

Synchronization in Biology

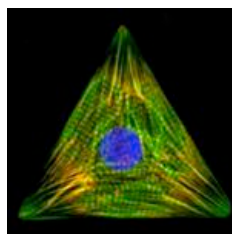
CS289 2018



Examples of Synchrony

Some videos

- Metronomes
- Fireflies
- Bees





Examples of Synchrony

- Physical systems
 - Pendulums, Clocks, Turbines – anything with periodicity
- Cells
 - Pacemaker cells (heart, neuro-muscular, defects)
 - Central pattern generators (“pattern” => locomotion)
 - *Many similar cells throughout our body!*
 E.g. Oesophagus and intestines (“waves”),
 chewing, breathing, running, walking.
 Often have to be “tuned”, e.g. baby sucking, bouncing
- Organisms
 - Fireflies, Cicadas chirping, Bees
 - People! Audiences clapping or doing the Mexican wave

Can all these systems be viewed with the same lens?

History

- Fireflies can synchronize
 - Why do they synchronize?
 - How was this discovered?
 - What did people think was going on?



“The apparent phenomenon was caused by the twitching or sudden lowering and raising of my eyelids. The insects had nothing whatsoever to do with it”

Philip Laurent, Science 1917

History

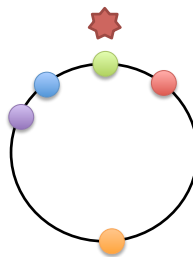


John and Elisabeth Buck, 1985

- Fireflies can synchronize
 - Why do they synchronize?
 - How was this discovered?
 - What did people think was going on?
- Buck and Buck experiments (1940s-1970s)
 - Extensive field studies and single firefly experiments
- Mathematical models
 - Peskin (1975), **Mirollo-Strogatz** (1990), Lucarelli-Wang (2004)
 - Many more models: Kuromoto models, Ermentrout-Kopell, etc

The Mirollo Strogatz Model

Describe a simplified version

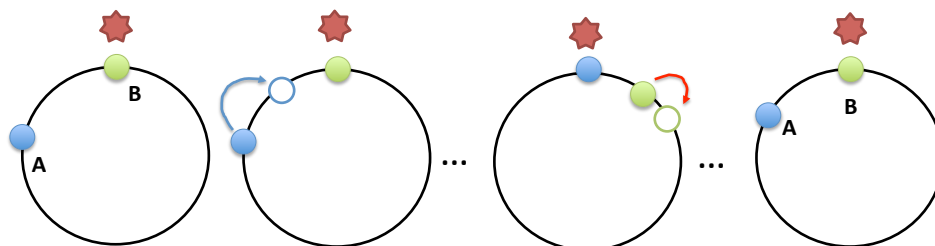


"The Race Track"

Each oscillator is a racer on the track.
 Assume all agents have the same speed.
 When someone crosses the top line a "gun" is fired
 Everyone who hears it "reacts" by instantaneous jump forward

What "reaction" function results in synchrony?

A Simple 2 Agent Model



At time zero, let B be arriving at the firing point

$$\text{Diff}(A \rightarrow B) = X$$

B's phase = 0
A's phase = $1-X$

Lets assume for now that $X < 1-X$

B fires, and A "jumps"
Let's assume jump is a function of A's phase

$$\begin{aligned} \text{New Diff}(A \rightarrow B) \\ = X - \text{jump}(1-X) \end{aligned}$$

A's new phase = $1-X + \text{jump}(1-X)$

Note: "Absorption" prevents A from passing B

Some time later, A fires, and B "jumps"

Note 1: At this point
B's phase = $\text{Diff}(A \rightarrow B)$

Note 2: In this particular model the "jump" is always positive.

$$\begin{aligned} \text{New Diff}(A \rightarrow B) \\ = X - \text{jump}(1-X) \\ + \text{jump}(X - \text{jump}(1-X)) \end{aligned}$$

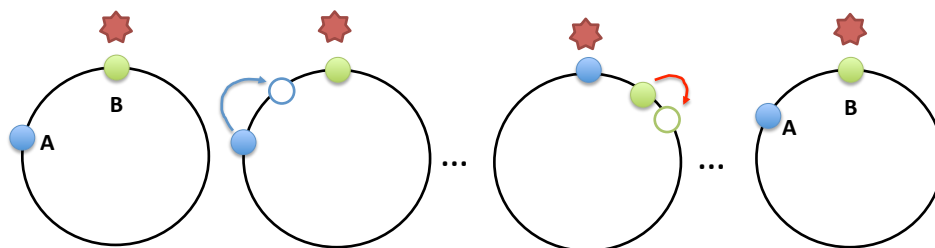
Some time later, we are back to the original situation where B is about to fire....

i.e. Return Map

Ideally,
A and B got closer
 $\text{New Diff} < \text{Old Diff}$

Did they?

A Simple 2 Agent Model



Let original $\text{Diff}(A \rightarrow B) = X$
Assume that $X < 1-X$

Return Map

$$\begin{aligned} \text{New Diff}(A \rightarrow B) &= X - \text{A's jump} + \text{B's jump} \\ &= X - \text{jump}(1-X) + \text{jump}(X - \text{jump}(1-X)) \end{aligned}$$

What jump function would NOT work?
What jump functions would?

We are given that $1-X > X$

Two criteria on jump
Positive and monotonically increasing
 $\text{jump}(w) > 0$ always
 $\text{jump}(w) > \text{jump}(w_0)$ if $w > w_0$,
then $1-X > X - \text{jump}(1-X)$
And $\text{jump}(1-X) > \text{jump}(X - \text{jump}(1-X))$
Proved!

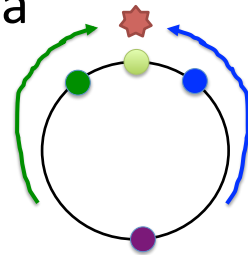
Overall Idea

The Return Map “pulls” the oscillators closer

Note that in this model

- Jumps are always positive.
- The “order” of oscillators never changes, because of absorption.
- **Pulling happens on both sides, but there is an unstable “fixed” point: $\text{ReturnMap}(X) = X$**

Can do this analysis for **N oscillators** too.



Unstable fixed point
at opposite phase

A good jump function has the following property:

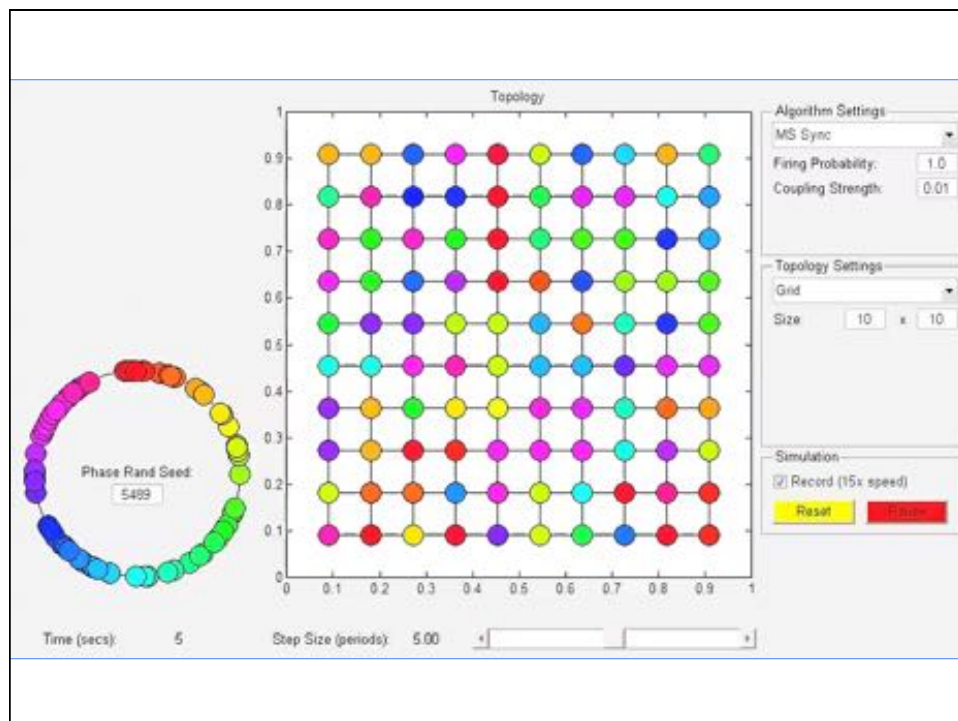
=> A group of oscillators gets “closer” with each round of firings.

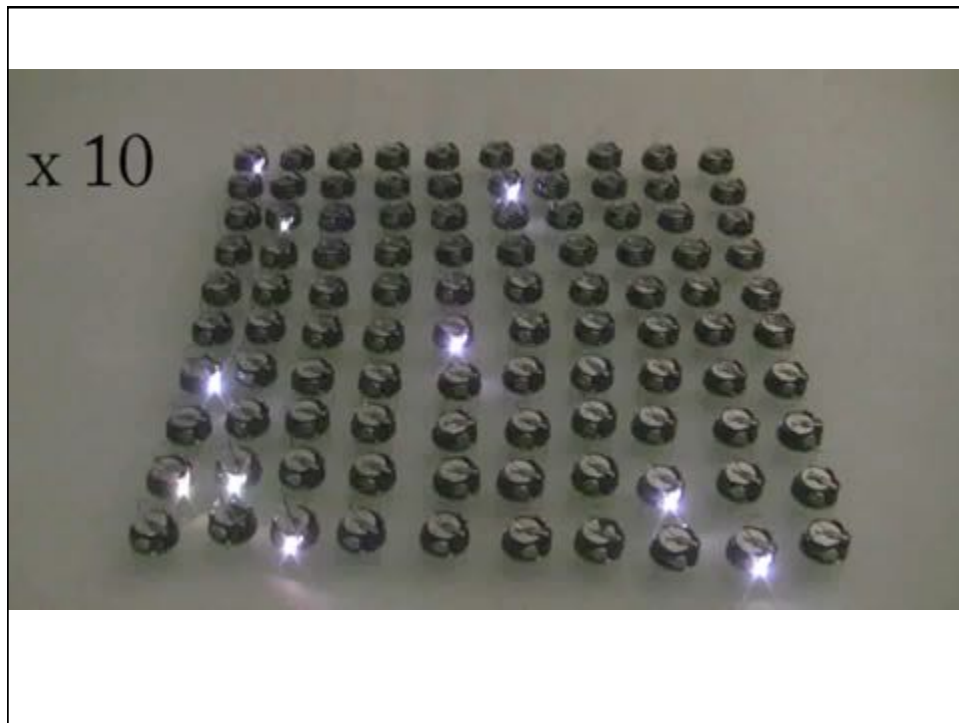
What Mirollo and Strogatz proved was that it is easy to find a good jump function!

(Note that in our model, a **monotonically increasing jump function** is the same as a **concavity requirement on a voltage function**)

Example: $\text{Jump}(X) = c \cdot X$ [equivalent to $f(X) = \ln(cX)$ in M&S model]

Intuitively, you jump more strongly the further you are from firing

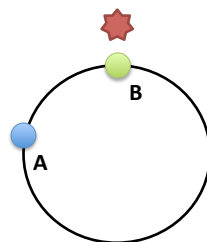




A Simple 2 Agent Model

Activity: Modify the "jump" behavior to

1. Maintain a fixed "delta" between the two
2. Desynchronize (opposite of sync)



Lots of interesting questions

- For the same model
 - What happens if: noise, heterogeneity, errors?
 - How can you break synchrony?
- Can this work for a network of oscillators?
 - How scalable or robust is this idea?
- Can I make other patterns?
 - E.g. waves, tripod gaits (think of millipedes!)

Using “heart” cells to create artificial pulsing tissues

