in Figure 5 that we also observe our key shift of innovation type when restricting the analysis to just patents that utilize subcategories present in 1860-1880. This concern is also weakened when combining the insights of the regression analysis reported in the prior paragraph with the strong localization effect for recombinations identified in Fact 5, discussed further below. The fact that the trends are moreover present in all categories of technologies speaks against this explanation, as the growth rates of technology fields have been quite different over the period studied.

**Fact 3:** *These forms of technological progress differ in their economic impact as measured by patent citations and firm-level employment growth regressions. The economic impact is highest for patents developing novel technologies, second for patents developing novel combinations, and weakest for reuse/refinement patents.*

Our first metric to analyze the differential impact of patents is the number of subsequent citations that they receive (e.g., Trajtenberg 1990, Harhoff et al. 1999, Jaffe et al. 2000, Hall et al. 2005). We analyze citations received to 2000 for patents granted between 1950 and 1990. The citation count differences are very stark. Novel technology patents receive an average of 19 citations and a median of 11. Novel combination patents receive an average of 9 citations and a median of 6. Reuse/refinement patents receive an average of 6 citations and a median of 4. In addition to higher mean citation rates, the variance in citation outcomes is substantially higher for novel technology patents. Similar conclusions are reached about the relative strength of patent types when examining the number of claims on patents, the measured originality of technology inputs, and the measured generality of technology inputs. Self-citation (forward and backward) is highest for reuse/refinement patents and weakest for novel technology patents. Unreported regressions confirm that these differences hold-up with controlling for fixed effects for patent class x grant year.<sup>4</sup>

As a second approach, Table 1 documents firm-level employment growth regressions using the sample from Akcigit and Kerr (2010). This sample matches the USPTO patent database into the Census Bureau’s operating data on U.S. firms. The operating data are taken from the Longitudinal Business Database (LBD) and provide employment levels for firms in each year starting in the late 1970s. As a representative year, the data include 108 million workers and 5.8 million establishments in 1997. We organize our sample around five-year blocks. The three periods included in the regressions are 1978-1982, 1983-1987, and 1988-1992. We calculate for each firm its employment growth to the following period, and we use 1993-1997 data to calculate growth for 1988-1992. Following Davis et al. (1996), we calculate employment growth relative to the average of the two periods.<sup>5</sup>

We merge patent records using firm name and location matching algorithms that build upon Balasubramanian and Sivadasan (2011) and Kerr and Fu (2008). Akcigit and Kerr (2010) provide extensive details on this dataset and the patent matching procedure. Columns 1-3 using a conservative strategy for matching patents to firms (i.e., the proximity of the names in the two files to be declared a match), while Columns 4-6 use a more aggressive strategy. The

<sup>4</sup>See also Packalen and Bhattacharya’s (2012) analysis of innovative key phrases in patent documents.

<sup>5</sup>The formula is  $EmpGr_{f,t} = [Emp_{f,t+1} - Emp_{f,t}] / ([Emp_{f,t+1} + Emp_{f,t}] / 2)$ . This growth variable is bounded between (-2, 2) for continuing firms, reduces the impact of outliers, and protects against mean reversion. For our core sample, the mean of  $EmpGr_{f,t}$  is 0.128 and the standard deviation is 0.597. Firm employment  $Emp_{f,t}$  for each period is calculated as a mean across the years where positive employment is observed. Our patterns below are robust to several variants of defining employment growth (e.g.,  $\ln[Emp_{f,t+1}/Emp_{f,t}]$ ).

regressors to explain the firm’s employment growth to the next period are the firm’s current employment, the firm’s total patenting in the period, and the distribution of the firm’s patents across innovation types,

$$EmpGr_{f,t} = \eta_{i,t} + \gamma_E \ln(Emp_{f,t}) + \gamma_P \ln(Patents_{f,t}) + \beta NovelShare_{f,t} + \epsilon_{f,t},$$

where  $f$  and  $t$  index firms and five-year periods. We include a vector of industry-year fixed effects  $\eta_{i,t}$  for the firm. Industries are assigned to firms at the two-digit level of the Standard Industrial Classification system using industries in which firms employ the most workers. We cluster standard errors at the firm level; regressions are unweighted.<sup>6</sup>

Our focus is on  $NovelShare_{f,t}$ , which gives the fraction of the firm’s patents that are either novel technologies or novel combinations. There is clear evidence that having a greater share of a firm’s patents be novel technology or novel combination patents is associated with higher firm employment growth. In Column 2, a 20% increase in this share (i.e., moving from 25% to 45%) has the same impact as a 10% increase in the firm’s overall level of patenting. Column 5 would suggest an even larger comparative response (equivalent to a 40% increase in firm’s overall level of patenting). Columns 3 and 6 show the differential importance of novel technologies, although the infrequent occurrence of novel technologies during the 1978-1992 sample period lowers the precision of these estimates.

**Fact 4:** *Technology use is maintained for a long time after original invention. More than 80% of technologies ever developed are still used during the course of a decade today. As a consequence, patents granted today still build extensively on technologies first observed in the 1800s.*

Inventors continue to use a large portion of developed technologies for some time into the future. Figure 2c provides the log count of cumulative technologies developed by year, the log count of technologies utilized within a year, and the log count of technologies created by year. The count of technologies used within a year has continued to rise in parallel with the cumulative count developed. Figure 2d likewise provides two fractions. The first is the share of technologies exploited in a grant year compared to the cumulative numbers of technologies developed to that point. The second is a decade-level share that allows for localized variation year-to-year in which technologies are utilized. The latter measure in particular suggests that even today, more than 80% of technologies ever developed are utilized for a new patent at least once during a ten-year span.

As a consequence of this longevity, technologies developed in earlier periods remain relevant for current invention. Looking at patent grants after 1950, roughly one quarter of technologies that are used come from the four time periods of 1830-1869, 1870-1909, 1910-1949, and 1950+. Given the large increase in the number of patents granted and the growing number of technologies listed per patent, many technologies developed long ago still find the majority of their "uses" come after 1950.

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<sup>6</sup>This growth specification implicitly contains firm fixed effects. First, the dependent variable is employment growth versus the level of employment. Second, the innovation literature often characterizes patents as stock variables. By focusing on contemporaneous patent counts and type distributions, we are implicitly looking at changes in innovation stocks to the current period from the previous period. Econometric specifications tend to find contemporaneous R&D investments have the most important impact for rates of technology formation (e.g., Pakes and Griliches 1980, Hausman et al. 1984, Hall et al. 1986).

**Fact 5:** *Technological combinations are localized in nature. For example, about 70% of the technologies combined with a focal technology are born within 20 years of the focal technology’s birth year. As a consequence, combinatorial possibilities do not extend over the full range of inventions.*

Combinations of earlier technologies display cohort effects in how they match with one another. This can be observed on several dimensions, and we focus most of our description and quantification on the historical period when a technology was born (which will most closely align with our model’s construction). Independent of when a patent is granted, technologies are clustered by time era. Thus, a patent being filed in 1995 is more likely to use exclusively technologies from the 1870s or from the 1960s than to mix-and-match across the two time periods. This pattern suggests cohorts of technology development exist. Combinatorial processes occur within localized streams or cohorts, but there is less potential for cross-pollination over cohorts.

Figure 3a shows this broad pattern using all patents granted since 1950. We treat each technology listed on a patent as a separate observation for these graphs. We then group these technologies by their period of origin: 1830-1869, 1870-1909, 1910-1949, and 1950+. Given the extensive number of technologies developed in early U.S. history, roughly a quarter of our observations fall into each of these bins despite only looking at patents granted after 1950. For each group, we then plot the density function of the origin period of technologies to which the group is matching. These densities sum to 100% across the horizontal axis. These patterns exhibit substantial localization, with, for example, a technology from the 1830-1869 group being much more like to combine with a technology from the 1860s than from the 1960s, even on patents granted after 1950.

Beneath this overall process, there is an interesting and important divergence across patent types. The matching processes for novel combination and reuse/refinement patents look very similar to the aggregate picture in Figures 3c and 3d, respectively. On the other hand, the combinatorial process for novel technology patents is quite distinct in Figure 3b, which is the subject of Fact 6.<sup>7</sup> These same patterns are also visible when isolating earlier time periods. The technology advancement picture looks similar within each patent category from Hall et al. (2001), with the exception of some deviations in the Miscellaneous category. Thus, at a given point in time, the combinatorial process for existing technologies is undertaking a matching function that is cohort specific. As a simple statistic, 71% of realized combinations for a technology are to other technologies born within +/- 20 years.

The cohort-age matching process evident is also evident in citation patterns. We examine citation patterns on patents granted after 1947, where citations are restricted to those that occur between two U.S. domestic patents. Table 2’s distributions are stark. Patents granted after 1947 that use a particular technology cohort are mostly likely citing other patents that use the same technology cohorts. Presented results use the primary technology’s cohort or the median cohort among technologies on patent. We find similar results when using other strategies (e.g., min/max). We also find similar patterns if restricting the sample to patents granted after 1975.<sup>8</sup>

<sup>7</sup>Figure 3b documents the pairwise matching of all technologies listed on novel technology patents granted 1950 and after. As such, it is possible to be observing two technologies from the 1830-1869 cohorts that are matched together. There would at least be one additional technology on these patents that would be novel.

<sup>8</sup>As a corollary, each patent type tends to cite its own type more frequently. For novel technology patents,

As a second example to the time dimension, combinations are also concentrated within major technology groups. Excluding patents with just one technology code, 56% of patents pull their technologies from one of the 36 sub-categories of the USPTO system, 35% pull from two categories, and 8% pull from three categories. Less than 2% of patents combine technologies from four or more sub-categories. This pattern is also evident when using the six category-level boundaries, and these patterns are stable over time. As a third example, we undertook Monte Carlo simulations of actual combinations relative to random counterfactuals of based upon the technology set exploited in the same year. In every period, the observed combinations are statistically less than the random counterfactuals at the 1% confidence level, indicative of localized combinations (independent of specific time or classification boundaries).

**Fact 6:** *Novel technologies are born on patents using the most recent technologies. Novel technology patents typically have more technologies listed on the patent than contemporaneous novel combination or reuse/refinement patents.*

Our final set of observations returns to the deviation for novel technologies evident in Figure 3b. While some evidence remains that this cohort matching is important, this process clearly becomes second-order to a novel technology development process that builds extensively upon the most recent technologies. A corollary to these features is that novel technologies tend to be developed on patents that mostly contain young technologies.

Figure 4a continues this depiction of novel technology patents by showing the cohort distribution of technologies listed on patents granted in the 1990s, 1970s, 1950s, and 1930s that developed novel technologies. To maximize the sample, we use worldwide patent grants for this purpose. The patterns very clearly pick off the leading edge, in large part due to the technologies that are being observed for the first time. Figure 4b alternatively plots the cohort distribution of primary technologies listed on patents granted in the 1990s, 1970s, 1950s, and 1930s that developed new technologies after excluding the novel technologies themselves (which are by definition in the grant year period). The same higher relative importance of recent cohorts is observed. We find similar results when examining a weaker condition where only the "rate" at which inventors pull from potential new technologies is higher for recent periods.

Novel technologies tend to be born on patents that have a higher technology count than contemporaneous novel combinations, as shown in Figure 4c. The high number of technologies present for patents that introduce novel technologies is quite distinctive piece for theories to match (e.g., compared to novel technologies being born in isolation and then being combined with others).<sup>9</sup> Likewise, other traits about novel technologies are distinctive. Novel technologies are born as the primary technology on a patent 32% of the time, and there tends to be only one novel technology born on a patent that has a novel technology. For most patents that include a novel technology (90%), we observe at least one of the bilateral combinations of tech-

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19% of cites are to novel technology patents, 70% to novel combination patents, and 11% to reuse/refinement patents. For novel combination patents, these shares are 3%, 81%, and 16%, respectively. For reuse/refinement patents, these shares are 2%, 68%, and 30%, respectively. These patterns are very similar if restricting to citations with an age lag of less than 20 years.

<sup>9</sup>The transition from novel technology to novel combinations is smoother when differentiating among novel combinations. That is, the technology counts for novel combinations patents that build upon technologies that are one year old are closer in year  $t$  to the size of novel technology patents in that year than the novel combination patents where all technologies are ten years old.

nologies on the patent beforehand. However, we observe the total sequence of technologies in only a minority of cases (29%), and many instances of sequences come later. Once a technology is born, its subsequent combinatorial uses are mostly independent of its birth conditions. That is, a technology born on 2-technology and 6-technology patent in the same year are utilized in very similar ways after birth.<sup>10</sup>

Figure 4d closes with a feature that we utilize later. The figure compares the evolution of two groups of patents that exclusively use technologies from a selected time period. The black group considers patents that include only technologies discovered during the 1860-1880 decades (i.e., inclusive of 1889). By definition, these patents are only observed after 1860. These patents can be observed after 1890 in novel combinations or reuse/refinement roles; by definition they cannot be observed as novel technologies. The green group similarly considers patents that only include technologies discovered during the 1940-1960 decades. The figure plots by decade the distribution of these two groups across patent types. There is a very important feature in this figure—the non-monotonic share of reuse/refinement patents holds within the 1860-1880 group. This is a powerful model element to check against, as simple explanations that rely on overall macro trends or larger technology pools will have difficulty explaining why the behavior of this specific group changes even though all of the underlying technologies have been in existence since the 1860-1880 decades. Second, it is important that both groups are converging to the same distribution types once the novel technology phase for the 1940-1960 cohort has passed.

### 3.3 Empirical Boundaries

We focus on these key facts for our model, to provide a comprehensive yet digestible number of elements. It is worth noting two features not emphasized in this work. First, unreported analyses investigate institutional variation in these patterns using patent assignee information available from 1920 onwards. Universities have the highest novel technology share, followed by government and military entities. Novel technology shares then decline by assignee size among industrial assignees. The frequency of novel technologies is lowest among individual inventors. While these institutional differences are intriguing and worthy of future study, especially to consider the long tails of innovation’s impact, we do not emphasize them in this paper as variations in these shares across types are small.<sup>11</sup> As institutional features cannot explain the broad changes that we emphasize in this work, we economize on the model in this regard by considering individual inventors making profit-maximizing choices.

Second, discussions of long-run technology development often focus on general purpose technologies (GPT) like steam power, electricity, and similar. Jovanovic and Rousseau (2005) and Bresnahan (2011a) provide recent reviews. We do not observe GPTs in our data work, and Moser and Nicholas (2004) discuss the difficulty in discerning GPTs through patent data. David (1990) further highlights the difficulty in setting their timing to economic growth. GPTs

<sup>10</sup>It is likewise useful to note that a positive correlation exists between a greater cohort time span among a patent’s technologies and the number of technologies that the patent lists. This will be evident in the model below and has a conceptually similar underpinning to the rising counts in Figure 4c for novel patents.

<sup>11</sup>For example, in the post 1975 period, all of the above assignee types have patent type distributions that fall within the ranges: <1% novel technologies, 76%-80% novel combinations, and 19%-24% reuse/refinements. The  $R^2$  when regressing invention type on a set of indicator variables to model unassigned patents, industrial patents with four assignee size categories, university patents, and government patents is 0.0003.



are correspondingly absent from our model. In many respects, however, the localized nature of the combinatorial processes that we observe closely parallel discussions of GPTs and their long-run impact. We stop short of labeling one cohort of patents to be from a specified GPT family, partly by necessity and partly by theoretical design, but the localized nature of technological progress that we describe is consistent with waves of technological development standing upon GPTs. The formulation of our model embeds the distinct nature of path dependence emphasized by David (1985) for recombinations.

Finally, our analysis is clearly restricted to patented inventions, and Moser (2012) shows that many important historical innovations are not patented. The robustness of our key facts when looking within technology categories, when restricting to patent classes established by 1910, when restricting to patents combining subclasses from 1840-1860, and similar exercises helps alleviate some of the concern that changes in "what is patented" drive these trends. This exercise, however, will always be incomplete when building upon just patent data, and we can only assert the consistency of our patterns within the patent dataset itself.

### 3.4 Comparison to Strict Combinatorial Growth

Our final step before presenting the model is to compare these data properties against a strict combinatorial framework to fix ideas about the required differences for the model to match the data. Consider empirically what one might observe if given  $n$  technologies to use by themselves and then to combine into two-way combinations, three-way combinations, etc. If lower-order combinations are easier to accomplish and always available, one would observe a sequential process of technological development that fills up layers that are getting exponentially larger in size:  $n \ll n!/(2! * (n - 2)!) \ll n!/(3! * (n - 3)!) \ll \dots$ . When looking at the data, one would anticipate seeing 1) very similar counts of uses of each technology within the set  $n$  and 2) a relatively low total count of layers needed to achieve roughly the United States' total novel patent count during this period. For example, with  $n = 100$ , four layers of combinatorial power are sufficient to achieve the total novel patent count observed since 1836, and 96% of these observations will be in the last technology layer of four-way combinations, as shown in Figure 5a. If  $n = 25$ , nine layers are required to reach this volume.

Even though combinatorial processes are clearly at work in our data trends, the data in many ways look very different to these simple calculations. Figure 5b plots the distribution of the 3.3 million novel patents granted by 2004 to U.S. inventors (i.e., excluding reuse/refinement). This graph is clearly dependent upon our current point in time and would look different in 50 years (shifting rightward with high likelihood). This graph shows that the majority of the novel uses of our technologies ( $n > 150,000$ ) are made thus far on patents with two or three technologies. A long and relatively thick tail then persists, with over 4% of novel uses including ten technologies on patents.

Thus, the data differ from the stylized calculations in several important ways. First, inventions are moving much faster up the higher-order combinations of technologies than our simple examples would suggest. To some degree, this gap can be closed by forcing a more localized combinatorial processes, as highlighted in Fact 5. This localization will not close the whole gap, however, as it will still be the case that the distribution shares would be expected to be increasing in the number of technologies combined, regardless of how localized the combinatorial process is, and this pattern would be at odds with Figure 5b. Thus, the model will need to embody features that push the technology counts higher in multiple ways, which is reflective

of the way that novel technologies are born in Fact 6. As the discussion has highlighted, these features are also important for maintaining a realistic growth rate to technology development.

Second, and somewhat ironically given the divergence just mentioned, the model needs to deliver greater variability at the individual technology level in terms of uses. The simplest combinatorial processes would simply use all technologies equally, which is the underlying assumption built in Figure 5a. Yet, Figure 5c shows a very skewed distribution in terms of how many times actual technologies are used in novel ways. Of the 151,208 technologies developed, almost half (71,864) are used fewer than 10 times for novel purposes. At the other extreme, 194 technologies are used 200+ times. Thus, while the model needs to provide a break on the aggregate combinatorial expansion, it also needs to introduce greater heterogeneity in the underlying uses of technologies. In summary, to match the U.S. data well, the model needs to embrace the combinatorial power that is often absent in growth models, with Weitzman (1998) as the major exception, but it needs to do so in a new way in order to hit these micro-data properties.

## 4 Theoretical Model

In this section, we build a new theoretical growth model to operationalize the empirical findings that we have shown so far. Since the new ingredient is the combinatorial aspect of innovations, we will try to keep the production side as simple as possible and assume that firms purchase new innovations from inventors who combine existing knowledge. This new structure comes with a technical difficulty—keeping track of the combinatorial possibilities is cumbersome. We will circumvent this problem by introducing a Poisson process that will turn the problem into a memoryless process.

The following section describes the production side of the economy.

### 4.1 Preferences and Final Good Technology

Time is discrete. There is a representative household with risk-neutral individuals that discount the future with a discount factor  $\beta \in (0, 1)$  which delivers the Euler equation<sup>12</sup>

$$\beta = \frac{1}{1 + r_t},$$

where  $r_t$  is the interest rate at time  $t$ . Individuals consume a unique final good  $Y_t$ , which is also used for producing intermediate goods ( $K_t$ ) and new technology adoption ( $X_t$ ), therefore the resource constraint of this economy is simply

$$Y_t = C_t + K_t + X_t.$$

The final good is produced by labor and a large set of intermediate goods  $j \in \{1, \dots, J\} \equiv \mathcal{J}$  with the production technology

$$Y_t = \frac{L_t^\gamma}{1 - \gamma} \sum_{j \in \mathcal{J}} q_{jt}^\gamma k_{jt}^{1-\gamma},$$

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<sup>12</sup>This can be formally derived through the household problem as  $\max_{C_t, A_{t+1}} \sum_{t=0}^{\infty} \beta^t C_t$  subject to  $A_{t+1} + C_t = (1 + r_{t+1}) A_t + w_t$ ,  $\forall t$ , where  $A_t$  is the asset holding of the household and  $w_t$  is the wage income.

where  $J \in \mathbb{Z}_{++}$  denotes the number of varieties in the economy. In this specification,  $k_{jt}$  is the quantity of intermediate good  $j$ ,  $q_{jt}$  is its quality, and  $\gamma \in (0, 1)$ . We normalize the price of the final good  $Y$  to be one in every period without loss of generality. The final good is produced competitively with input prices taken as given. Labor is supplied inelastically,  $L_t = 1$ . Time subscripts  $t$  will henceforth be omitted where the meaning is obvious.

Each intermediate good  $j$  is owned by an infinitely-lived firm. The firm produces each intermediate good at the marginal cost  $\zeta$  such that

$$k_j = \frac{K_j}{\zeta},$$

where  $K_j$  is the final good spent on the production of intermediate good  $j$ . For simplicity, we set  $\zeta \equiv 1 - \gamma$ , without loss of generality.

The maximization problem of the final goods producer generates the inverse demand  $p_j = q_j^\gamma k_j^{1-\gamma}$ ,  $\forall j \in \mathcal{J}$ . The constant marginal cost of producing each intermediate variety is  $1 - \gamma$ , and the profit maximization problem of the monopolist  $j$  is

$$\pi(q_j) = \max_{k_j \geq 0} \left\{ q_j^\gamma k_j^{1-\gamma} - (1 - \gamma) k_j \right\}, \quad \forall j \in \mathcal{J}.$$

The first order condition yields an optimal quantity and price for intermediate good  $j$

$$k_j = q_j \text{ and } p_j = 1. \tag{1}$$

The realized price is a constant markup over the marginal cost and is independent of the individual product quality. Thus, the profit for each active good is

$$\pi(q_j) = \pi q_j, \tag{2}$$

where  $\pi \equiv \gamma$ . The maximization in the final goods sector, together with (1), implies a wage rate

$$w = \frac{\gamma}{1 - \gamma} \bar{q}$$

where  $\bar{q} \equiv \sum_{j \in \mathcal{J}} q_j$  is the average quality index in this economy.

As evident in (2), profits are increasing in the quality, therefore firms are willing to increase the quality of their products. Quality improvements—which are also the technological progress in this model—happen through purchases of new innovations from inventors. New innovations contribute to the firm's quality as a function of the novelty of the innovation  $s_j$  such that

$$q_{j,t+1} = q_{j,t} + s_j \bar{q}_t.$$

In this specification  $\bar{q}_t$  ensures that the marginal gains from innovations scale up with the economy and does not vanish in the long run. We will describe the determination of the novelty of innovations ( $s_j$ ) in the next subsection.

Every period, one firm is matched to one inventor randomly to purchase an innovation. There are  $I \in \mathbb{Z}_{++}$  inventors and  $J \in \mathbb{Z}_{++}$  firms in the economy such that  $I \leq J$ , therefore the probability that a firm meets an inventor is

$$\nu \equiv I/J.$$

The inventor has a patent of novelty  $s_j \bar{q}$  which she sells for price  $P(s_j \bar{q})$ . Let us denote the value of a firm of current quality  $q_j$  by  $V(q_j, \bar{q}_t)$ . For simplicity, we assume that the inventor has the full bargaining power and makes a take-it-or-leave-it offer to the firm such that

$$P(s_j \bar{q}_t) = V(q_j + s_j \bar{q}_t, \bar{q}_{t+1}) - V(q_j, \bar{q}_{t+1}).$$

Timing of the events is as follows:

- period  $t$  begins,
- inventors innovate and firms produce with their initial qualities,
- firms and inventors match randomly and patent sales take place,
- firm qualities improve according to the purchased patents, and
- period  $t$  ends.

Now we can express the firm's value function according to this timing convention as

$$V(q_j, \bar{q}_t) = \pi q_j + \beta \left[ \nu \mathbb{E} \{V(q_j + s_j \bar{q}_t, \bar{q}_{t+1}) - P(s_j \bar{q}_t)\} + (1 - \nu) V(q_j, \bar{q}_{t+1}) \right]. \quad (3)$$

This value function is intuitive. In every period, firm collects  $\pi q_j$ ; then before the next period starts, the firm will receive an opportunity with probability  $\nu$  to purchase a new idea of novelty  $s_j$  at the price  $P(s_j \bar{q}_t)$ . Finally, the firm discounts the future period by  $\beta = (1 + r)^{-1}$ . Then the following proposition characterizes the equilibrium value and price.

**Proposition 1** *The value function in (3) takes the following form*

$$V(q_j, \bar{q}_t) = \Pi q_j,$$

and the price is simply

$$P(s_j \bar{q}_t) = \Pi s_j \bar{q}_t,$$

where  $\Pi \equiv \frac{\pi}{1-\beta}$ .

**Proof.** See Appendix. ■

This result implies that the value of a firm is only a function of its current quality  $q_j$ . The reason for this is the fact that a new inventor makes a take-it-or-leave-it offer, in which case it extracts the full surplus from the firm.

## 4.2 Innovations and Technological Progress

Now we describe how innovations take place in this model. In any period  $t$ , the economy consists of a set of technologies  $\mathcal{M}_t \equiv \{0, 1, \dots, M_t\}$  that have been invented by the current period. Following motivating Fact 4, we assume technologies live forever. We denote the index of the latest technology by  $M_t$  and any single member by  $m \in \mathcal{M}_t$ . Inventors combine these available technologies to produce a higher quality product. As described in the motivating empirical evidence, we consider three types of innovations:

- novel technologies ( $NT$ ),
- novel combinations ( $NC$ ), and
- reuse/refinement ( $RU$ ).

*Novel technology* innovations produce a new technology that did not exist among the current  $M_t$  technologies. *Novel combinations* do not produce a new technology but combines a subset of  $\mathcal{M}_t$  that have never been combined before. Finally, *reuse/refinement* simply repeat a subset of  $\mathcal{M}_t$  that has already been combined before.

Not all technologies can be combined together. Technologies have to be compatible with each other to be combined successfully. Each technology  $m$  has a pool of technologies  $\mathcal{C}_m$  that are currently known to be compatible with  $m$ . This economy is illustrated in Figure 6.

The model can be visualized as the following matrix.

EXAMPLE: SNAPSHOT OF THE ECONOMY AT A POINT IN TIME										
Primary Tech $m$ :	1	2	...	10	30	80	...	112	$M_t = 113$	
$\mathcal{C}_m =$		3	1		7	21		59	99	80
	P	10	9		5	33		73	104	85
	O	8	25		18	37		101	108	92
	O	21	13			26		83	113	100
	L	6				29			91	105
		18				41				110
State: $(m, n, z)$	$(1, 7, C)$			$(10, 4, C)$			$(80, 5, H)$			$(113, 7, H)$

The header of each column represents the ID number of each primary technology  $m$ . Below each ID, there is a column representing the pool of compatible technologies  $\mathcal{C}_m$  that can be combined within the pool. According to this example, currently there are  $M_t = 113$  technologies invented.

Innovations differ in terms of their economic impact. In line with our motivating Fact 3, we assume that novel technologies and novel combinations have higher impact than reuse/refinement innovations. In particular, for some  $\eta^H > \eta^L$  we assume

$$s_j = \begin{cases} \eta^H & \text{if novel technology} \\ \eta^H & \text{if novel combination} \\ \eta^L & \text{if reuse/refinement.} \end{cases}$$

This structure allows us to group the current invention pools into two sets: hot technologies ( $\eta^H$ ) and cold technologies ( $\eta^L$ ).

Each individual is born with an expertise in a field  $m \in \mathcal{M}_t$ . In addition, they receive an additional single idea  $m^*$  from a uniform distribution:

$$m^* \sim U(m' : m' \notin \mathcal{C}_m, \max\{1, m - \tau\} \leq m' \leq m + \tau), \quad (4)$$

where  $\tau \in \mathbb{Z}_{++}$  is the degree of localization of knowledge. This localization embeds motivating Fact 5. If  $m^* > M_t$ , then this idea is a new technology which has not existed in the economy before.

Implementing a new idea requires a cost

$$X(m, m^*) = \alpha \bar{q}_t |\min\{m^*, M\} - m|,$$

where  $\alpha > 0$  is a constant (see Figure 7). This captures the fact that an idea is cheaper to implement if it is closer to the inventor's expertise.

In every period, an inventor who is born with the expertise  $m$  and a new idea  $m^*$  has the following two options:

1. Use the existing pool  $\mathcal{C}_m$ : In this case, the inventor who is born to technology  $m$  can use the existing pool to do innovation. Depending on the state of the pool (hot, cold) the inventor can generate  $\eta^H$  or  $\eta^L$  without incurring any cost.
2. Implement  $m^*$ : This might lead to one of two outcomes.
  - (a) *Novel combination, NC*: This is the case when  $m^* \leq M$ . The inventor adds a technology into the pool that already exists in the economy but not in the local pool. The inventor pays a cost  $X(m, m^*)$ , as illustrated in Figure 8.
  - (b) *Novel technology, NT*: This is the case when  $m^* > M$ . The inventor generates a technology that is new to the whole economy by paying a cost  $X(m, M)$ , which is illustrated in Figure 9. When this happens, the inventor increases  $M_t$  by 1 and replicates the same technology cluster  $\mathcal{C}_m$  below  $m^*$  as shown in Figure 10. This process reflects motivating Fact 6.

### 4.3 Equilibrium

Now we focus on the implementation decision of the inventor. As long as the product line is hot, the inventor will focus on novel combinations since they are costless and generate a high return. When the cluster turns cold or stale, the inventor will try to implement her idea and achieve a novel combination or novel technology innovation if and only if the additional return to implementation is higher than the cost incurred:

$$X(m, m^*) < P(\eta^H \bar{q}_t) - P(\eta^L \bar{q}_t),$$

which can be rewritten as

$$\alpha \bar{q}_t |m^* - m| < \Pi \bar{q}_t \eta^H - \Pi \bar{q}_t \eta^L.$$

This yields the range of ideas that will be implemented by the inventor who operates on a cold technology pool

$$|m^* - m| < \frac{\Pi}{\alpha} [\eta^H - \eta^L].$$

Let the largest  $m^*$  that is implemented by the inventor with expertise  $m$  be denoted by  $\hat{m}^*$  such that

$$\hat{m}^* \equiv \max \left\{ m^* : |m^* - m| < \frac{\Pi}{\alpha} [\eta^H - \eta^L] \right\}. \quad (5)$$

Note that when  $m$  is close to 1, i.e.,  $m < \tau$ , the uniform distribution is not symmetric around  $m$ . Recall that new ideas are distributed as in (4) and that  $n$  represents the number of technologies thus far added into a given local pool. Therefore conditional on being in a cold technology pool, for  $m > \tau$ , the probability of adding a new technology into the pool (when  $n$  is small and  $\tau$  is large) is

$$\Pr(NC, m) = \begin{cases} \frac{\hat{m}^* + M - 2m}{\hat{m}^* - m} & \text{if } \hat{m}^* > M \\ \frac{2\tau}{\tau} & \text{otherwise.} \end{cases}$$

Now we can also express the probability of implementing a new technology. For any given cold product line with a location  $m$ , the new idea  $m^*$  will generate a novel technology if

$$M < m^* < m + \frac{\Pi}{\alpha} [\eta^H - \eta^L].$$

Therefore the probability of generating a novel technology (when  $n$  is small and  $\tau$  is large) is

$$\Pr(NT, m) = \begin{cases} \frac{\hat{m}^* - M}{2\tau} & \text{if } \hat{m}^* > M \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

**Proposition 2** *Probability of generating a novel technology increases weakly in  $m$ .*

**Proof.** This follows directly from (6) and the definition of  $\hat{m}^*$  in (5). ■

#### 4.4 Combinatorial Aspect

Now we are ready to describe the combinatorial aspect of the model. The way we will capture combinatorial innovation without having to keep track of the technology IDs is as follows. Let us denote the measure of inventors that are at technology  $m$  by

$$\mu_t = I/M_t.$$

When there is a new technology arriving in a cluster (column), the cluster enters into *hot* stage such that the profits are  $\Pi \bar{q}_t \eta^H$  (as opposed to the *cold* stage where profits are  $\Pi \bar{q}_t \eta^L$ ). Then, each cluster enters into cold stage at the rate

$$\text{cool down rate}(n) = \mu \xi / 2^{n-1}, \quad (7)$$

where  $\xi \geq 1$ . This captures the combinatorial possibilities and the logic here is as follows. Consider a technology cluster with  $n$  different technologies where one of the  $n$  technologies is just introduced to the cluster. This new technology can be combined with the remaining  $n - 1$  technologies in  $2^{n-1}$  different ways (considering also the possibility that the technology can be used alone). If the inventor produces one combination in every period, it would take her  $2^{n-1}$  periods to exploit all the possible combinations. If  $1/\xi$  fraction of those combinations are not viable (with  $\xi \geq 1$ ), then it would take only  $2^{n-1}/\xi$  periods to produce all viable combinations. Finally, if the inventor comes up with an innovation with probability  $\mu$  every period, then it would take her  $2^{n-1}/\xi\mu$  periods to invent all possible combinations. Using a cool-down rate of  $\mu\xi/2^{n-1}$  in (7) gives exactly the same expected duration of a hot product line.

Then the transition matrix is:

TABLE: TRANSITION MATRIX			
	FROM	TO	TO
<i>Panel A: Cold to Hot</i>			
STATE:	$(m, n, C)$	$(m, n + 1, H)$	$(m, n, C)$
WITH PROB:		$\Pr(NC, m)$	$1 - \Pr(NC, m)$
<i>Panel B: Hot to Cold</i>			
STATE:	$(m, n, H)$	$(m, n, C)$	$(m, n, H)$
WITH PROB:		$\mu\xi/2^{n-1}$	$1 - \mu\xi/2^{n-1}$

**Lemma 1** *In this economy,*

- (a) *the number of technologies ( $n$ ) in a given technology cluster weakly increases over time,*
- (b) *the total number of technologies ( $M_t$ ) weakly increases over time,*
- (c) *the share of inventors per technology area ( $\mu$ ) weakly decreases over time,*
- (d) *the fraction of new-technology patents start from 100% at time  $t = 0$  with  $M_0 = 1$ .*

This last bullet point matches Figure 1b (see Figure 11), reflecting the initialization of the economy. Since there are insufficient technologies to combine, the inventors initially produce only novel technologies. A simple proposition shows that the model conforms trivially with motivating Fact 2:

**Proposition 3** *The number of technologies combined in a patent increases over time.*

**Proof.** This follows from Lemma 1a. ■

Now we need to keep track of the frontier  $M_t$ . The number of novel technologies will be equal to

$$\Delta M_t = \mu_t \sum_{m \in \mathcal{M}_t} \mathbb{I}_{(m=c)} \Pr(NT, m) \quad (8)$$

where  $\mathbb{I}_{(m=c)}$  is an indicator function that assumes a value of 1 if the technology cluster is cold and  $\mu$  is the measure of individuals that are born to technology  $m$ . The above law of motion implies the following. The necessary condition to produce a novel technology is to operate in a cold product line, hence the indicator function  $\mathbb{I}_{(m=c)}$ . Moreover, the cost must be sufficiently low and the new idea must come from  $m^* > m$ , which happens with probability  $\Pr(NT, m)$ .

Note that the number of novel technology innovations in (8) depends on two main economic factors: First, the number of inventors working at the technology frontier  $\mu_t = I/M_t$  becomes less and less over time. This is because the same number of individuals are spread over a larger technology base. Second, the fraction of cold product line decreases (captured by the indicator function). This is due to the fact that as time evolves, a larger number of combinatorial possibilities keep the product line hot, which makes the profit-maximizing inventor focus on the novel combinations rather than on developing novel technologies.



**Proposition 4** *Consider a technology  $m$ . The probability that  $m$  will be used in a novel technology decreases over time.*

**Proof.** This immediately follows from the fact that the frontier is moving farther away from any given technology over time. ■

**Proposition 5** *Consider an inventor who has an expertise in the frontier technology  $m = M$ . The probability of her doing novel combinations is weakly increasing over time, and the likelihood of pursuing a novel technology is weakly decreasing.*

**Proof.** This follows from Lemma 1a and the fact that the cool-down rate  $\mu\xi/2^{n-1}$  is decreasing in  $n$ . ■

The intuition for this last result comes from the fact that initially there are very few technologies to combine and therefore most inventors just implement their new ideas. However, over time as more technological combinations become possible, inventors rely more on the combinatorial innovations rather than the development of novel technologies. This is captured by the fact that the transition rate from hot to cold (i.e.,  $\mu\xi/2^{n-1}$ ) decreases. Hence more novel combinations are done.

## 4.5 Discussion of the Model

In this model, novel technology innovations generate spillovers on future innovations. In particular, any novel technology that is introduced into the economy generates many other possibilities for novel combinations. This is not internalized by novel technology inventors, which implies that the decentralized economy is inefficient. If considered, government policy should be directed towards frontier (novel technology) innovations. Interestingly, this inefficiency worsens over time due to the increasing novel combination possibilities associated with any novel technology innovation. The calibrated model will consider these features more closely.

Our model generates predictions consistent with empirical Facts 1 and 2. These are illustrated in Figures 11-14 which provide a numerical example for our model, without a thorough calibration. At time  $t = 0$ , the lack of existing technologies prevent any profitable combination possibilities, therefore inventors initially invest their effort in developing novel technologies. Hence, the first period is associated with 100% novel technology patents, which then fades away over time (Figure 11). Because of the increase in the technology base but still shallow technology pools, the share of reuse/refinement patents rises initially (Figure 12). However, as the number of technologies increases and pools deepen, the power of novel combinations grows. Novel combinations become pervasive in the economy as shown in Figure 13. As a consequence, the average number of technologies that are combined in a single patent rises as shown in Figure 14.

As we discussed above, the slow down of the novel technology innovations depends on two factors: constant population and growing combinatorial possibilities. This result would still go through, although somewhat mitigated, if the population were to grow at a constant rate. This is because the second effect (combinatorial effect) is growing exponentially which would always dominate the positive impact of population growth. Therefore constant population provides analytical tractability.

## 5 Model Calibration and Analysis

[This section is not yet complete.]

## 6 Conclusions

It is now well recognized that innovation and technological progress are central factors in long-term economic growth. It is likewise well appreciated in many circles that these advances often come through recombinations of existing technological building blocks, rather the endless creation of independent, novel technologies. Economic historians see this in their accounts of the great and small innovations over the past two hundred years, and management scholars see this in their depictions of firm innovation processes today. While economic growth theorists since Weitzman (1998) have also recognized this property, it has not been extensively incorporated into general equilibrium models. This descends from several factors: limited measurement of how important these combinatorial channels are, the difficulty of incorporating combinatorial growth processes into models, the susceptibility of unconstrained combinatorial logic to generate unrealistic growth rates, and similar factors.

We attempt to close this gap through our combination of empirics and theory. We find that combinatorial processes are critical for understanding innovation and how technological change endogenously perpetuates itself. We identify, moreover, that the empirical evidence displays properties and regularities that are amenable for growth theory. Our work also shows a very important and sustained shift in the types of innovations that occur over time, with novel combinations becoming ever more important. If these trends continue, the important findings about combinatorial innovations developed by economic historians and management scholars may, in fact, be more relevant for tomorrow's innovation and for today's or the times past.

Revisiting Kaldor (1961), Jones and Romer (2012) develop six facts to guide the next generation of economic growth models. Examples include the rising human capital per worker worldwide and the enormous increases in market size. Given its relatively sparse empirical account, it is not surprising that combinatorial features did not make this "short list". Nor does our paper establish it as such. But, the long-run patent data for the U.S. economy appear to be screaming the importance of a better understanding of how technology building blocks are combined, and we hope that this project prompts further research into these essential topics.

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*Theoretical Appendix*

**Proof of Proposition 1.** Conjecture

$$V(q_j, \bar{q}_t) = Aq_j + B\bar{q}_t.$$

Then the price will be

$$P(s_j \bar{q}_t) = As_j \bar{q}_t.$$

Substituting this into (3) we get

$$Aq_j + B\bar{q}_t = \pi q_j + \beta \mathbb{E} \left[ \begin{array}{l} \nu \mathbb{E} \{Aq_j + As_j \bar{q}_t + B(1 + g_t) \bar{q}_t - As_j \bar{q}_t\} \\ + (1 - \nu) [Aq_j + B(1 + g_t) \bar{q}_t] \end{array} \right]$$

where

$$g_t \equiv \frac{\bar{q}_{t+1} - \bar{q}_t}{\bar{q}_t}$$

is the growth rate of the average quality at time  $t$ . Hence

$$Aq_j + B\bar{q}_t = \pi q_j + \beta [Aq_j + \bar{q}_t B (1 + \mathbb{E}g_t)].$$

Now equating the coefficients:

$$\begin{aligned} Aq_j &= \pi q_j + \beta Aq_j \\ B\bar{q}_t &= \bar{q}_t \beta B (1 + \mathbb{E}g_t) \end{aligned}$$

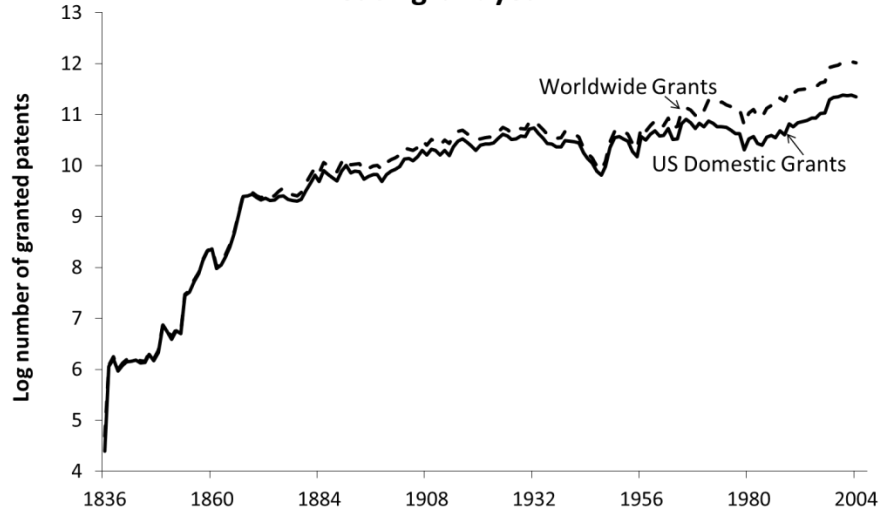
which imply

$$A = \frac{\pi}{1 - \beta} = \Pi \quad \text{and} \quad B = 0.$$

■

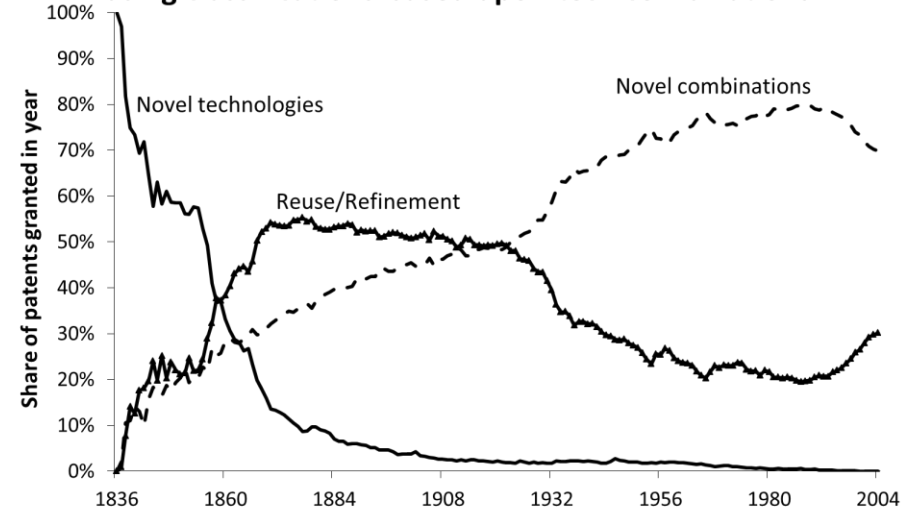
# Figure 1: Evolution of patent technology types

**1a: Number of patents granted by the USPTO in each grant year**



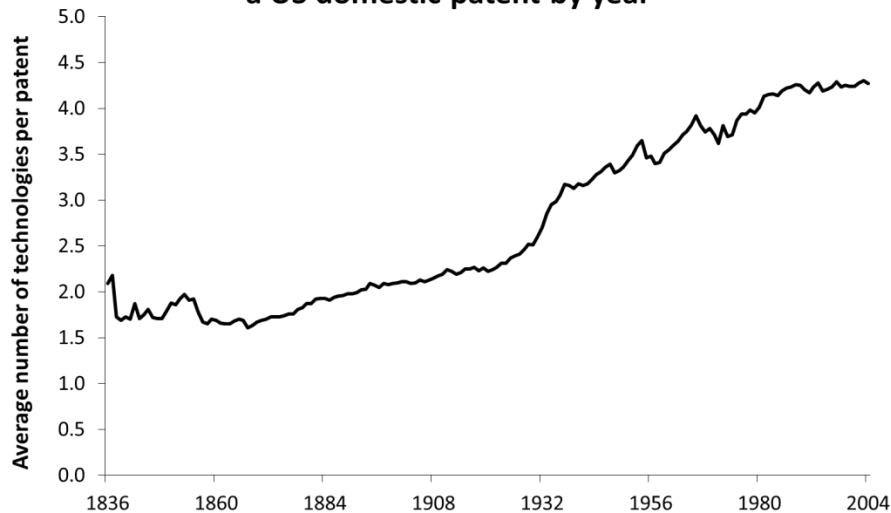
Notes: Figure provides the log count of patents granted by the USPTO in each year. The solid line is for patents with inventors who are residing in the United States at the time of their patent filing.

**1b: Shares of US domestic patents by grant year using classifications based upon tech combinations**



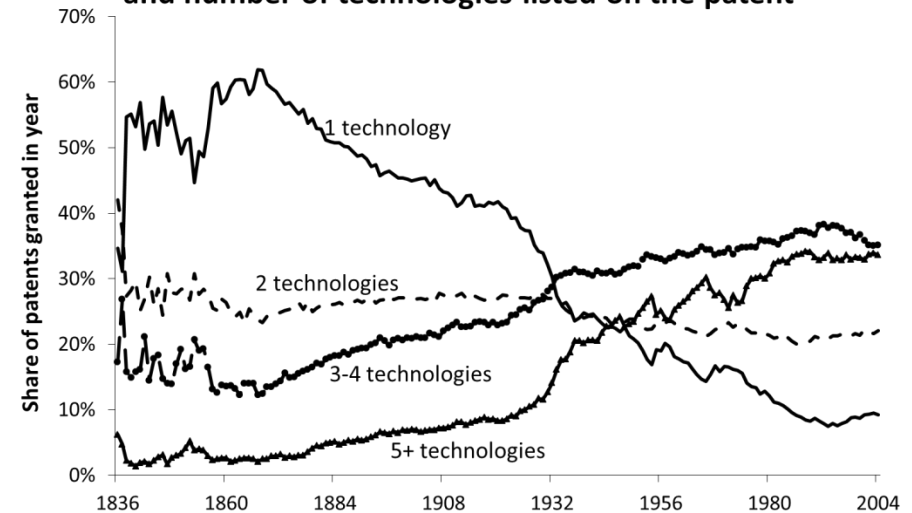
Notes: Novel technology patents contain a new technology. Novel combination patents combine previously observed technologies in a new way. Reuse/Refinement patents repeat a prior technology combination.

**1c: Average number of technologies combined into a US domestic patent by year**



Notes: Figure provides the average number of technologies combined into patents by grant year. The sample is restricted to patents with inventors residing in the United States.

**1d: Shares of US domestic patents by grant year and number of technologies listed on the patent**

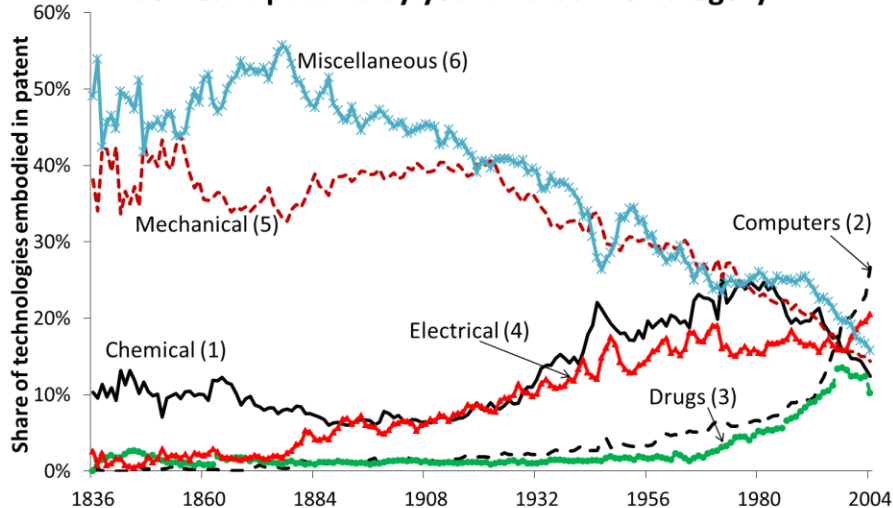


Notes: Figure documents the annual share of patents granted by the number of technologies listed on the patent. The sample is restricted to patents with inventors residing in the United States.



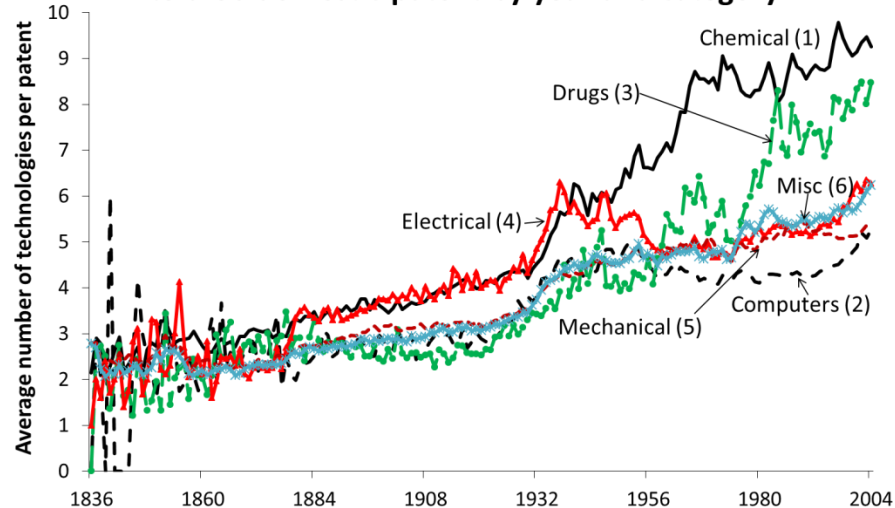
# Figure 2: Patterns by technology and cumulative over time

**2a: Share of technologies combined into US domestic patents by year and USPTO category**



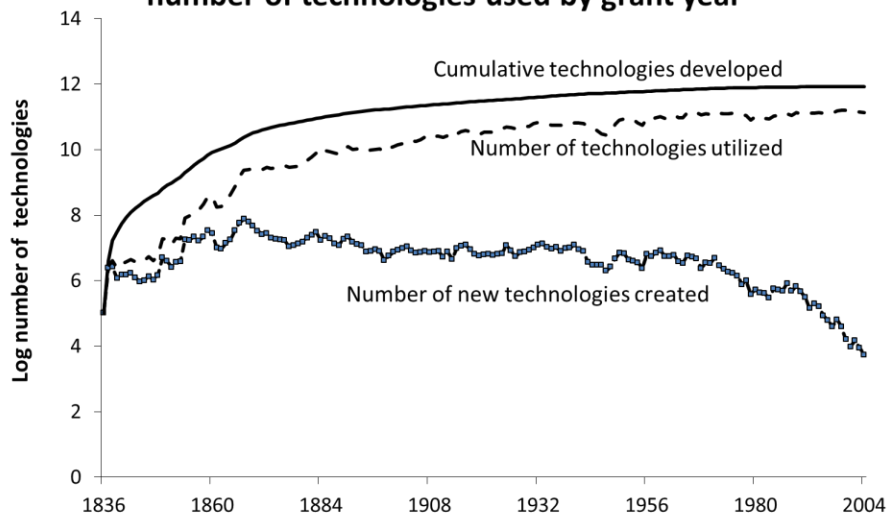
Notes: Figure provides the share of technologies combined into US domestic patents by the six main USPTO patent categories. Shares are calculated across patent-technology observations.

**2b: Average number of technologies combined into a US domestic patent by year and category**



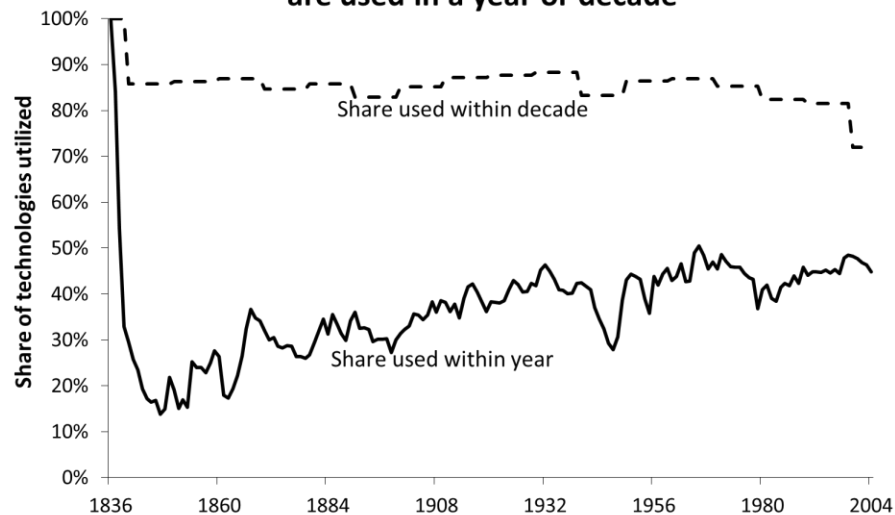
Notes: Averages are calculated by applying the total technology count of a patent to each patent-technology observation and collapsing. Technology counts will extend outside of the focal category.

**2c: Number of technologies ever developed and number of technologies used by grant year**



Notes: Figure shows for each year the cumulative number of technologies developed by US domestic inventors and the number used for inventions in that year.

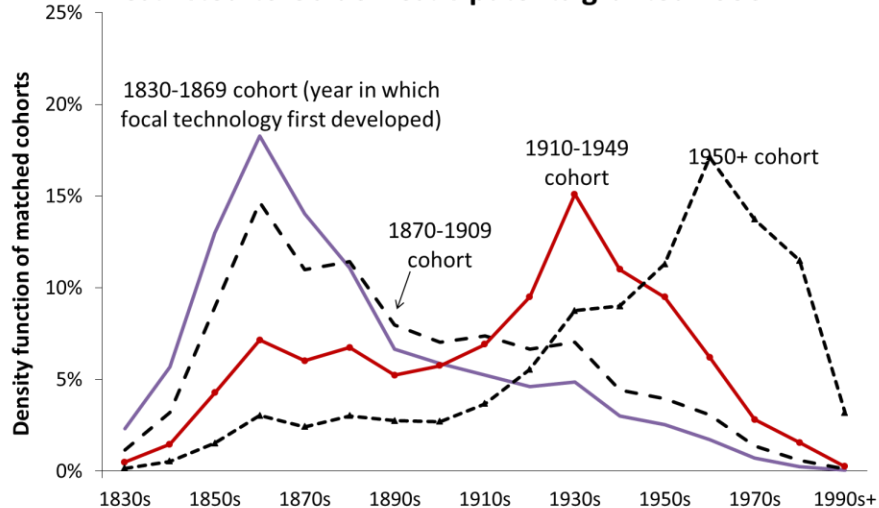
**2d: Shares of technologies developed to date that are used in a year or decade**



Notes: The solid line is the share of technologies exploited in a grant year compared to the cumulative numbers of technologies developed to that point. The dashed line is a decade-level share of technologies utilized during decade compared to total count of technologies developed by end of decade.

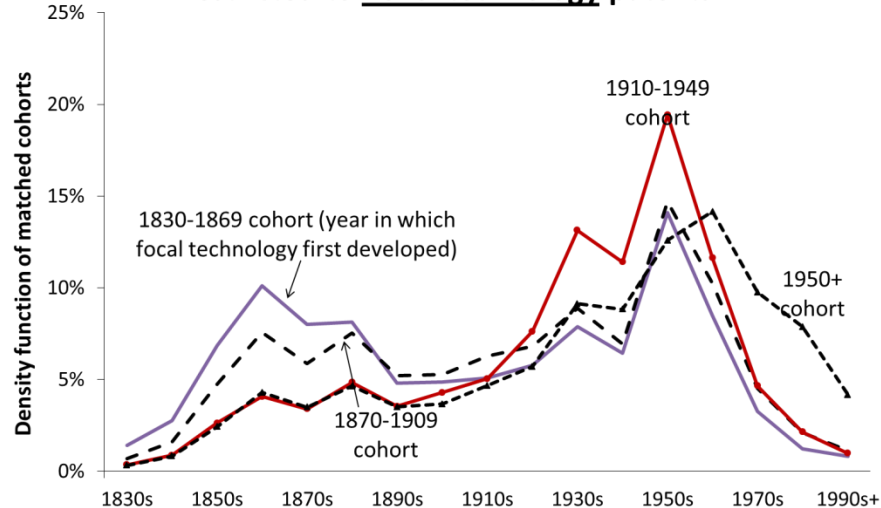
# Figure 3: Cohort matching process for patent technologies

**3a: Density function of paired technology ages restricted to US domestic patents granted 1950+**



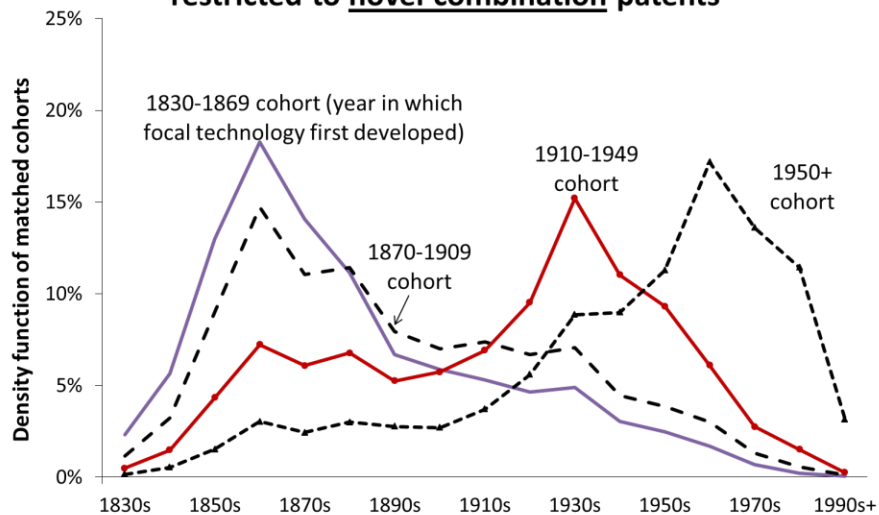
Notes: Figure provides the density function of technology cohorts listed on granted patents. The four density functions consider when the focal technology was first developed. Density functions sum to 100%.

**3b: Density function of paired technology ages restricted to novel technology patents**



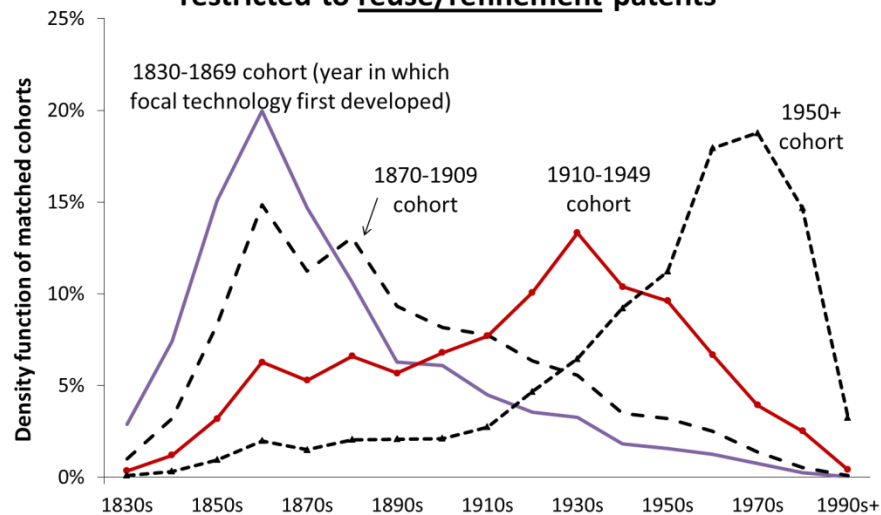
Notes: Sample includes all Novel Technology patents granted 1950 and after.

**3c: Density function of paired technology ages restricted to novel combination patents**



Notes: Sample includes all Novel Combination patents granted 1950 and after.

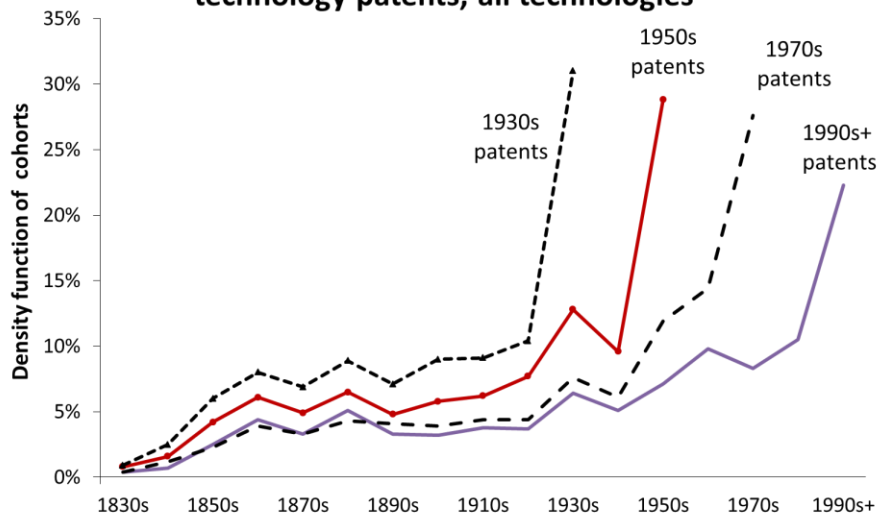
**3d: Density function of paired technology ages restricted to reuse/refinement patents**



Notes: Sample includes all Reuse/Refinement patents granted 1950 and after.

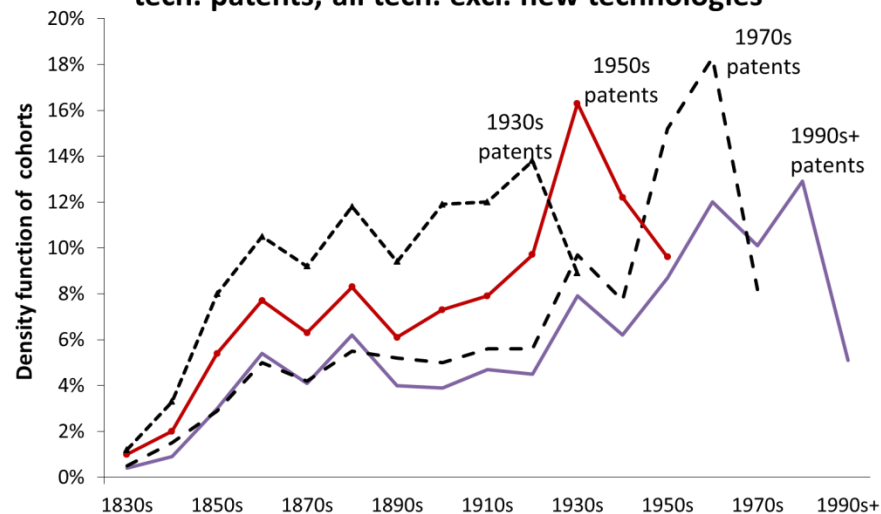
# Figure 4: The leading frontier of novel technology development

**4a: Distribution of technology cohorts on novel technology patents, all technologies**



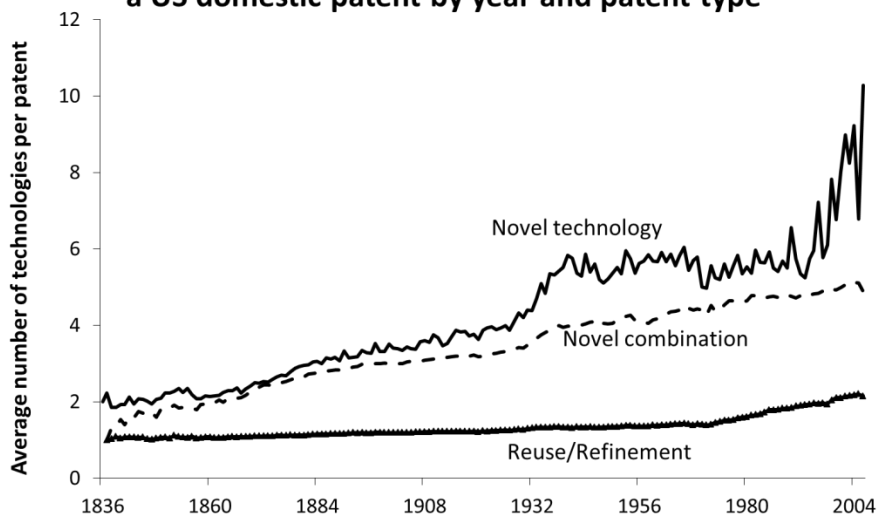
Notes: Figure provides density function of cohorts of all technologies listed on patents that contain a new technology. The density functions consider when the patent was granted. Density functions sum to 100%.

**4b: Distribution of technology cohorts on novel tech. patents, all tech. excl. new technologies**



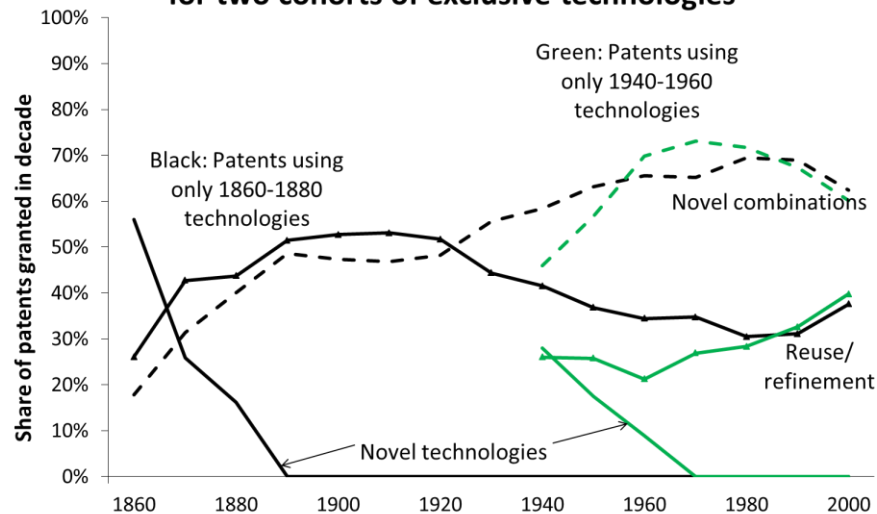
Notes: Figure provides density function of cohorts of all technologies other than new technologies listed on patents that contain a new technology.

**4c: Average number of technologies combined into a US domestic patent by year and patent type**



Notes: Figure provides the average number of technologies combined into patents by grant year and patent type.

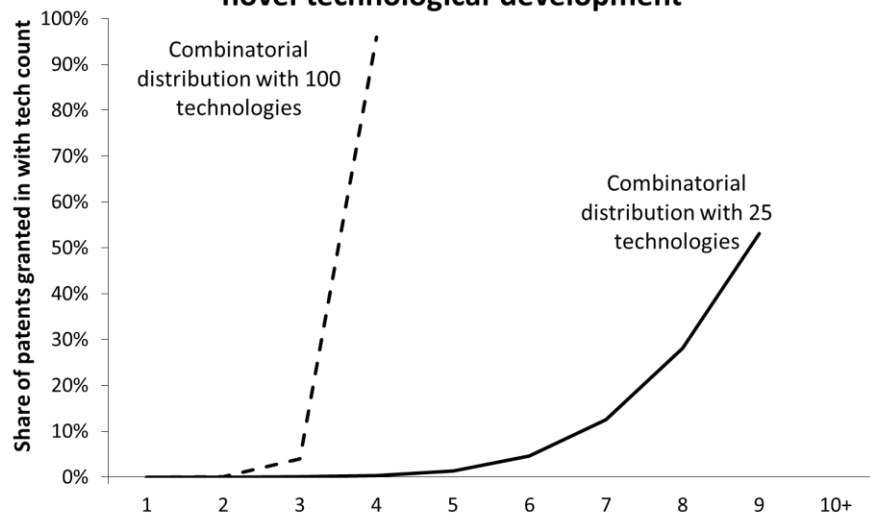
**4d: Comparing the evolution of invention forms for two cohorts of exclusive technologies**



Notes: Figure compares the evolution of patent cohorts. The black group considers patents that include only technologies discovered during 1860-1880. The green group similarly considers patents that only include technologies discovered during 1940-1960. Shares by patent type and grant decade are displayed.

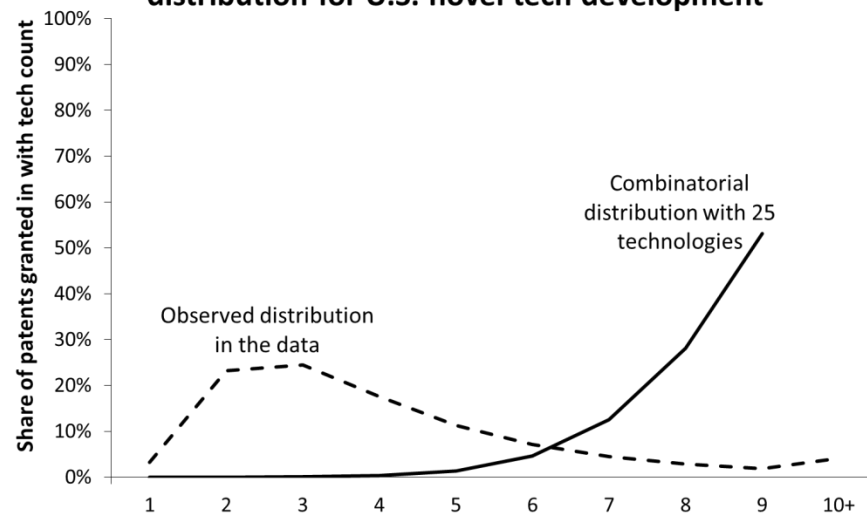
# Figure 5: Comparisons to strict combinatorial processes

**5a: Strict expressions of combinatorial power for novel technological development**



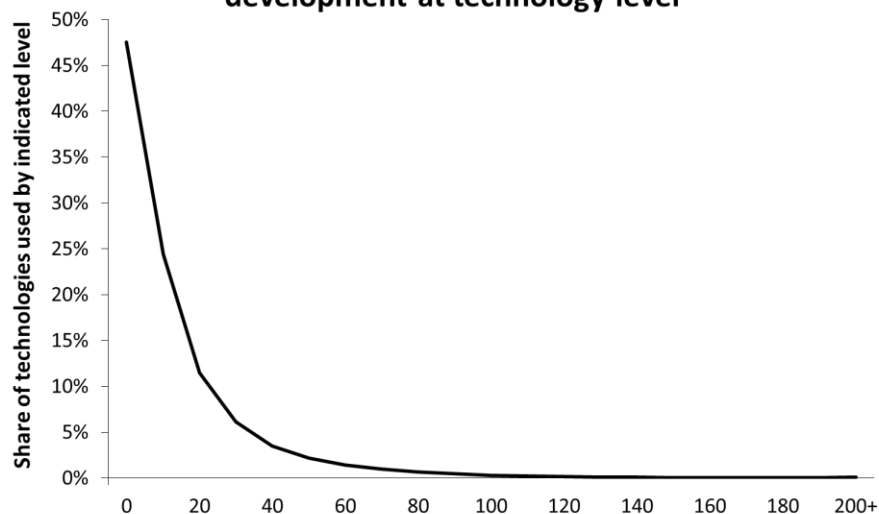
Notes: Figure illustrates combinatorial power with counts of 25 and 100 technologies. For  $n \in \{25, 100\}$  and  $j \in \{1, 2, \dots\}$ , combinatorial power is expressed as  $n! / (j! * (n-j)!)$  until roughly the number of novel technologies observed for the United States is achieved. The lines present the resulting distributions in share terms.

**5b: Comparisons of Figure 5a to the observed distribution for U.S. novel tech development**



Notes: Figure plots the observed distribution of the 3.3 million novel patents granted by 2004 to U.S. inventors (i.e., excluding reuse/refinement patents). The technology count embedded in the realized distribution is over 150,000 technologies.

**5c: Distribution of unique uses for U.S. novel tech development at technology level**



Notes: Figure plots the observed distribution of the number of unique novel uses (novel technology or novel combination) across the more than 150,000 technologies developed during the period. 71,864 technologies are used less than ten times in novel ways, while 194 technologies are used 200+ times in novel ways.

Count	Data		n=25		n=100	
1	105,239	3.2%	25	0.0%	100	0.0%
2	754,588	23.2%	300	0.0%	4,950	0.1%
3	795,195	24.5%	2,300	0.1%	161,700	4.0%
4	569,048	17.5%	12,650	0.3%	3,921,225	95.9%
5	365,301	11.2%	53,130	1.4%		
6	230,033	7.1%	177,100	4.6%		
7	144,055	4.4%	480,700	12.5%		
8	91,736	2.8%	1,081,575	28.1%		
9	59,194	1.8%	2,042,975	53.1%		
10+	134,215	4.1%				
	3,248,604		3,850,755		4,087,975	

Data distribution excludes reuse/refinement patents and foreign patents. Data singleton count 105,239 is less than the total n of 151,208 due to some technologies never appearing alone.

FIGURE 6: ILLUSTRATION OF THE ECONOMY

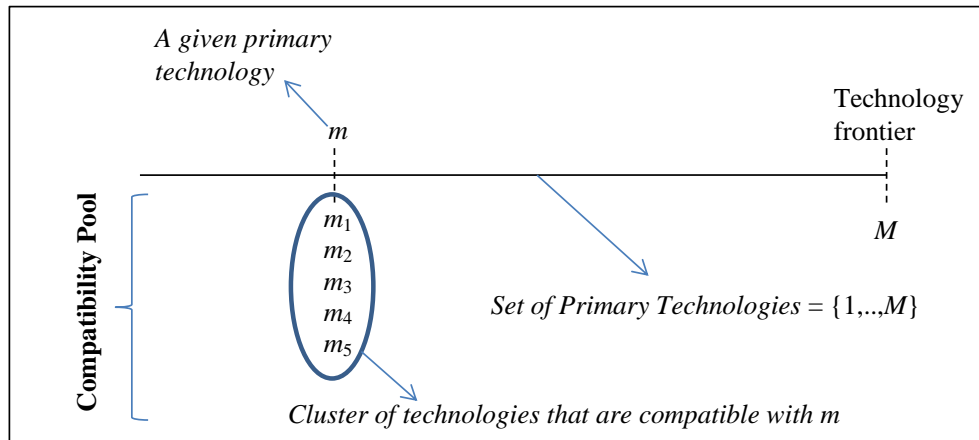


FIGURE 7: ILLUSTRATION OF THE ECONOMY

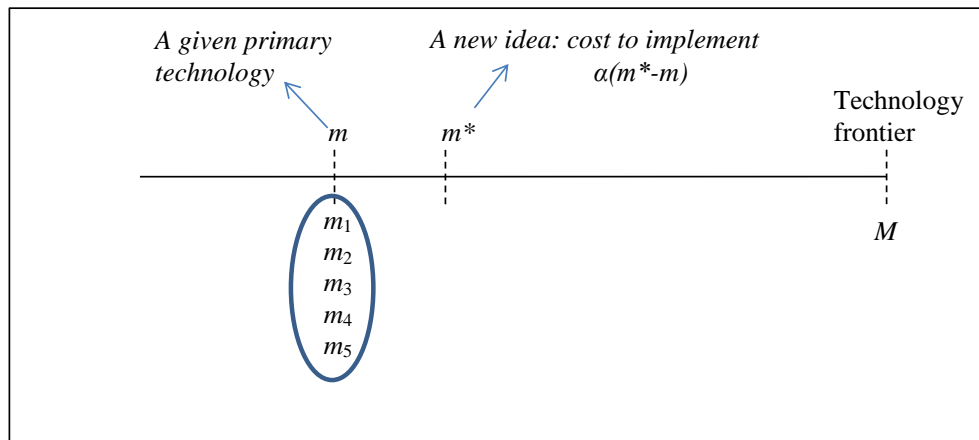


FIGURE 8: ILLUSTRATION OF THE ECONOMY

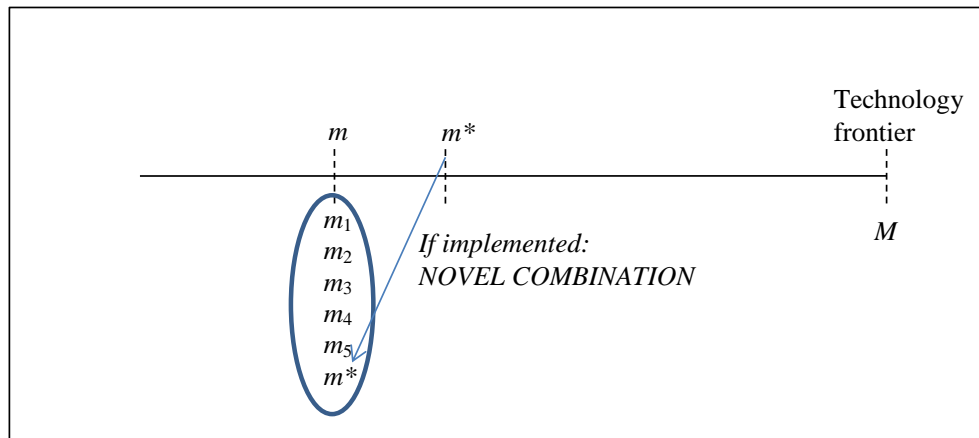


FIGURE 9: ILLUSTRATION OF THE ECONOMY

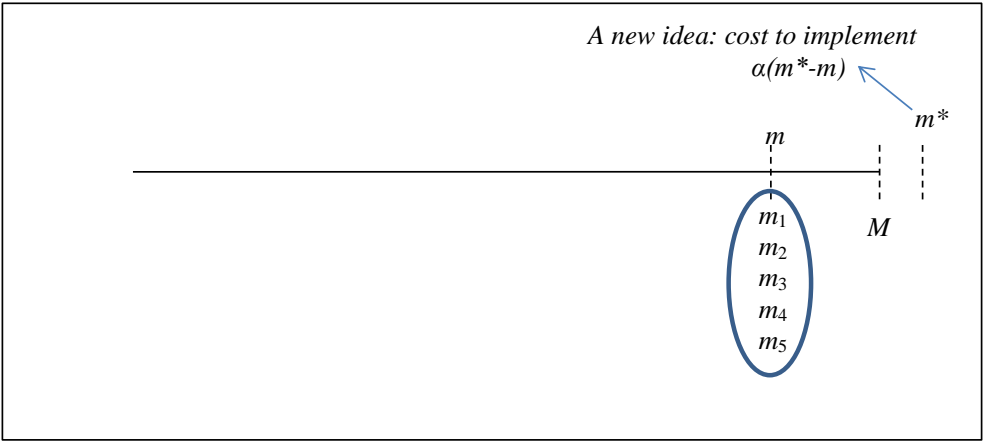
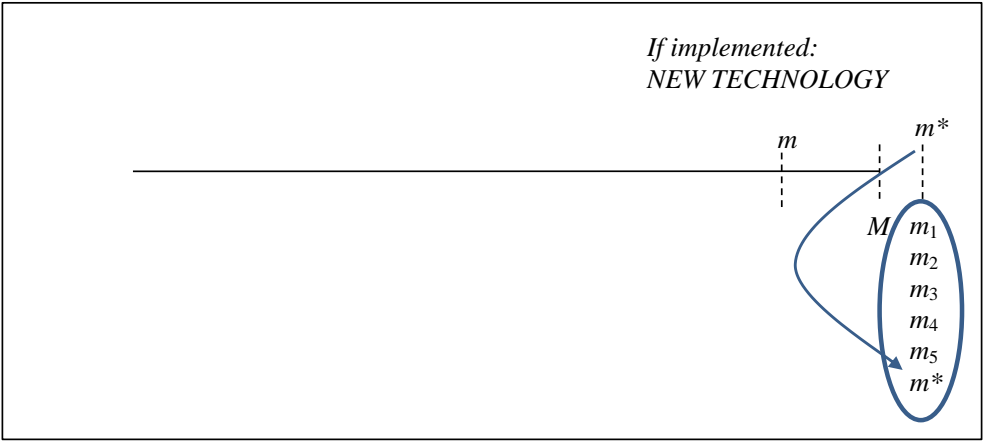


FIGURE 10: ILLUSTRATION OF THE ECONOMY



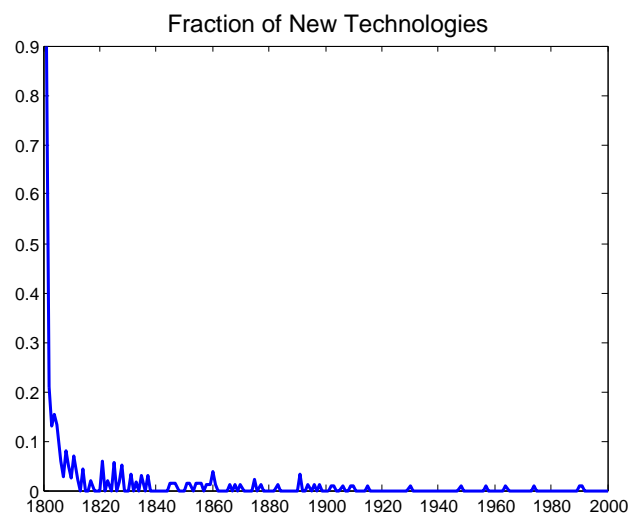


FIGURE 11: NEW TECHNOLOGIES

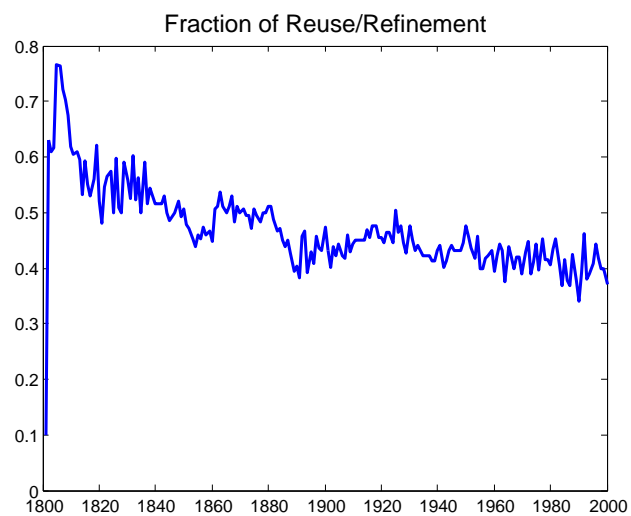


FIGURE 12: REUSE/REFINEMENT

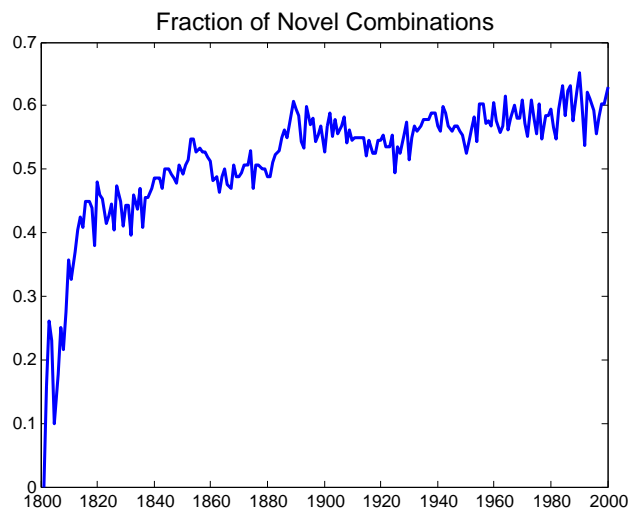


FIGURE 13: NOVEL COMBINATIONS

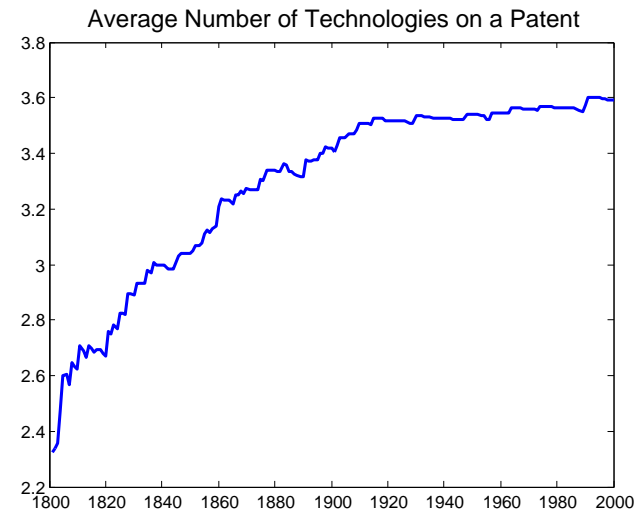
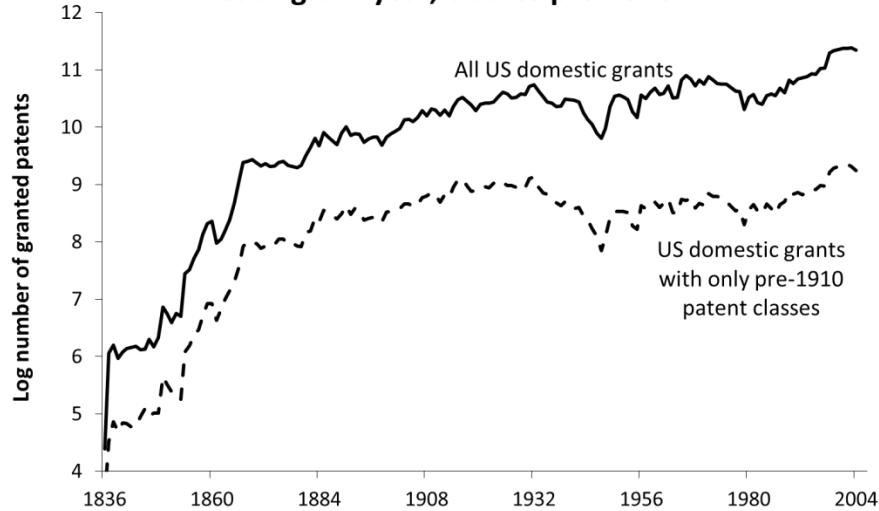


FIGURE 14: AVERAGE NUMBER OF TECHNOLOGIES

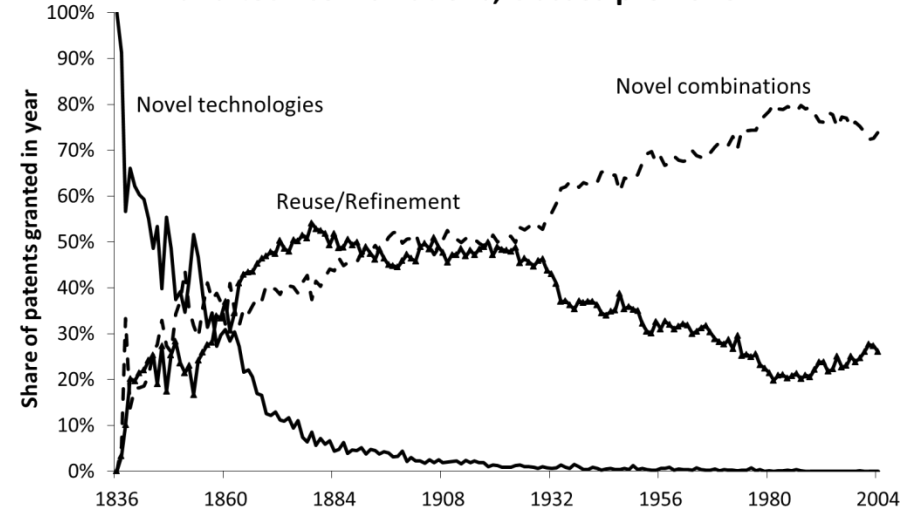
# Appendix Figure 1: Figure 1 with patents using pre-1910 classes

**1a: Number of patents granted by the USPTO in each grant year, classes pre-1910**



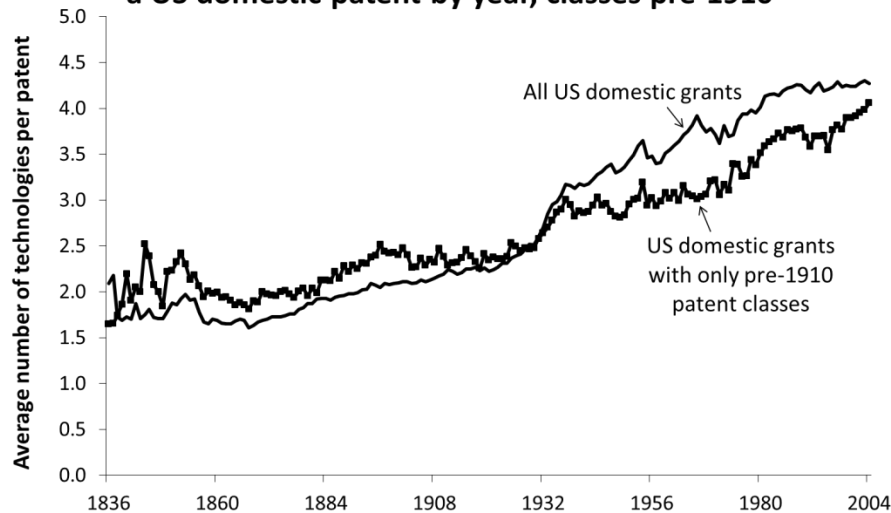
Notes: Figure provides the log count of patents granted by the USPTO in each year. The sample is patents with inventors who are residing in the United States at the time of their patent filing.

**1b: Shares of US domestic patents by grant year and tech combinations, classes pre-1910**



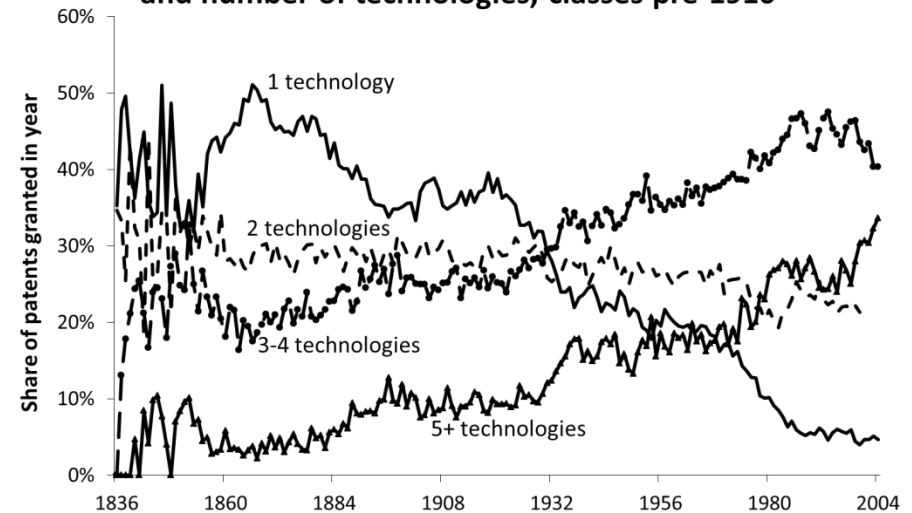
Notes: Novel technology patents contain a new technology. Novel combination patents combine previously observed technologies in a new way. Reuse/Refinement patents repeat a prior technology combination.

**1c: Average number of technologies combined into a US domestic patent by year, classes pre-1910**



Notes: Figure provides the average number of technologies combined into patents by grant year. The sample is restricted to patents with inventors residing in the United States.

**1d: Shares of US domestic patents by grant year and number of technologies, classes pre-1910**



Notes: Figure documents the annual share of patents granted by the number of technologies listed on the patent. The sample is restricted to patents with inventors residing in the United States.



**Table 1: Innovation type and employment growth of firm**

	Conservative matching strategy			Aggressive matching strategy		
	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable is employment growth of firm						
Log employment	-0.065 (0.002)	-0.065 (0.002)	-0.065 (0.002)	-0.055 (0.002)	-0.055 (0.002)	-0.055 (0.002)
Log patents	0.049 (0.003)	0.048 (0.004)	0.048 (0.004)	0.016 (0.003)	0.014 (0.002)	0.014 (0.002)
Type distribution of firm's patents (relative to reuse/refinement patents):						
Share of patents that are not reuse/refinements		0.023 (0.011)			0.030 (0.009)	
Share of patents that are novel technology			0.106 (0.079)			0.134 (0.073)
Share of patents that are novel combinations			0.023 (0.011)			0.029 (0.009)
Industry-period effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	29,496	29,496	29,496	43,435	43,435	43,435

Notes: Table quantifies the relationship between firm employment growth and the type distribution of a firm's patents. In particular, it shows the higher growth effects from patents that are novel technology discoveries or technology combinations relative to reuse/refinement patents. The sample includes all firms in the U.S. Census Bureau data that are matched to patent data during the 1975+ period. The data center on three time periods: 1978-1982, 1983-1987, and 1988-1992. Firm growth is measured as  $[\text{emp}(t+1) - \text{emp}(t)]^2 / [\text{emp}(t+1) + \text{emp}(t)]$  across these periods. Patent-type distributions are determined by inventions made by the firm in the current period. Earlier and later periods than 1978-1992 are used in constructing these variables. Estimations include industry-period fixed effects, cluster standard errors at the firm level, and are unweighted.

**Table 2: Distribution of citations, 1947-2004 patents**

Cohort of citing technology	Cohort of cited technology			
	Each row sums to 100%			
	1836-1869	1870-1909	1910-1949	1950+
	(1)	(2)	(3)	(4)
A. All patents, cohort of primary technology				
1836-1869	<b>0.538</b>	0.305	0.118	0.039
1870-1909	0.253	<b>0.505</b>	0.176	0.066
1910-1949	0.123	0.224	<b>0.488</b>	0.165
1950+	0.051	0.104	0.204	<b>0.641</b>
Total	0.250	0.305	0.242	0.203
B. All patents, median technology cohort on patent				
1836-1869	<b>0.600</b>	0.334	0.056	0.010
1870-1909	0.241	<b>0.571</b>	0.159	0.029
1910-1949	0.060	0.251	<b>0.553</b>	0.137
1950+	0.019	0.079	0.242	<b>0.660</b>
Total	0.243	0.355	0.244	0.158

Notes: Table documents the cohort age effect among patent citations for inventions granted after 1947.