CHAPTER 4

Assortative Mating in Marriage Markets

Chapter 3 argued that an efficient marriage market assigns imputed incomes or "prices" to all participants that attract them to suitable polygamous or monogamous marriages. Imputed prices are also used to match men and women of different qualities: some participants, we have seen, choose to be matched with "inferior" persons because they feel "superior" persons are too expensive. Obstacles to the efficient pricing of participants arise when the gains from marriage cannot readily be divided or when one spouse (usually the husband) is given more power than the other. Bride prices, dowries, divorce settlements and other capital transfers evolved partly to overcome such obstacles.

This chapter shows that an efficient marriage market usually has positive assortative mating, where high-quality men are matched with high-quality women and low-quality men with low-quality women, although negative assortative mating is sometimes important. An efficient market also tends to maximize the aggregate output of household commodities, so that no person can improve his marriage without making others worse off.

As we have seen, the mating of superior men and women is an implicit form of polygamy, which can substitute for explicit polygamy.

This chapter proves the converse, that explicit polygamy is an implicit form of positive assortative mating, which can substitute for the mating of superior persons. Consequently, the mates of polygynous males tend to be of a lower average quality than the mates of equally superior monogamous males.

Equilibrium Conditions for Assortative Mating with Monogamy

Identical men receive the same income in an efficient marriage market regardless of whom they marry or whether they choose to remain single. Since marriages with superior women produce larger outputs, superior women receive higher incomes in efficient markets. If all marriages were monogamous, an assumption maintained in this section, the difference between the incomes of the jth woman and the ith woman would be:

$$Z_j^f - Z_i^f = (Z_{mj} - Z^m) - (Z_{mi} - Z^m) = Z_{mj} - Z_{mi}, \qquad (4.1)$$

where Z_k^f is the equilibrium income of the kth woman, Z^m is the equilibrium income of men, and Z_{mk} is the marital output of the kth woman and any man. Superior women receive a premium that is determined by their additional productivity as wives.

The analysis is considerably more complicated when both men and women differ; incomes then depend on how they are sorted into different marriages. Moreover, the optimal sorting in turn is determined by the set of equilibrium incomes. This appearance of circularity is resolved by recognizing that both are determined simultaneously in the marriage market. In an efficient marriage market superior persons tend to marry one another and are compensated for their higher productivity.[1]

The commodity outputs produced by single persons and by all possible monogamous matings between an equal number of men and women (unequal numbers are considered later in this chapter) are shown by the following matrix:

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1. The discussion in the remainder of this section is based on Becker, 1973 and 1974a.

$$
\begin{array}{c} \\ \\ M_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ M_N \end{array}
\begin{array}{c}
\begin{array}{cccc} & F_1 & \cdots\cdots & F_N \end{array} \\
\begin{bmatrix}
 & Z_{s1} & \cdots\cdots & Z_{sN} \\
Z_{1s} & Z_{11} & \cdots\cdots & Z_{1N} \\
\cdot & \cdot & & \cdot \\
\cdot & \cdot & & \cdot \\
\cdot & \cdot & & \cdot \\
\cdot & \cdot & & \cdot \\
\cdot & \cdot & & \cdot \\
\cdot & \cdot & & \cdot \\
Z_{Ns} & Z_{N1} & \cdots\cdots & Z_{NN}
\end{bmatrix}
\end{array}
\qquad (4.2)
$$

where $F_1, \ldots, F_N$ and $M_1, \ldots, M_N$ refer to women and men of different qualities. Since the complementarity between men and women and the differences between their comparative advantages imply that both men and women are better off married, the row and column giving single outputs can be ignored and attention focused on the $N \times N$ matrix of marital outputs.

There are $N!$ ways to select one entry in each row and column, or $N!$ different sortings that permit each man to marry one woman and vice versa. The aggregate marital output produced by any sorting can be written as

$$Z^k = \sum_{i_k \in M,\, j_k \in F} Z_{ij}, \qquad k = 1, \ldots, N! \qquad (4.3)$$

If a sorting that maximizes total output is numbered so that its entries lie along the diagonal, the maximum total output can be written as

$$Z^* = \sum_{i=1}^{N} Z_{ii} = \max Z^k \geq Z^k \quad \text{for all } k. \qquad (4.4)$$

If each person is a utility maximizer and chooses the mate who maximizes his utility, the optimal sorting must have the property that persons not married to each other could not marry without making at least one of them worse off. In game theoretic language, the optimal sorting is in the core, since no (monogamous) coalition outside the core could make either of its members better off without making the other worse off.

Utility is monotonically related to commodity income; therefore a noncore marriage cannot produce more than the sum of the incomes that its two mates would receive in the core. If it could produce more,

and if any division of output were feasible,[2] a division could be found that would make each better off, thereby contradicting the optimality of the core. If the sorting along the diagonal were in the core, this condition states that

$$Z_i^m + Z_j^f \geq Z_{ij} \quad \text{for all } i \text{ and } j, \qquad (4.5)$$

where the accounting identity between output and income implies that

$$Z_i^m + Z_i^f = Z_{ii}, \qquad i = 1, \ldots, N. \qquad (4.6)$$

Condition (4.5) immediately excludes any sorting from the core that does not maximize aggregate commodity output, for otherwise at least one man and one woman would be better off with each other than with their mates assigned by the core. Conversely, any sorting that does maximize aggregate output must be part of the core.[3] Moreover, the theory of optimal assignments, which has the same mathematical structure as the sorting of persons by marriage, implies that generally more than one set of incomes satisfies conditions (4.5) and (4.6) for a sorting that maximizes aggregate output (for a proof see Koopmans and Beckmann, 1957, p. 60).

The solution can be illustrated with the following 2×2 matrix of outputs:

$$
\begin{array}{ccc}
 & 5 & 2 \\
 & F_1 & F_2 \\
3\; M_1 & 8 & 4 \\
5\; M_2 & 9 & 7
\end{array}
\qquad
\begin{array}{cc} & \begin{array}{cc} F_1 & F_2 \end{array} \\ \begin{array}{c} M_1 \\ M_2 \end{array} & \begin{bmatrix} 8 & 4 \\ 9 & 7 \end{bmatrix}. \end{array}
\qquad (4.7)
$$

Although the maximum output of a marriage is produced by a marriage between M_2 and F_1, the optimal sorting is (M_1,F_1) and (M_2,F_2). For if $Z_1^m = 3, Z_1^f = 5, Z_2^m = 5,$ and $Z_2^f = 2$, then M_2 and F_1 have no incentive

2. Bride prices and dowries introduce considerable flexibility into the effective division of output, even when the apparent division is inflexible. I shall discuss this point later in the chapter.

3. If M_i married F_j and M_p married F_i in an optimal sorting k that does not maximize total output, condition (4.5) requires that $Z_i^m + Z_i^f \geq Z_{ii}$, for all i. Hence, by summation,

$$Z^k = \sum_{\text{all marriages in } k} Z_i^m + Z_i^f \geq \sum_i Z_{ii} = Z^*,$$

where Z^*, the maximum total output, must exceed Z^k because Z^k is less than the maximum by assumption. Thus we have contradicted the assumption that an optimal sorting can produce less than the maximum total output. It is easily shown in the same way that all sortings that maximize total output must be part of the optimal sortings.

to marry, since $Z_2^m + Z_1^f = 10 > 9$; neither do M_1 and F_2, since $Z_1^m + Z_2^f = 5 > 4$.

This example illustrates that the marriage market chooses not the maximum output of any single marriage but the maximum sum of the outputs over all marriages, just as competitive product markets maximize the sum of the outputs over all firms. Put another way, the marriage market acts as if it maximizes not the gain from marriage compared to remaining single for any particular marriage, but the total gain over all marriages.[4] Of course, the commodity output maximized by households is not to be identified with national output as usually measured, but includes the quantity and quality of children, sexual satisfaction, and other commodities that never enter into measures of national output.

The process of discovering optimal sortings is greatly simplified by this conclusion that aggregate output is maximized, because any sorting that maximizes aggregate output is an optimal sorting and must be able to satisfy condition (4.5), a condition that would be difficult to verify directly. I should emphasize, moreover, that the optimality of maximizing aggregate output is a theorem, not an assumption about behavior.[5] Each man and woman is assumed to be concerned only about his or her own "selfish" welfare, not about social welfare. In pursuing their selfish interests, however, they are unknowingly led by the "invisible hand" of competition in the marriage market to maximize aggregate output.

Mating of Likes

Psychologists and sociologists have frequently discussed whether persons with like or unlike traits mate, and biologists have occasionally assumed positive or negative assortative mating instead of random mating for nonhuman species. However, none of these disciplines have developed a systematic analysis that predicts for different traits

4. Clearly, $\Sigma_{i=1}^{N}[Z_{ii} - (Z_{si} + Z_{is})]$ is maximized when $Z^k = \Sigma Z_{ii}$ is maximized, because Z_{si} and Z_{is} (single commodity outputs) are given and are independent of marital sortings.

5. Goode (1974) confuses theorem with assumption in his comment on an earlier paper of mine.

whether likes or unlikes tend to mate.[6] My analysis implies that the mating of likes (or unlikes) takes place when such pairings maximize aggregate commodity output over all marriages, regardless of whether the trait is financial (wage rates, property income), biological (height, race, age, physique), or psychological (aggressiveness, passiveness). This analysis is also applicable to matching workers with firms, students with schools,[7] farms with farmers, customers with shopkeepers, and worker preferences for different kinds of working conditions with firms supplying these conditions.

Assume that men and women differ only in the quantitative traits A_m and A_f respectively, and that each trait has a positive marginal productivity:

$$\frac{\partial Z(A_m,A_f)}{\partial A_m} > 0 \quad \text{and} \quad \frac{\partial Z(A_m,A_f)}{\partial A_f} > 0. \tag{4.8}$$

The major theorem on assortative mating is that a positive sorting of large A_m with large A_f and small A_m with small A_f maximizes aggregate output if, and only if, increasing both A_m and A_f adds more to output than the sum of the effects of separate increases in A_m and A_f. For an increase in A_m would reinforce and raise the effect of an increase in A_f. Similarly, a negative sorting of large A_m with small A_f and small A_m with large A_f maximizes output when increasing both adds less to output than the sum of the effects of separate increases. All sortings have the same aggregate output when increasing both has the same effect as separate increases. This can be formally stated as the following theorem, which is proved in note A of the appendix to this chapter.

6. In an interesting discussion Winch (1958, pp. 88–89) assumes that each person tries to maximize utility ("In mate selection each individual seeks within his or her field of eligibles for that person who gives the greatest promise of providing him or her with maximum need gratification") and, especially in chap. 4, stresses complementary needs as a determinant of mating. However, he brings in "eligibles" as a deus ex machina and, more importantly, nowhere shows how mating by complementary needs produces equilibrium in the marriage market.

7. This sorting is analyzed for Japanese firms by Kuratani (1973). Hicks (1957, chap. 2) asserts, without offering any proof, that more able workers are employed by more able firms. Black and Black (1929, pp. 178 ff.) discuss the sorting of merchants and locations with a few numerical examples. Rosen (1978) gives a valuable, more recent discussion.

Theorem Positive assortative mating—mating of likes—is optimal when

$$\frac{\partial^2 Z(A_m,A_f)}{\partial A_m \partial A_f} > 0, \tag{4.9}$$

because aggregate output is then maximized. Negative assortative mating—mating of unlikes—is optimal when the inequality is reversed.

Consider, as an example, the matrix of outputs between two men and two women:

$$\begin{array}{cc} & \begin{array}{cc} F_1 & F_2 \end{array} \\ \begin{array}{c} M_1 \\ M_2 \end{array} & \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \end{array}, \quad \text{with } A_{m_2} > A_{m_1} \text{ and } A_{f_2} > A_{f_1}. \tag{4.10}$$

If $Z_{22} - Z_{12} > Z_{21} - Z_{11}$ because A_m and A_f are complements, then $Z_{11} + Z_{22} > Z_{12} + Z_{21}$. A positive sorting between A_m and A_f would maximize aggregate output, because increasing both A_m and A_f adds more to output than do separate increases in A_m and A_f.

This theorem indicates that higher-quality men and women marry each other rather than selecting lower-quality mates when these qualities are complements: a superior woman raises the productivity of a superior man and vice versa. The mating of likes or unlikes is optimal as traits are complements or substitutes, because superior persons reinforce each other when traits are complements and offset each other when traits are substitutes. This theorem also implies that the gain from marriage to a woman of a given quality is greater for a superior man when traits are complements, and is greater for an inferior man when traits are substitutes.[8] I shall use this implication later to determine who remains unmarried when the total number of men and women of different qualities is equal.

The theorem can be used to analyze the optimal sorting of particular

8. The gain to M_i from marrying F_j rather than remaining single is

$$G_i = (Z_{ij} - Z_{is}) - Z_j^f,$$

where Z_j^f is the given income of F_j, and Z_{is} is the income of M_i if he remains single. The term in parentheses increases (or decreases) with the quality of M_i when A_m and A_f are complements (or substitutes); see note 16.

financial, biological, or other traits. For example, if men and women differ only in given market wage rates—each man and each woman is assumed to be identical in all other market and household traits—aggregate output is maximized by a perfect negative assortative mating of these wage rates, which maximizes the gain from the division of labor. Low-wage women should spend more time in household production than high-wage women because the time of low-wage women is less valuable, and low-wage men should spend more time in household production than high-wage men. By mating low-wage women with high-wage men and low-wage men with high-wage women, men and women with cheaper time are used more extensively in household production, and those with expensive time are used more extensively in market production.[9]

All differences in the output of commodities that are not related to differences in money incomes must be related to differences in nonmarket productivity—to differences in intelligence, education, health, strength, fecundity, height, personality, religion, or other traits. Consider now optimal sortings when men and women differ only in nonmarket productivity. Since an increase in productivity increases output by reducing the cost of production, the optimal sorting of most nonmarket traits tends to be positive because of the inverse or "harmonic" relation between commodity output and its cost of production:

$$Z = \frac{S}{\pi(w_m, w_f, p, A_m, A_f)}, \tag{4.11}$$

where S is money full income; π is the average cost of producing the household commodity Z; w_m and w_f are the given wage rates; p is the price of goods; and A_m and A_f are traits of men and women respectively.

Since changes in A_m and A_f do not affect S because money income is given, then

9. The proof of this proposition (Appendix; note B) assumes that all men and women are in the labor force, and that an increase in the husband's wage rate does not increase the hours worked by his working wife. The second assumption is consistent with the available evidence (see for example Cain, 1966), but the first is not, since some women never participate in the labor force after they marry (Heckman, 1981). A perfect negative sorting might not be the only optimal sorting when some married women do not participate (see the discussion in Becker, 1973, pp. 827–829).

$$\frac{\partial^2 Z}{\partial A_m \partial A_f} > 0 \quad \text{if } 2\pi^{-1}\pi_{a_m}\pi_{a_f} > \pi_{a_m,a_f}, \quad \text{where } \frac{\partial \pi}{\partial A_i} = \pi_{a_i} < 0, \quad \text{for } i = m,f. \tag{4.12}$$

Condition (4.12) necessarily holds if A_m and A_f have either independent or reinforcing effects on average costs, for then $\pi_{a_m,a_f} \leq 0$; moreover, (4.12) might hold even if they have offsetting effects. Therefore, positive assortative mating is optimal not only when nonmarket traits have reinforcing effects on costs, but a less obvious and more impressive conclusion is that it is also optimal when the traits have independent effects on costs and may be optimal even when they have offsetting effects, because of the harmonic relation between output and cost of production.

This tendency toward complementarity between traits that affect nonmarket productivity can be seen more transparently by considering a couple of special cases. The cost function would be multiplicative and separable if the elasticity of output with respect to either trait were independent of goods and time:

$$\pi = b(A_m,A_f)K(w_m,w_f,p). \tag{4.13}$$

Hence,

$$\frac{\partial^2 Z}{\partial A_m \partial A_f} > 0 \quad \text{as } 2b^{-1}b_m b_f > b_{mf}, \tag{4.14}$$

which must hold if $b_{mf} \leq 0$ and might hold even if $b_{mf} > 0$. This is the same as condition (4.12) except that b does not depend on wage rates or on the substitutability between the household time of husbands and wives. Positive assortative mating is optimal even when the traits of husbands and wives have independent effects on b ($b_{mf} = 0$) because output is harmonically related to b.

The separability assumption embodied in Eq. (4.13) is too strong; most traits affect output partly by raising the efficiency of the time supplied to a household. A simple, if extreme, way to incorporate this relation is to assume each trait affects output only by augmenting the effective amount of household time. Appendix note C proves the plausible result that positive assortative mating is still optimal as long as the elasticity of substitution between the household time of men and women is not very high. Negative assortative mating is optimal for traits augmenting kinds of time that are easily substitutable between men and women.[10] Consequently, positive assortative mating is to be expected when the effective amount of time is augmented; the time of men and the time of women have generally not been close substitutes because of women's investments in and other orientation toward child rearing and men's investments in and other orientation toward market activities. Note, however, that the substitutability between the time of men and women increases as demand shifts away from the quantity of children to the quality of children (Chapter 5).

Does our analysis justify the popular belief that more beautiful, charming, and talented women tend to marry wealthier and more successful men? Note D of the appendix shows that it does: a positive sorting of nonmarket traits with property income always, and with earnings usually,[11] maximizes aggregate commodity output. Higher values of nonmarket traits tend to have larger absolute effects on output when combined with higher money incomes, because from Eq. (4.11) the optimal commodity output depends on the ratio of money (full) income to costs.

The simple correlations between intelligence, education, age, race, nonhuman wealth, religion, ethnic origin, height, place of origin, and many other traits of spouses are positive and strong (see Winch, 1958, chap. 1; Vandenberg, 1972). A small amount of evidence suggests that simple correlations between some psychological traits, such as propensities toward dominance, nurturance, or hostility, may be negative (Winch, 1958, chap. 5; Vandenberg, 1972). The correlation between spouses by intelligence is especially interesting, since it is as high as that between siblings (Alström, 1961). Apparently the marriage market, aided by coeducational schools and other devices, is more efficient at sorting than is commonly believed.

The evidence of positive simple correlations for most traits, and of negative correlations for some, is certainly consistent with my theory of sorting. A more powerful test of the theory, however, requires evidence on partial correlations when other traits are held constant. Even

10. Perhaps, therefore, dominant and deferential persons tend to marry (Winch, 1958, p. 215) because the dominant person's time can be used when the household encounters situations calling for dominance, and the deferential person's time can be used when deference is needed.

11. By "usually" I mean that a positive sorting with earnings always maximizes aggregate output when an increase in the nonmarket trait does not reduce the hours worked by the *spouse*, and that a positive sorting might maximize output even when the hours are reduced. I return to this point in note D of the appendix.

when age and wage rates are held constant, the correlation between years of schooling is high: +0.53 for white families and virtually the same (+0.56) for black families.[12] Moreover, persons who marry out of their race, religion, age cohort, or education class have relatively high probabilities of divorce, even when other traits are held constant (see Becker et al., 1977). This is additional evidence that a positive sorting by education and by these other traits is optimal, because the analysis in Chapter 10 implies that divorce is more likely when mates are mismatched.

The evidence on divorce cited above also supports the somewhat surprising implication of the theory derived earlier, that a negative sorting by wage rates is optimal. Divorce is more likely when the wife's wage rate is high relative to that of her husband (again several other variables are held constant). The optimality of a negative sorting is also implied by the larger fraction of women who are married in American states that have higher wages of males and *lower* wages of females (age, years of schooling, the sex ratio, the fraction Catholic, and other variables are held constant—see Freiden, 1974; Preston and Richards, 1975; Santos, 1975) or by the larger fraction of households headed by unmarried women in metropolitan areas where women have higher earnings relative to men (Honig, 1974).[13]

Although the direct evidence on the correlation between the wage rates of spouses is less comforting because it is significantly positive even when age and education are held constant: +0.32 for whites and +0.24 for blacks (calculated from the 1967 Survey of Economic Opportunity mentioned in note 12), this evidence is seriously biased in that marriages are excluded if the wife did not participate in the labor force. Since a woman is more likely to participate when her wage rate is high relative to her husband's, a positive correlation between wage rates for those marriages where both participate is consistent with a negative correlation for all marriages. Indeed, estimates by H. Gregg Lewis (unpublished) and by Smith (1979) indicate that a positive "observed" correlation implies either a negative "actual" correlation (about −0.25

12. A 20-percent random sample of the approximately 18,000 married persons in the 1967 Survey of Economic Opportunity was analyzed. Families were excluded if the husband or the wife was either older than 65 or unemployed, or if the wife was employed for less than 20 hours in the survey week.

13. However, the causation may run the other way, from marital status to labor force participation to wage rates, because wage rates *become* higher when women participate more continuously in the labor force.

according to Lewis' estimate) or a much weaker actual correlation (about +0.04, according to Smith) for all marriages, because a relatively small fraction of married women have participated.[14] Consequently, when the evidence on the wage rates of husbands and wives is suitably interpreted, it also is not grossly inconsistent with a negative sorting.

Sorting with Unequal Numbers of Men and Women

A person enters the marriage market if he expects his marital income to exceed his single income. Therefore, the incomes imputed by the marriage market determine not only the sorting of persons marrying, but also determine who remains single because they cannot do as well by marrying. For example, men and women delay marriage until the complementarity between the sexes and the differences in their comparative advantages in producing children and other household commodities become sufficiently important that they would be better off married. The reason for the typical early marriages of women is that their biology, experiences, and other investments in human capital have been more specialized than those of men to the production of children and other commodities requiring marriage or its equivalent (Chapters 2 and 3).

Some men are forced to remain single if the number of men, N_m, exceeds the number of women, N_f, and if polygamy is not permitted. Those men remain single who cannot compete against other men for the scarce women because they gain less from marriage than the other men do. The equilibrium sorting of the men and women marrying must still maximize aggregate commodity income, for all other sortings violate the equilibrium condition of Eq. (4.5).

If men and women differ in the traits A_m and A_f respectively, positive

14. These adjusted correlations may also be misleading in view of the fact that wages are determined partially by investments in human capital. Women who spend less time in the labor force invest less in market-oriented human capital and thereby reduce their earning power. On the other hand, the positive correlation between the wage rates of husbands and wives who are both participating may really be measuring the predicted positive correlation between a husband's wage rate (or his nonmarket productivity) and his wife's nonmarket productivity. Many unobserved variables, like intelligence, raise both wage rates and nonmarket productivity.

or negative sorting is optimal, as these traits are complements or substitutes. The $N_m - N_f$ men with the lowest qualities remain single when A_m and A_f are complements, because lower-quality men then tend to gain less from marriage and would be outbid for wives by higher-quality men.[15] Similarly, the $N_m - N_f$ highest-quality men remain single when A_m and A_f are substitutes because they tend to be outbid for wives by lower-quality men. This analysis generalizes the Ricardian theory of the extensive margin by permitting idle land (the analogue of single persons) to be productive.

Consequently, the lowest-quality members of the redundant sex remain single when there is positive assortative mating of those marrying, and the highest-quality members remain single when there is negative sorting. Since positive sorting is more likely, lower-quality persons are more likely to remain single. For example, if A_m refers to the property income of men and A_f to the nonmarket productivity of women, and if men are redundant, lower-income men remain single because they gain less from marriage to these women than higher-income men do.

Consider men of three different qualities: M_k, M_j, M_g (ordered from highest to lowest), and women of three different qualities: F_k, F_j, F_g (similarly ordered), when the traits of men and women are complements. If M_k and F_k, M_j and F_j, and M_g and F_g were to marry each other in the equilibrium sorting, the following equilibrium conditions would have to hold:

$$Z_k^m + Z_j^f > Z_{kj}, \qquad Z_j^m + Z_g^f > Z_{jg}, \tag{4.15}$$

where Z_k^m and Z_j^m are the equilibrium incomes of M_k and M_j when married to F_k and F_j respectively, and Z_j^f and Z_g^f are the equilibrium incomes of F_j and F_g when married to M_j and M_g respectively. If M_k and F_k were the only kinds of persons in the marriage market, an increase in the number of M_k would lower the income of married M_k to his single level, Z_{ks}, in order to induce the redundant men to remain single. When these other kinds of persons are also in the marriage market, however, Z_k^m could not be lowered to Z_{ks} without violating the first inequality[16] in (4.15). Some of the lower-quality F_j would be induced to marry the redundant M_k, and the new equilibrium income[17] of M_k would exceed Z_{ks}. The redundant M_k bump lower-quality M_j, since the traits of men and women are complements. Some M_j become redundant when they are bumped out of their marriages, their incomes fall, and some F_g are induced to marry these redundant M_j.

The bumping of lower-quality men out of their marriages through competitive reductions in the incomes of higher-quality men continues until the incomes of the lowest-quality men are reduced to their single levels. Since these men no longer gain from marriage, some are willing to remain single.

Thus an increase in the number of men of a particular quality tends to lower the incomes of all men and raise those of all women because of the competition in the marriage market between men and women of different qualities. Moreover, if the optimal sorting were positive because the traits of men and women were complements, some low-quality men

15. To show that none of the $N_m - N_f$ lowest-quality men could marry if A_m and A_f were complements, assume the contrary: that M_i marries F_j and that M_k remains single, $A_{m_k} > A_{m_i}$. If this sorting were optimal,

$$Z_{ij} + Z_{ks} > Z_{kj} + Z_{is}, \quad \text{or} \quad Z_{ij} - Z_{is} > Z_{kj} - Z_{ks}.$$

By the definition of complementarity,

$$Z_{ij} - Z_{ig} < Z_{kj} - Z_{kg}, \quad \text{when } A_{m_k} > A_{m_i} \text{ and } A_{f_j} > A_{f_g}.$$

It seems plausible that the same inequality would tend to hold if remaining single is substituted for marrying the lower-quality women: if Z_{is} and Z_{ks} replace Z_{ig} and Z_{kg} respectively. If so, the first inequality above would contradict the assumption of complementarity between A_m and A_f, and M_i could not replace M_k in the optimal sorting if $A_{m_k} > A_{m_i}$. A similar argument shows that none of the highest-quality men could marry if A_m and A_f were substitutes.

16. The left-hand side of the first inequality in (4.15) is maximized (given Z_k^m) when $Z_j^m = Z_{js}$, or when $Z_j^f = Z_{jj} - Z_{js}$. If also $Z_k^m = Z_{ks}$, that first inequality would become

$$Z_k^m + Z_j^f = Z_{ks} + Z_{jj} - Z_{js} > Z_{kj},$$

which can be written as

$$Z_{ks} - Z_{js} > Z_{kj} - Z_{jj}.$$

If positive assortative mating were optimal, the traits of men and women would be complements, and this last inequality could not be satisfied if M_j were of lower quality than M_k. Hence $Z_k^m > Z_{ks}$. Similarly, if negative assortative mating were optimal, these traits would be substitutes, and the inequality could not be satisfied if M_j were of higher quality than M_k.

17. If some M_k marry F_k and some marry F_j, the income of an M_k must be the same whether he marries an F_k or an F_j:

$$Z_k^m + Z_k^f = Z_{kk}, \qquad Z_k^m + Z_j^f = Z_{kj}.$$

The incomes of M_k, F_k, and F_j are not uniquely determined by these two equations (see the more extensive discussion in Becker, 1973), but the premium to F_k must equal her marginal productivity:

$$Z_k^f - Z_j^f = Z_{kk} - Z_{kj}.$$

would be bumped out of the marriage market and other men would be bumped into "inferior" marriages—that is, into marriages with lower-quality women.

This analysis shows that the equilibrium income and mate assigned to any person by the optimal sorting depend not only on his traits but also on the traits of everyone else in the marriage market (that is, they depend on the *relative* as well as the *absolute* level of traits). For example, an increase in the number of male college graduates would reduce the incomes of male high-school graduates and the education of mates assigned to them. On the other hand, even a significant increase in the education of any particular man might have little effect on the mate assigned to him if the education of all other men also significantly increased. To take an actual example, this analysis explains why higher-income men in the United States have married at younger ages and have had more stable marriages than lower-income men, yet sizable secular increases in average incomes have not had such strong effects on the average age at marriage or on the average stability of marriages (see Keeley, 1974; Becker et al., 1977, p. 1173).

Differences in Preferences, Love, and the Optimal Sorting

When there is a single homogeneous household commodity, as I have assumed so far in this chapter, each person could be said to have the same utility "function," defined most simply by the quantity of that commodity. However, the utility functions or preferences of different persons could differ vastly when there are many separate commodities. Would the preferences of men and women in the marriage market then be an additional, perhaps even a crucial, variable determining the equilibrium sorting—along with income, education, race, and other traits—or would preferences have no effect on the equilibrium sorting, no matter how much they differed?

The answer depends entirely on the cost of production in different households. If each commodity were produced at a constant relative cost that was the same in all households, the total output produced by a marriage of M_i and F_j could be measured unambiguously by

$$Z_{ij} = {}_1Z_{ij} + {}_2v\,{}_2Z_{ij} + \ldots + {}_nv\,{}_nZ_{ij}, \tag{4.16}$$

where Z_{ij} is their total output measured in units of the commodity ${}_1Z$, ${}_kZ_{ij}$ is their output of the kth commodity, ${}_kv$ is the cost of producing a

unit of ${}_kZ$ relative to the cost of producing a unit of ${}_1Z$, and this relative cost is assumed to be the same for all households. Since the output of each commodity is consumed by the mates, then

$$\begin{aligned} Z_{ij} &= \sum_{k=1}^{n} {}_kv\,({}_kZ_i^m + {}_kZ_j^f) \\ &= \sum {}_kv\,{}_kZ_i^m + \sum {}_kv\,{}_kZ_j^f \\ &= Z_i^m + Z_j^f, \end{aligned} \tag{4.17}$$

where ${}_kZ_i^m$ and ${}_kZ_j^f$ are the quantities of the kth commodity consumed by M_i and F_j respectively.

Since Eq. (4.16) can be used to convert any given total output into the commodity mix best suited to particular preferences, each person would maximize utility—that is, consume more of each commodity—by choosing the mate who helped maximize his aggregate income, regardless of his preferences or those of different possible mates.[18] In particular, M_i and F_j would marry each other if their total output exceeded their combined aggregate incomes from marrying other persons or from remaining single, no matter how radically their preferences differed.[19]

On the other hand, preferences could well affect the equilibrium sorting if costs were not the same in all households. In particular, persons with similar preferences have an incentive to marry each other if costs are lower when the consumption patterns of mates are more similar, as they would be when some commodities are jointly consumed, when production of commodities is more efficient at a larger scale, or when specialized consumption capital lowers the costs of particular commodities.[20] Conversely, persons with different preferences have an incentive to marry each other if there are decreasing returns to scale. *Therefore, preferences are more likely to be positively than neg-*

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18. Robert Michael has reminded me of the nursery rhyme:

Jack Sprat could eat no fat,
His wife could eat no lean;
And so, betwixt them both, you see,
They licked the platter clean.

19. For example, if M_i only wanted to consume ${}_2Z$ and F_j only wanted to consume ${}_1Z$,

$$Z_i^m + Z_j^f = {}_2v\,{}_2Z_i^m + {}_1Z_j^f = {}_2v\,{}_2Z_{ij} + {}_1Z_{ij} = Z_{ij}.$$

20. The gain from investments in the consumption capital of a particular commodity is greater when more of that commodity is consumed (Chapter 2).

atively sorted—as are most other traits—because both joint consumption and specialized consumption capital encourage the matching of persons with similar preferences.

Many readers may be wondering whether romantic attachments have any place in my analysis, or is "love" too emotional or irrational to be analyzed by the economic approach? Although marriage for love has been much less important in other societies than in contemporary Western societies, love marriages do not have to be ignored; aspects of such marriages can be analyzed by the economic approach. Marriage for love is discussed more extensively in Chapters 8 and 11; here I show only that the effect of love on the equilibrium sorting is analytically a special case of the effect of differences in preferences.

It can be said that M_i loves F_j if her welfare enters his utility function, and perhaps also if M_i values emotional and physical contact with F_j. Clearly, M_i can benefit from a match with F_j, because he could then have a more favorable effect on her welfare—and thereby on his own utility—and because the commodities measuring "contact" with F_j can be produced more cheaply when they are matched than when M_i has to seek an "illicit" relationship with F_j. Even if F_j were "selfish" and did not return M_i's love, she would benefit from a match with someone who loves her, because he would transfer resources to her to increase his own utility. Moreover, a marriage involving love is more efficient than other marriages, even when one of the mates is selfish, and increased efficiency benefits the selfish mate also. These results and other aspects of altruism and love are discussed in Chapter 8, where it is shown that marriages involving love are likely to be part of the equilibrium sorting because in market terms they are more productive than other marriages.

Assortative Mating with Polygamy

The model of household production developed in Chapter 3 assumes that the output of the ith male married to the jth female is

$$Z_{ij} = n(\alpha_i, \beta_j) Z[p(\alpha_i) x_m, \ell(\beta_j) x_f], \tag{4.18}$$

with $\partial n/\partial\alpha > 0$, $\partial n/\partial\beta > 0$, $dp/d\alpha > 0$, and $d\ell/d\beta > 0$, where the effective resources of men and women with efficiencies α_i and β_j are $p(\alpha_i)x_m$ and $\ell(\beta_j)x_f$ respectively. The basic theorem in this chapter states that superior men are mated with superior women and inferior men with inferior women if

$$\frac{\partial Z}{\partial\alpha\partial\beta} = \frac{\partial^2 n}{\partial\alpha\partial\beta} Z + \left(\frac{\partial n}{\partial\alpha}\right)\left(\frac{\partial Z}{\partial x_f}\right) x_f \frac{d\ell}{d\beta} + \left(\frac{\partial n}{\partial\beta}\right)\left(\frac{\partial Z}{\partial x_m}\right) x_m \frac{dp}{d\alpha}$$
$$+ n \frac{\partial^2 Z}{\partial x_m \partial x_f} x_m \cdot x_f \left(\frac{dp}{d\alpha}\right)\left(\frac{d\ell}{d\beta}\right) > 0. \tag{4.19}$$

Sufficient conditions for this inequality are that $\partial^2 n/\partial\alpha\partial\beta > 0$ and $\partial^2 Z/\partial x_m \partial x_f > 0$.

A given quantity of spouse resources can be obtained by marrying either one superior person—that is, a person with relatively large ℓ or p—or several inferior persons. Hence less assortative mating would be expected with polygamy than with monogamy, because polygamy can match the total resources of a superior man or woman by substituting several inferior for one superior mate. One small piece of evidence indicating that positive sorting is weaker with polygyny is that the simple correlation coefficient between the education of husbands and wives is only +0.37 for the polygynous men of Maiduguri, whereas it is more than 0.5 in the United States (Grossbard, 1978, p. 30).

The effect of polygamy on the degree of assortative mating is more complicated when an improvement in efficiency mainly raises the output from given inputs of male and female resources (given by the function n). To begin an analysis of this case, assume that all men and all women have the same resources ($p = \ell = 1$), and that only men can have several mates. More efficient men probably tend to marry more efficient women even if efficient men are polygynous, because $\partial^2 n/\partial\alpha\partial\beta > 0$ implies that the effect on output of a more efficient wife is greater when her husband is also more efficient.

Although positive assortative mating is a substitute for explicit polygyny, superior men still are more likely to be polygynous. They would be inclined to marry several women who might differ in quality.[21] The most inferior men cannot attract wives when superior men

21. The marginal product of β_1-women with α_1-men is reduced when β_2-women marry α_1-men, in that fewer resources remain to spend on β_1-wives. An α_1-man with w_{11} β_1-wives and w_{12} β_2-wives maximizes

$$Z_{1,w_1,w_2} = w_{11} n(\alpha_1,\beta_1) Z\left(\frac{x_m^1}{w_{11}}, x_f\right) + w_{12} n(\alpha_1,\beta_2) Z\left(\frac{x_m^2}{w_{12}}, x_f\right),$$

subject to $x_m^1 + x_m^2 = x_m$. The equilibrium condition is

$$\frac{\partial Z_{1,w_1,w_2}}{\partial x_m^1} = n(\alpha_1,\beta_1)\frac{\partial Z}{\partial x_m^1} = n(\alpha_1,\beta_2)\frac{\partial Z}{\partial x_m^2}.$$

Therefore the marginal product of x_m is the same with β_2-wives as with β_1-wives.

attract several in marriage markets with the same number of men and women. Since all women tend to marry, the average woman would marry a man "above" her in ability and skill if men and women in the marriage market are of equal average ability and skill. Of course, the average woman would marry above her even with monogamy, and even when all men and women marry, if the average man has invested more than the average woman has (see Table 3.1). Therefore, our analysis readily explains why women have typically married "up" and men have typically married "down" in both monogamous and polygamous societies.[22]

Inflexible Prices, Dowries, and Bride Prices

The analysis of equilibrium sorting developed in this chapter has assumed that all divisions of outputs between mates are feasible. The equilibrium division in any marriage, possibly not unique, is determined from conditions (4.5) and (4.6) and results from efforts by all participants in the marriage market to maximize their own commodity income. An important property of these equilibrium conditions is that each person prefers to be matched with the mate assigned by the equilibrium sorting than with any other person, for the reason that he would receive a lower income with anyone else. Moreover, the equilibrium sorting, and hence these preferences for mates, are not fixed but depend on the number of persons with particular traits and other variables.

If the division of output in any marriage were determined not in the marriage market but in other ways, and if a person would receive the same fraction of the output of all possible matches, then

$$Z_i^m = e_i Z_{ij} \quad \text{for all } j, \qquad Z_j^f = d_j Z_{ij} \quad \text{for all } i, \tag{4.20}$$

where $e_i + d_j \neq 1$ if joint consumption or monitoring costs are significant, and e_i and e_j or d_j and d_k may not be equal because the shares of different men or women may differ. Appendix note E shows that a perfect positive assortative mating would maximize aggregate output and would be an equilibrium sorting because persons not assigned to each other would be made worse off if they married. This chapter has shown that a perfect positive assortative mating also tends to be the equilibrium sorting and to maximize output when the division of each output is determined by market equilibrium. Therefore, permitting the marriage market to determine the division of output and imposing that division by Eq. (4.20) frequently give the same sorting.

22. For example, Hindu women were not permitted to marry mates of lower status, whereas Hindu men could (Mandelbaum, 1970); also, Islamic women are not supposed to marry mates of lower status (the doctrine of *kafā'a*), while Islamic men can (Coulson, 1964, pp. 49, 94).

My approach to the marriage market contrasts sharply with other formal models of marital sorting (see Gale and Shapley, 1962; Stoffaës, 1974). These models, like the model given by Eq. (4.20), assume that each person has a *given* ranking of potential mates that determines rather than is determined by the equilibrium sorting. Unlike the rankings implied by (4.20), however, in these models different persons may not rank potential mates in the same way—for example, M_i may prefer F_j, who prefers M_k, who prefers F_i. If rankings were not the same, an "optimal" sorting could only try to minimize the overall conflict between feasible and preferred matches.[23]

These models can be said to assume implicitly, while the model given by (4.20) assumes explicitly, that the division of output in any marriage is not determined by the marriage market and is completely rigid. An individual usually would not prefer the mate assigned him by the optimal sorting, because marital prices are not permitted to eliminate the inconsistencies among the preferred choices of different persons. If the division of marital output were determined by the marriage market, the ranking of potential mates would not be given; it would depend on how the outputs produced with different mates were divided. That is to say, if marital prices were flexible, the problem formulated and solved by these models would be irrelevant to actual marital sortings.[24]

The division of marital output may seem to be inflexible, however, in

23. Gale and Shapley (1962) require optimal assignments to be "stable"; that is, persons not assigned to each other could not be made better off by marrying each other, a requirement that is closely related to condition (4.5).

24. It might be relevant, however, to markets that do not use prices to determine assignments. For example, Gale and Shapley (1962) also discuss the assignment of applicants to different universities, and Chapter 9 considers the mating of nonhuman species where each entity maximizes the survival of its genes.

that commodities like housing space, children, conversation, and love are jointly consumed (they are "family commodities"). Consumption by one person does not reduce by an equal amount the quantity available to other household members. Moreover, some mates may be able to obtain more than their equilibrium share of output by shirking their duties as a result of the division of labor between mates and the cost of monitoring behavior (Chapters 2 and 8). In addition, men have sometimes been given legal control over the assignment of shares (see Weitzman, 1974, pp. 1182 ff.).

Consider the marriage market represented in Figure 3.1, which contains homogeneous women and homogeneous men. If the number of men exceeds the number of women, $N_m' > N_f$, the equilibrium income of men and women would equal $Z^{*m} = Z_{ms}$ and $Z^{*f} = Z_{mf} - Z_{ms}$ respectively. Suppose, however, that the division of output is inflexible for the reasons just given, and, in particular, that the marital income of women cannot exceed $\bar{Z}^f < Z^{*f}$; hence the marital income of men would equal $\bar{Z}^m = Z_{mf} - \bar{Z}^f > Z^{*m}$. Since all the available men want to marry at that income, the scarce women must be distributed among the more numerous men. The distribution of wives among men is not likely to be purely random, for men would try to raise their chances of getting married. They could try to guarantee prospective wives more than $\bar{Z}^f$, but such guarantees might not be easily enforced.

One alternative would be to give a capital or lump-sum transfer to a woman as an inducement to marriage. Since men offering larger transfers would obtain wives more easily, competition among men for the scarce women would bid up the transfers until all men again were indifferent between marriage and remaining single. They would be indifferent when the transfer equaled the present value of the difference between Z^{*f} and $\bar{Z}^f$, the difference between the equilibrium and the actual income of married women. The same reasoning shows that transfers would be from women to men if married men received less than their equilibrium incomes. Transfers to women are called "bride prices," and those to men are called "dowries."

The analysis would be basically the same if payments went to parents (not to the children marrying) because parents "owned" their children and transferred them to other families through marriage (Cheung, 1972). The capital value of children transferred to other families would still equal the present value of the difference between their equilibrium marital income and their actual income. Bride prices then

not only compensate parents for the transfer of their "property," but also induce them to invest optimally in daughters if girls with appropriate accumulations of human capital command sufficiently high prices.

The difference between actual and equilibrium income of wives is probably greater when their equilibrium income is a larger share of marital output (a larger share may not be as readily appropriated by wives). Therefore, the frequency and magnitude of bride prices should be greater when the equilibrium share of wives is greater, as in the following situations: in societies with a larger supply of men relative to women; for never married as opposed to divorced women;[25] in societies with a higher incidence of polygyny; and in patrilineal societies (Schneider, 1969) because husbands have more control over the division of marital output, especially over children, in such societies.

This analysis also implies that bride prices would have to be returned, at least in part, when a wife divorces without cause or when a husband divorces with cause—say, because his wife is unfaithful or barren (see the evidence in Goode, 1963, pp. 155 ff.). A husband divorcing without cause, however, would forfeit most of the bride price, especially if he had been married for a number of years.[26]

Consequently, even when the actual division of marital output diverges greatly from the equilibrium division, bride prices and dowries raise or lower marital incomes to the levels mandated by the equilibrium sorting. My assumption that marital *incomes* are flexible appears highly reasonable, therefore, when the purpose of bride prices and other capital transfers contingent on marriage is understood. Models that assume a rigid division of income greatly underestimate human ingenuity and experience in making the terms of marriage flexible and responsive to market conditions.

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25. Divorced women would command lower prices because they tend to be older than single women, and because they may have been divorced as a result of their deficiencies as wives, including sterility (see Chapter 10). The evidence in Goldschmidt (1973) and in Papps (1980) indicates that bride prices in Uganda and Palestine have been lower for divorced women.

26. Goode has shown that Moslem men usually forfeit most of the bride price when they divorce without cause. In this way bride prices and other capital transfers insure divorced women against losses on their specialized investments in children; see the further discussion of divorce and divorce settlements in Chapter 10.