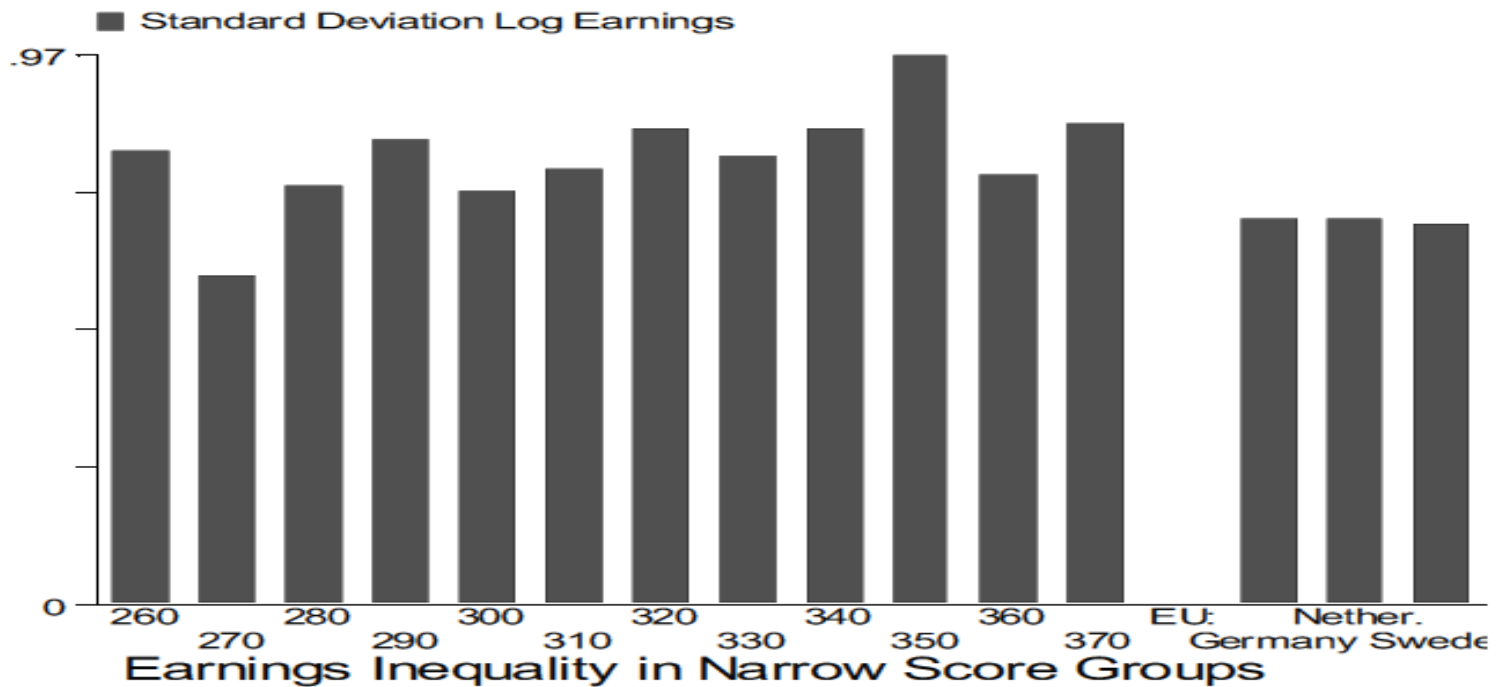


## Lecture 6: Economics/Math of Search and Dispersion of Prices/Wages

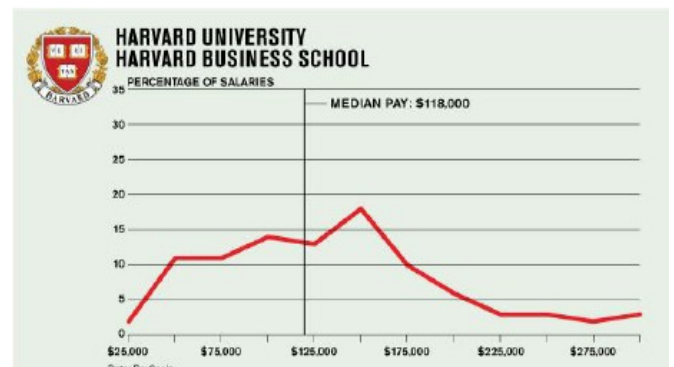
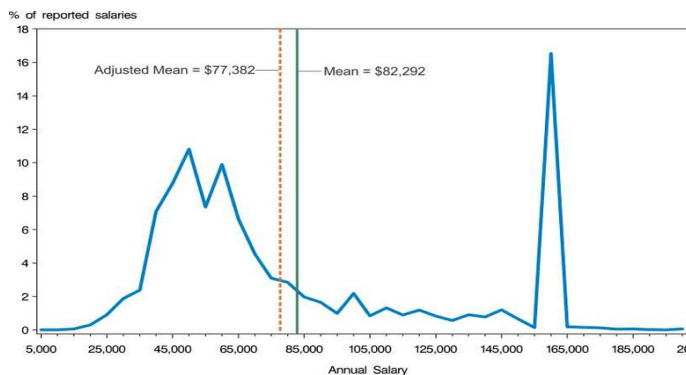
Market clearing model predicts a single price/wage for comparable workers/goods, but data show huge dispersion. The dispersion in wages among advanced countries is highest in the US, even within narrowly defined skill groups.

**Figure 3: Wage inequality in Narrow US Score Groups**



**Source:** National Adult Literacy Survey for US; International Adult Literacy Survey for other countries. We break the NALS sample of US workers into groups based on test score. For example, group 260 includes all persons with a score of 258-262. The average number of observations in each group is 286. We compare earnings inequality within each group to earnings inequality of each country. The average standard deviation of log earnings in these twelve groups is .79. The comparable figure for the four countries is Germany .68, Netherlands .67, Sweden .68, and US .86 (in the NALS, or .93 in the IALS).

Lawyers: National Association of Law Placement, Class of 2014 How about a narrower group? HBS class of 2013.



Law evidence is from “Lawyer Salaries Are Wierd” [www.biglawinvestor.com/bimodal-salary-distribution-curve/](http://www.biglawinvestor.com/bimodal-salary-distribution-curve/) .

Full professors of life sciences: Gini coefficient (standard measure of inequality that varies from 0 to 1) 0.120 in 1973 to 2006 0.250. In 2005 Gini for Sweden for all workers with wide range of skills was 0.260. Places of work differ in characteristics/amenities—> compensating differentials – and workers differ in ability, quality of education, etc so perhaps the high dispersion is due to unmeasured attributes of work and skill.

So look at identical item – books 2001 Clay, Krishnan, Wolf find:

**TABLE I  
SUMMARY STATISTICS**

	<b>NYT Bestseller</b>	<b>Former NYT Bestseller</b>	<b>Computer Bestseller</b>	<b>Former Computer Bestseller</b>	<b>Random Book</b>
Number of observations	25,681	28,342	26,870	16,661	63,879
Number of books	136	122	82	69	181
Average weeks on list	15.0	NA	15.6	NA	NA
Percent hardcover	52.9%	52.5%	18.3%	20.3%	66.9%
Percent fiction	56.6%	58.2%	0.0%	0.0%	24.3%
<i>Prices</i>					
Publisher's recommended price	\$17.28	\$17.97	\$43.08	\$51.55	\$37.92
Unit price	\$11.83	\$13.48	\$33.57	\$40.23	\$34.39

## Our Undercover Shoppers Found Big Price Differences for Computers and Peripherals

<b>Product</b>	<b>Low price</b>	<b>Average price</b>	<b>High price</b>
Dell Inspiron 17 5000 laptop with 17.3" screen, 2TB of storage, Intel Core i7-7500U processor, and 16GB of memory	\$799	\$929	\$1,289
Galaxy Tab S2 with 8" screen, 32GB solid-state storage, and 3GB of memory	\$299	\$367	\$451
HP Pavilion desktop with 1TB storage, Intel Core i3-6100T processor, and 8GB of memory	\$430	\$449	\$484
HP Zbook 17 G3 laptop with 17.3" screen, 512GB solid-state storage, Intel Core i7-6700HQ processor, and 16GB of memory	\$2,149	\$2,301	\$2,517
Lenovo ThinkPad X1 Yoga laptop with 14" touchscreen, 256GB solid-state storage, Intel Core i7-6500U processor, and 8GB of memory	\$1,260	\$1,707	\$1,999
Microsoft Surface Pro 4 with 12.3" touchscreen and 256GB solid-state storage	\$900	\$1,113	\$1,326
Samsung Notebook 9 with 15.6" screen, 256GB solid-state storage, Intel Core i7-6500U processor, and 8GB of memory	\$870	\$934	\$1,000
Canon PIXMA MX922 printer	\$73	\$125	\$207
HP Officejet Pro 8710 printer	\$104	\$137	\$283
Logitech MK520 wireless keyboard & mouse	\$30	\$47	\$87
Netgear Nighthawk AC1900 wireless router	\$157	\$173	\$205

<https://www.checkbook.org/boston-area/computer-stores/articles/Which-Stores-Offer-the-Lowest-Computer-Prices-5816>

AIR FARE PRICES: Bobby McWhatter 1818 PAPER using Kayak search tool. Built database of 1,000 observations LAX to NY

	mean	SD	SD/Mean	Pmax/Pmin
afternoon coach	\$319.04	71.74	0.22	2.30
premium	\$2,596.92	585.93	0.23	1.92

### Why such high dispersion?

- 1) Error in measurement: True single wage/price, but we measure transaction imperfectly: Airlines have different quality. NOT FOR BOOKS nor exact same computer
- 2) Few people buy at higher prices. BUT Amazon can be high priced, allows you to buy from other booksellers, some with lower prices. Why buy from them? Shipping ... ease of credit card ... etc.
- 3) Costs of search that should produce some dispersion.
- 4) Price discrimination in imperfect market: sends different messages/prices to different persons based on estimate of what they will pay.
- 5) Behavioral "status quo" effects found in savings plans and other behavior where people don't change easily – you just go to the store you went to last time.

### SEQUENTIAL SEARCH/RESERVATION WAGE MODEL (vs Fixed Sample Search)

Assume a single-peaked landscape of prices, where peak is lowest price. You know the distribution has a peak; you may or may not know the peak value but you don't know its location. Each search has a cost.

The optimum strategy is to determine a **RESERVATION WAGE (RW)** so that you accept first  $W > RW$ . This is a **SEQUENTIAL SAMPLING MODEL** in which you compare the marginal gain of another search against the expected marginal cost of that search.

With a *uniform distribution from 0 to 1* it doesn't take many searches for expected value to approach the max .

1 search expected to have 1/2 maximum so the marginal gain is  $= 1/2$  Max

2 searches expect to have 2/3rds Max so the marginal gain is  $(2/3 - 1/2) = 1/6$  Max

3 searches expect to have 3/4ths Max so the marginal gain is  $(3/4 - 2/3) = 1/12$  Max

n searches the expected maximum max is  $[n/(n+1)]$  1, with marginal gain of  $[n/(n+1)] - [(n-1)/(n)] = 1/(n)(n+1)$

If maximum is 30 and each search costs 2.5, balancing constant marginal cost against declining payoff from extra search gives **reservation Wage of 19**. If you have max of 18 the chance of getting a higher value would be  $12/30 = 2/5$  and the extra value varies from 1 to 12 for an average of  $78/12$  or 6.5. Since  $2/5 \times 6.5 = 2.6 >$  marginal cost of 2.5, you would expect to do better on the next search. If your max is 19 the chance of a higher value would be  $11/30$ . The extra over 19 varies from 1 to 11 for an average of  $66/30$  or 2.2 which is  $< 2.5$ . So your reservation wage would be 19.

Alternative is **FIXED SAMPLE** design based on *expected* values. This decides optimal # of searchers **before** the search process. It ignores the information from each search. Choosing # searchers at the outset means you calculate *expected* marginal gain from one search as 15; from two searches as 5; from three as 2.5 so you would search three times. But information you get as you proceed says you can do better. Good luck and you hit max on first shot, **STOP**. Bad luck with 1,2, 3 on the first three draws, **DON'T STOP**.

**SEQUENTIAL SAMPLING** dominates fixed sampling. Stop if you beat the reservation wage – ie when the cost of extra search exceeds expected gain. **VALUE IN SEARCHES**. But sequential could lead to problem with hypothesis testing in science: you want a statistically significant result – in your sample of 40, you estimate that if you had 20 more people and they showed the same distribution of outcomes as the 40, you would reduce the standard error by 20% and pow your results would like better. If that did not work, just keeping adding observations.

### What if you do not know distribution? Secretary Problem

You want to hire person with the highest “skill” from a known number of applicants but you do NOT know the distribution of skill. You accept/reject on the spot, and cannot go back and accept someone you had rejected. Similar problem: At a party with 100 folks you want to ask the most attractive one out but you don't know the distribution of attractiveness. If you pass on someone, they will not go out with you. What do you do?

Answer for both cases is to: **FOLLOW OPTIMAL STOPPING RULE**: Use first R of N applicants to determine a **reservation wage/value** for skill/attractiveness/whatever – the maximum of the R -- and then select the first applicant whose value exceeds the maximum R. For any distribution, this gives you the highest probability of getting the best. And the probability you get the best with this strategy  $\rightarrow 1/e \sim 37\%$  as  $N \rightarrow$  infinite. With smaller N you do better.

**Consider Three Candidates**, with ranking 1,2,3, and they could appear as: 1 2 3 1 3 2 2 1 3 2 3 1 3 1 2 3 2 1. (6 possible results: 3 possible for place 1, 2 for place 2, and 1 for place 1 so 3!).

If you randomly choose the first (or 2<sup>nd</sup> or 3<sup>rd</sup>) person who comes you have 1/3rd chance of getting best. To do better use the first person as a “base”/reservation wage and pick next one with a better score. If the best comes first 1,2,3 or 1,3,2 or last 3,2,1 you lose; but if you get **2,1,3 or 2,3,1 or 3,1,2 you win. This means win in 1/2 of the cases.**

The gain is that 1 is first 2 times (1/3rd) but is 2<sup>nd</sup> 2 times and is 3<sup>rd</sup> in the 2,3,1 case. The extra bump occurs when you get a 2<sup>nd</sup> choice value first, and then reject others until you get the top person..

**Four Candidates**: 1/4th of cases you will get the top by chance, so we want to beat 1/4th

1 2 3 4	1 2 4 3	1 3 2 4	1 3 4 2	1 4 2 3	1 4 3 2	You lose
2 1 3 4	2 1 4 3	2 3 1 4	2 3 4 1	2 4 1 3	2 4 3 1	You win
3 1 2 4	3 1 4 2	3 2 1 4	3 2 4 1	3 4 1 2	3 4 2 1	you win on 3124 and 3142, 3412
4 1 2 3	4 1 3 2	4 2 1 3	4 2 3 1	4 3 1 2	4 3 2 1	you win on 4123, 4132

So you win on 11/24 giving a probability of success of 0.458.

Would you do better letting candidates 1 and 2 pass and then choosing the first who beats the max of those 2? What if you have 100 candidates? Key is to determine **how many to use in the “discovery stage” to determine “RW”**.

To find R, calculate the probability of winning if you choose 1,2..., R for discovery. You lose if the best candidate is among the first R, or if not among the first R, is preceded by a candidate who beats out the max in discovery phase. Calculate probabilities of winning for R, maximize wrt R and use the best outcome to determine the reservation wage,

Example for calculating the probability of winning with 10 candidates and R of 3: Max of 1,2,3 sets reservation wage.

4<sup>th</sup> : 1/10th chance that this is the highest value

Fifth: 1/10th chance x chance that 4<sup>th</sup> is lower than first three:  $\frac{3}{4}$  (think why?) so this is  $\frac{1}{10} \times \frac{3}{4}$

Sixth: 1/10th chance x chance that fifth is lower than first four:  $\frac{3}{5}$  so  $\frac{1}{10} \times \frac{3}{5}$

nth: 1/10th chance x chance that nth is lower than first n-1 th

In general, the Probability of Winning if you have R discovery searches at

R+1:  $\frac{1}{n}$  since there is a  $\frac{1}{n}$  chance that R+1st of n is the maximum

R+2:  $\frac{1}{n}$  conditional on R+1 not being the maximum.  $\frac{R}{(R+1)}$ , so  $\text{Prob}(R+2 \text{ is max}) = \frac{1}{n} \left( \frac{R}{R+1} \right)$ .

R+3:  $\text{Prob}(R+3 = \text{max}) = \frac{1}{n} \left( \frac{R}{R+2} \right)$

Nth:  $P(N\text{th} = \text{max}) = \frac{1}{n} \left( \frac{R}{n-1} \right)$  because preceding (n-1) have lower value than some element in R

Havil **Gamma: Exploring Euler's Constant** shows that general rule as sample n gets bigger is to take  $\frac{1}{e}\%$  of the n observations for discovery, then pick first observation with value > the highest in discovery set.  $\frac{1}{e}$  is about .37 so sometimes called 37% rule. The chance you get the highest value is  $\sim \frac{1}{e} \ln e = \frac{1}{e}$  also!. Can calculate expected rank of the person you get and other statistics.

This is a **STOPPING RULE problem**: Take sequence of random variables (stock prices, offers on house, patient needs for transplant) and decide when to stop to maximize the reward **with no other information**. Stopping rules are critical in small scale Phase II clinical trials: if the medicine cures 2, 3, ... N patients, you want to get to Phase III trials and get the medicine out quickly; if 1, 2, patients who take med die, you stop fast.

**F. Thomas Bruss's ODDS-ALGORITHM** gives optimal stopping rules for such problems. Examples: throw die 12 times, when a 4 comes up you must declare "this is last 4". If it is last 4, you win. If a 4 comes later you lose. Selling car, get offers and want best. You get "high" offer but prefer higher one if it were truly coming.

Key assumptions: each offer, roll of die is independent, so you never know which is best. You want a rule that gives the biggest chance that the one you chose as the last observation is indeed the top offer.

Analysis for the die case: What is chance any given throw is a 4?  $\frac{1}{6}$ . If you get a 4 on first throw, is it likely to be the last 4? No because with 11 more throws very likely to get another 4. How about a 4 on the 12<sup>th</sup> throw? You win only if the 12<sup>th</sup> comes up with a 4, so you know the probability of that— $\frac{1}{6}$  same as on first throw. Don't do first and don't do 12<sup>th</sup> ... somewhere in between.

Odds algorithm is based on **odds ratio**  $r_k$  of event = probability  $p_k$  of the event over converse  $r_k = \frac{p_k}{(1-p_k)}$ . In the die case,  $p_k = \frac{1}{6}$ ,  $q_k = \frac{5}{6}$  so  $r_k = \frac{1}{5}$ . Since  $r_k$  is the same for all k, the odds algorithm solution is simple

**THE ODDS ALGORITHM**: Sum the odds in reverse order  $R_s = r_n + r_{n-1} + r_{n-2} + \dots$  until this sum reaches or exceeds 1. If this happens at s, s is the stopping threshold and the rule is to pick the first 4 that comes up in the throws from s+1 on and declare it to be the last 4

With  $r_k = \frac{1}{5}$  you have at period

	12	11	10	9	8	7	6
$p_k$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$		
$q_k$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6}$		
$r_k$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$		
so the sum $R_s$ is	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1	which says pick first 4 that occurs from 9 to 12	
and Product $Q_k =$	$\frac{5}{6}$	$(\frac{5}{6})^2$	$(\frac{5}{6})^3$	$(\frac{5}{6})^4$	$(\frac{5}{6})^5$		

**THE ODDS THEOREM**: Sum the odds in reverse order and form the product  $Q_k$  of chance that event did not occur  $q_k = 1 - p_k$ . The odds algorithm/strategy maximizes the probability of stopping on the winning value with a probability of winning of  $Q_s R_s$ . In the dice case this is  $(\frac{5}{6})^5 = 0.402$ . If  $R_s \geq 1$ , the win probability of stopping on the winning probability  $\geq \frac{1}{e} = 0.378$

**Example 2: Selling the car.** What is the chance that any given offer is the highest THUS FAR? If you have k offers, chance that **any given offer** is highest will be  $\frac{1}{k}$  – ie if you have two offers  $\frac{1}{2}$  chance first or second is highest; if you have three it is  $\frac{1}{3}$ <sup>rd</sup>, etc. Now apply the theorem. Say you have 7 potential offers  $r_k = \frac{p_k}{q_k}$  which varies with p.

Period	7	6	5	4	3
$p_s$	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$
$r_n$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$
$R_n$	$\frac{1}{6}$	$\frac{11}{30}$	$\frac{37}{60}$	$\frac{171}{180}$	$\frac{261}{180}$

So pick the best offer from 5<sup>th</sup> offer on ie 5, 6, or 7.  $Q_3 = (\frac{2}{3})(\frac{3}{4})(\frac{4}{5})(\frac{5}{6})(\frac{6}{7}) = \frac{2}{7} = 0.286 \times \frac{261}{180} = 41\%$

### Odds-algorithm

Write  $p_k$ ,  $q_k$  and  $r_k$  in three lines and write each line in reverse order, that is, beginning with  $k = n$ :

- (i)  $p_n, p_{n-1}, p_{n-2}, \dots$
- (ii)  $q_n, q_{n-1}, q_{n-2}, \dots$
- (iii)  $r_n, r_{n-1}, r_{n-2}, \dots$

Each  $r_k$  is the quotient of the numbers above it. Now we sum up the odds in line (iii) until the value 1 is reached or just exceeded. This yields the sum  $R_s = r_n + r_{n-1} + \dots + r_s \geq 1$  with a stopping index  $s$  (if the sum of odds never reaches 1 then we set  $s = 1$ ). Then we compute from (ii) the product  $Q_s = q_n q_{n-1} \dots q_s$ . This is all we need for the main result.

**Optimal strategy and win probability.** The optimal strategy is to stop from  $s$  onwards on the first opportunity (if any).

The optimal win probability  $W$  is the product of  $R_s$  and  $Q_s$  that is

$$W = R_s Q_s.$$

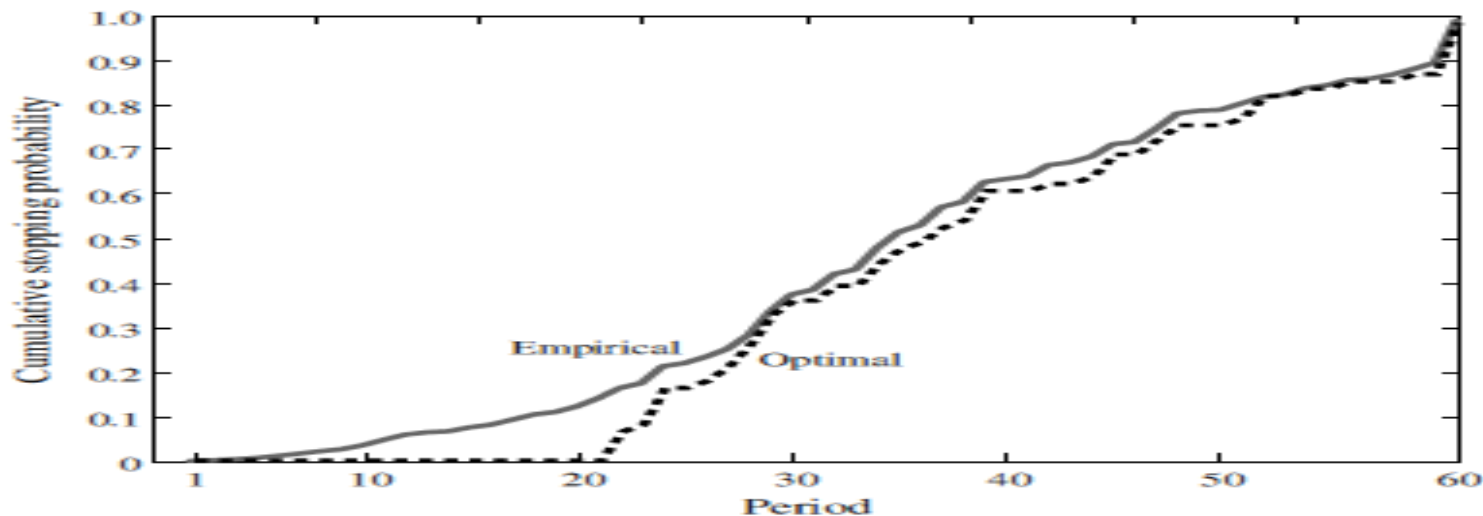
Note that the odds-algorithm gives us the optimal strategy and optimal value at the same time. Moreover, in the general case no other method could possibly do this more quickly, that is, the algorithm is optimal itself.



If you do not know the probability of a success at each point, you have to estimate this using sequential updating, which is complicated but doable. See F. Thomas Bruss and Guy Louchard The odds algorithm based on sequential updating and its performance *Adv. in Appl. Probab.* Vol 41, No 1 (2009), 131-153.

**Do people follow the algorithm in decisions?** Seale and Rapoport 1997 *Organizational Behavior and Human Decision Processes* 69 pp 221-236 find that **people generally do not search enough**. J. Neil Bearden, Amnon Rapoport, Ryan O. Murphy “Sequential Observation and Selection with Rank-Dependent Payoffs: An Experimental Study” *Management Science* Vol. 52, No. 9, September 2006, pp. 1437–1449 find the same and analyze it in depth:

**Figure 1** Observed and Predicted Cumulative Distributions of Stopping Time Across Subjects by Period of Search for Experiment 1



### Neighborhood Search Algorithms without probability model

You are on a landscape but do not know its property. Say it is the space of people categorized by age, gender, education, # of times they watch wrestling on cable. You want to find the people who might buy your new product. If you look at the distance weighted by closeness, very near neighbor counts more than someone further away.

**Nearest Neighbor:** Observe the characteristics of people who buy it and assume that people with similar characteristics behave similarly -- ie a CORRELATED LANDSCAPE – and use nearest neighbor or K nearest neighbors buying performance to predict if they will buy or not. This works as follows: you have data about people whether they were/were not buyers labeled as Buy or Not. To decide which way a new person is likely to act we compare them to the people with similar attributes. Each of the people in data set is represented by a point in N-dimensional space. We assign to the new persons the behavior of the nearest neighbor. K-NN says look at your K nearest neighbors.

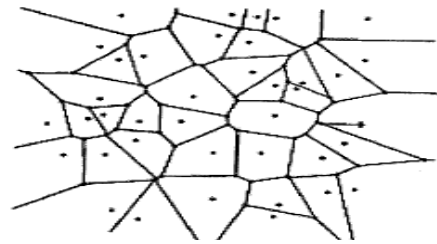
[www.comp.lancs.ac.uk/~kristof/research/notes/nearb/cluster.html](http://www.comp.lancs.ac.uk/~kristof/research/notes/nearb/cluster.html) gives simple nearest neighbor algorithm,

The US Census estimates the earnings of someone who refuses to tell how much they earn by “giving them” the earnings of the last person with specified characteristics. This is HOT DECK procedure. It goes wrong if they exclude



a key variable in the definition of the N-hood and you want to estimate the effect of that variable on earnings. Say it is union, if union wages higher and you do not include union in the hot deck, you will UNDERSTATE the union effect.

Voronoi diagrams have the property that for each site every point in the region around that site is closer to that site than to any of the other sites. They are convex polygons so that each point is closer to its central point than any other. Check (<http://www.msi.umn.edu/~schaudt/voronoi/voronoi.html>) In 2 dimensional space, we have:



What is the connection between Secretary's Problem/search policies that set up a reservation price and pick the first outcome better than the max in that set and Genetic Algorithm/search strategies that try to combine the good features of one strategy with the good features of another?

Secretary problem ASSUMES **uncorrelated landscape**. If correlated and persons with glasses average 20 points higher than those without glasses, and your reservation value was 50, what would you do if the first person above 50 did not wear glasses? Reject them on the notion the score was some weird aberration – or hire them with confidence – if someone from low-scoring group scores well, they must be really good. Cannot go back if you missed high value.

Genetic Algorithm and Simulated annealing and related search models ASSUME correlated landscapes. If you find high scorers in one area, likely to find high scorers near by. If we have a 2 dimensional lattice NS and EW then the trick for the GA is to find high values in North and high values in West and combine them to get the potential highest values in North West.

There is an implicit cost to search since there may be so many points to examine that you cannot check the whole universe. But you can go back and choose a higher point so you are not locked into “pick the final point”. Given that you are going to use early random searches to decide which areas/attributes look best, might you do better if you randomly search to get some reservation value for different areas and then do your combinations? In the simulated annealing this would be to make the “temperature” depend on early information.

**Class paper:** Ask people to guess the last 4 in a 12, 18, 24 throw experiment and see if they form an approximately optimal discovery stage? Are there random or systemic differences among people in the **size of the optimal discovery** stage? Ask people secretary/dating problem. Are there differences in behavior across the games? How could you exploit behavior that falls short of optimal and make huge sums of money?

## When to stop dating and settle down, according to math By Ana Swanson WP February 16, 2016

Committing to a partner is scary for all kinds of reasons. But one is that you never really know how the object of your current affections would compare to all the other people you might meet in the future. Settle down early, and you might forgo the chance of a more perfect match later on. Wait too long to commit, and all the good ones might be gone. You don't want to marry the first person you meet, but you also don't want to wait too long. This can be a serious dilemma, especially for people with perfectionist tendencies. But it turns out that there is a pretty simple mathematical rule that tells you how long you ought to search, and when to stop searching and settle down.

The math problem is known by a lot of names – “the secretary problem,” “the fussy suitor problem,” “the sultan's dowry problem” and “the optimal stopping problem.” Its answer is attributed to a handful of mathematicians but was popularized in 1960, when math enthusiast Martin Gardner wrote about it in Scientific American. In the scenario, you're choosing from a set number of options. For example, let's say there is a total of 11 potential mates who you could seriously date and settle down with in your lifetime. If you could only see them all together at the same time, you'd have no problem picking out the best. But this isn't how a lifetime of dating works, obviously.

One problem is the suitors arrive in a random order, and you don't know how your current suitor compares to those who will arrive in the future. Is the current guy or girl a dud? Or is this really the best you can do? The other problem is that once you reject a suitor, you often can't go back to them later. So how do you find the best one? Basically, you have to gamble. And as with most casino games, there's a strong element of chance, but you can also understand and improve your probability of “winning” the best partner. It turns out there is a pretty striking solution to increase your odds. The magic figure turns out to be 37 percent. To have the highest chance of picking the very best suitor, you should date and reject the first 37 percent of your total group of

lifetime suitors. (If you're into math, it's actually  $1/e$ , which comes out to 0.368, or 36.8 percent.) Then you follow a simple rule: You pick the next person who is better than anyone you've ever dated before.

To apply this to real life, you'd have to know how many suitors you could potentially have or want to have—which is impossible to know for sure. You'd also have to decide who qualifies as a potential suitor, and who is just a fling. The answers to these questions aren't clear, so you just have to estimate. Here, let's assume you would have 11 serious suitors in the course of your life. If you just choose randomly, your odds of picking the best of 11 suitors is about 9 percent. But if you use the method above, the probability of picking the best of the bunch increases significantly, to 37 percent—not a sure bet, but much better than random.

This method doesn't have a 100 percent success rate, as mathematician Hannah Fry discusses in a 2014 TED talk. There's the risk, for example, that the first person you date really is your perfect partner, as in the illustration below. If you follow the rule, you'll reject that person anyway. And as you continue to date other people, no one will ever measure up to your first love, and you'll end up rejecting everyone, and end up alone with your cats. (Of course, some people may find cats preferable to boyfriends or girlfriends anyway.)

Another, probably more realistic, option is that you start your life with a string of really terrible boyfriends or girlfriends that give you super low expectations about the potential suitors out there, as in the illustration below. The next person you date is marginally better than the failures you dated in your past, and you end up marrying him. But he's still kind of a dud, and doesn't measure up to the great people you could have met in the future. So obviously there are ways this method can go wrong. But it still produces better results than any other formula you could follow, whether you're considering 10 suitors or 100.

Why does this work? It should be pretty obvious that you want to start seriously looking to choose a candidate somewhere in the middle of the group. You want to date enough people to get a sense of your options, but you don't want to leave the choice too long and risk missing your ideal match. You need some kind of formula that balances the risk of stopping too soon against the risk of stopping too late. The logic is easier to see if you walk through smaller examples. Let's say you would only have one suitor in your entire life. If you choose that person, you win the game every time—he or she is the best match that you could potentially have. If you increase the number to two suitors, there's now a 50:50 chance of picking the best suitor. Here, it doesn't matter whether you use our strategy and review one candidate before picking the other. If you do, you have a 50% chance of selecting the best. If you don't use our strategy, your chance of selecting the best is still 50%.

But as the number of suitors gets larger, you start to see how following the rule above really helps your chances. The diagram below compares your success rate for selecting randomly among three suitors. Each suitor is in their own box and is ranked by their quality (1st is best, 3rd is worst). As you can see, following the strategy dramatically increases your chances of "winning"—finding the best suitor of the bunch. As mathematicians repeated the process above for bigger and bigger groups of "suitors," they noticed something interesting—the optimal number of suitors that you should review and reject before starting to look for the best of the bunch converges more and more on a particular number. That number is 37 percent. The explanation for why this works [gets into the mathematical weeds](#)—here's another great, [plain-English explanation of the math](#)—but it has to do with the magic of the mathematical constant  $e$ , which is uniquely able to describe the probability of success in a statistical trial that has two outcomes, success or failure. Long story short, the formula has been shown again and again to maximize your chances of picking the best one in an unknown series, whether you're assessing significant others, apartments, job candidates or bathroom stalls.

### Other variants of the problem

There are a few tweaks to this problem, depending on your preferences, that will give you a slightly different result. In the scenario above, the goal was to maximize your chances of getting the very best suitor of the bunch—you "won" if you found the very best suitor, and you "lost" if you ended up with anyone else. But a more realistic scenario, as [mathematician Matt Parker](#) writes, is that "getting something that is slightly below the best option will leave you only slightly less happy." You could still be quite happy with the second- or third-best of the bunch, and you'd also have a lower chance of ending up alone. If your goal is to just get someone who is good, rather than the absolute best of the bunch, [the strategy changes a little](#). In this case, you review and reject the square root of  $n$  suitors, where  $n$  is the total number of suitors, before you decide to accept anyone. As in the formula above, this is the exact point where your odds of passing over your ideal match start to eclipse your odds of stopping too soon. For our group of 11 suitors, you'd date and reject the first 30 percent, compared with 37 percent in the model above. All in all, this version means that you end up dating around a little less and selecting a partner a little sooner. But you have a higher chance of ending up with someone who is pretty good, and a lower chance of ending up alone. With a choice of 10 people, the method gets you someone who is 75 percent perfect, relative to all your options, according to [Parker](#). With 100 people, the person will be about 90 percent perfect, which is better than most people can hope for.

In 1984, a Japanese mathematician named Minoru Sakaguchi developed another version of the problem that independent men and women might find more appealing. In Sakaguchi's model, the person wants to find their best match, but they prefer remaining single to ending up with anyone else. In this case, you wouldn't start looking to settle down until reviewing [about 60.7 percent](#) of candidates. In this situation, you notice that, since you don't care too much if you end up alone, you're content to review far more candidates, gather more information, and have a greater chance of selecting the very best.

These models are theoretical, but support some of the conventional wisdom about dating. First, they offer a good rationale for dating around before deciding to get serious. Without a dating history, you really don't have enough knowledge about the dating pool to make an educated decision about who is the best. You might think your first or second love is truly your best love, but, statistically speaking, it's not probably not so. Second, when you choose to settle down really depends on your preferences. If you want to find someone who is pretty good and minimize your chances of ending up alone, you'd try to settle down relatively

early – after reviewing and rejecting the first 30 percent of suitors you might have in your lifetime. If your goal is to find the very best of the bunch, you would wait a little longer, reviewing and rejecting 37percent of the total. And if you would like to find your perfect match, but you are also okay with ending up single, you'd wait much longer, reviewing and rejecting 60.7 percent of the total before you start looking for your match.

These equations are also reassuring for those with fear of missing out, those who worry about committing to a partner because they don't know what they might be missing in the future. The math shows that you really don't have to date all the fish in the sea to maximize your chances of finding the best.

If you choose the first suitor you date every time, you will  
 win twice (2/6):

1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

If you choose the second suitor you date every time, you will  
 win twice (2/6)

1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

If you choose the third suitor you date every time, you will  
 win twice (2/6)

1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

BUT if you date and reject the first suitor, then choose the next suitor who is better than the first, you win three times (3/6)

X	2	3	(Choose none)
X	3	2	(Choose none)
X	1	3	
X	X	1	
X	1	2	
X	2	1	