

Math 223b : Algebraic Number Theory

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Meeting Time: MWF 3-4:15 **Location:** Science Center 310

Course Assistant: TBA **Section Time and Location:** TBA

Course Website: canvas.harvard.edu/courses/38193

Course Description

Math 223b is the second half of the graduate class field theory sequence. It covers global class field theory.

Tentative List of Topics

Review of global fields and introduction to adeles. Statements of global class field theory and applications. Brauer groups. Proofs of the main theorems of class field theory. L-functions and Chebotarev density. Complex multiplication and explicit class field theory for imaginary quadratic fields.

Prerequisites

The prerequisite for this class is Math 223a or permission of the instructor.

Textbook and References

There is no required textbook for the course. However, I will post PDF lecture notes on the Canvas website immediately after each class.

Other recommended references:

- *Class Field Theory course notes* by James Milne,
<http://www.jmilne.org/math/CourseNotes/CFT310.pdf>
 I really like Milne's course notes in general.
- *Algebraic Number Theory* by Jürgen Neukirch.
<https://link.springer.com/book/10.1007/978-3-662-03983-0>
 This is a really good overview of algebraic number theory from the basics through class field theory. Its main drawback is that it avoids use of cohomological techniques.
- *Class Field Theory: the Bonn Lectures* by Jürgen Neukirch.
<https://link.springer.com/book/10.1007/978-3-642-35437-3>
 More focused on the proofs of class field theory than Neukirch's other book with a more systematic treatment of Galois cohomology
- *Algebraic Number Theory* edited by Cassels and Frohlich. The classic text: the author list is a who's who of 20th century algebraic number theorists. A little uneven, and watch out for typos (there are well-documented online errata), but many chapters are very good.
- *Primes of the form $x^2 + ny^2$* by David Cox.
http://nrs.harvard.edu/urn-3:hul.ebookbatch.SAFAR_batch:9781118390184
 Good introduction to the statements of global class field theory (no proofs) and to the theory of complex multiplication.

Homework and Grading Policies

If you are taking the class for a grade, your final class grade will be based 80% on weekly homework and 20% on the final paper.

Homeworks will be assigned weekly, and will be due Fridays at midnight. You may submit them online before the start of class or bring a hard copy to class. All homeworks will be weighted equally and the lowest two homework grades will be dropped.

The final assessment for this class will be a 5-10 page paper on a topic related to the course material.

You are encouraged to discuss the homework problems with your classmates, but you must write them up independently. You should acknowledge everyone you worked with in your homework writeups, as well as any external sources you consulted.