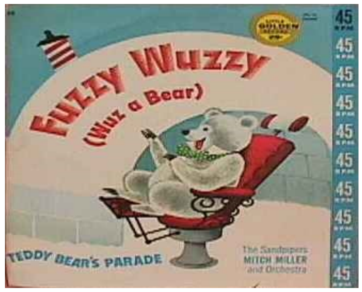
Lecture 18: Fuzzy Logic and Expert Systems Models

Fuzzy Wuzzy
Was a bear
Fuzzy Wuzzy
Had no hair
Fuzzy Wuzzy
Wasn't fuzzy, was he?



A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union,intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that the fuzzy sets be disjoint.

An expert system is a computer program that uses non-numerical domain-specific knowledge to solve problems as a human expert would solve them. It advises you to make a decision based on what experts would do. It codifies qualitative as well as quantitative information into a formal analysis. Before big data, the way to develop an expert system was to ask someone questions about how they made decisions: You might give them a list of factors and say something like -- does this affect your decision? By how much and turn the answers into numerics, make calculations with the numerics, and give advice. To the extent that experts evaluate evidence that comes in CONFIGURATIONS and involves non-linearities, mimicking the expert takes account of complex interactions among variables that are not readily captured by a statistical model.

OurWalmart, a workers group, seeking to improve conditions for Walmart's 1.3 million workers, developed Workit, an app that links to an AI intelligence "advisor" developed by IBM Watson, that provides information about rights and how workers have gained better conditions. The AI advisor uses the Walmart Company Handbook, labor law, and answers to workers' questions by lawyers/union experts to give advice.

Before Wall Street imploded John McCain was so sure that Alan Greenspan was the supreme expert that he suggested that when Greenspan retired, the country replace him with a simulcra to get Greenspan-like decisions.

But do you really want to trust experts? Before the 1929 Crash Irving Fisher predicted that *"Stock prices have reached what looks like a permanently high plateau."* Most of the Harvard Economics department kept saying prosperity was just around the corner. And most finance experts said capitalism had entered a new moderation right before the 2007-08 implosion of Wall Street.

On the other hand,in the 1950s Sumner Slichter, a Harvard B-School professor, made economic predictions based on his idiosyncratic knowledge. Here are SLICHTER type statements:

1: If the interest rate is somewhat low and the housing market is booming, stocks will drop a bit
2: If money supply increases a lot, and the debt ratio for consumers is rising, inflation will be moderate and the stock market will increase above inflation.

Paul Samuelson had a student compare Slichter's predictions with those from econometric models. If you bet that econometrics does better, you LOSE! The econometric model used the same equations in each forecast while Slichter changed models between periods. Knowing when to focus on the housing market as opposed to consumer confidence means the analyst changes weights in a model midstream-- per concept drift in data-streaming. Another way of envisaging what Slichter had was an ensemble of models, which makes his approach work better than a "same model fits all worlds" regression. Samuelson said "since Slichter cannot be cloned", we should improve econometrics. But if Slichter does better, isn't the right thing to do is to copy Slichter's mind set?

But how do you analyze the fuzzy terms that experts use to describe their assessments? High interest rates **usually** cause investment to fall; low interest rates **sometimes** lead to an investment boom. Opposing Wall Street **often** leads to political defeat. Need to quantify the fuzzy words to capture how the expert weighs evidence and reaches conclusions. A team of expert systems – the forest of tree models –might make decisions on the basis of their collective wisdom – and beat out Slichter to boot. Think Fuzzy.

1What is Fuzzy?

A way to transform vague human statement – "fuzzy" terms expressed with imprecise qualitative ways about what affects an outcome but that can be useful in decision-making and possibly predict more than **crisp** concepts: X causes Y; If Z, then the world turns into a red balloon with no ifs or buts.

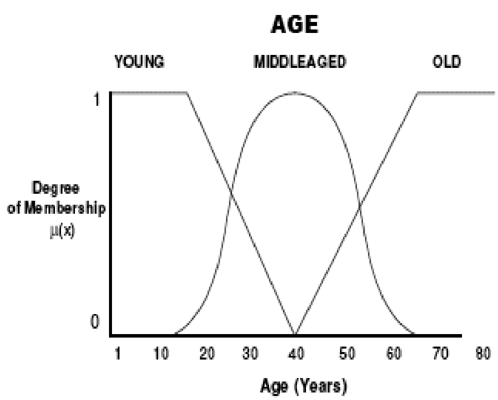
In fact, engineers use fuzzy control systems to run elevators. Fuzzy Logic and Neurofuzzy Applications in Business and Finance reports that over 30,000 scholarly papers use fuzzy. Tech Book stores are filled with fuzzy. As of April 2020 Google has 52.5M references to fuzzy logic and.Google scholar lists **1.86M** references to fuzzy logic: including *Twitter mood predicts stock market; A fuzzy logic control using a differential evolution algorithm aimed at modeling financial market dynamics;Using fuzzy logic to determine the vulnerability of marine species to climate change; Fuzzy Logic-Based Attenuation Relationships of Strong Motion Earthquake Record.* For wild read, Bart Kosko's Fuzzy Thinking.

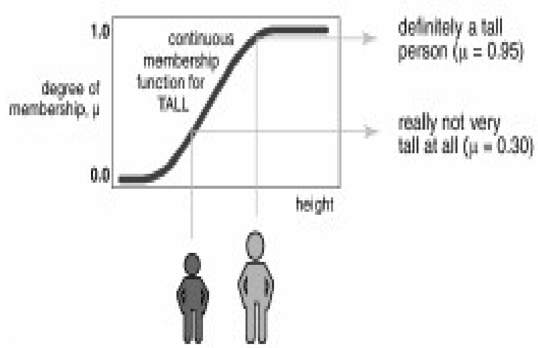
Fuzzy Sets. At the heart of fuzzy analysis is the distinction between crisp sets and fuzzy sets. Crisp sets are 0-1 dichotomies. You are tall/short. Happy/unhappy. Yes/ No. Sometimes forcing concepts into the 0/1 dichotomy is unnatural. Surveys about attitudes or perceptions offer options of the following form: Thinking about your social life at Harvard, would you say, you are: Very satisfied; Somewhat satisfied; Somewhat dissatisfied; Very dissatisfied? (4 pt scale). On a scale of 1 to 10, how satisfied are you? (Missing is "compared to what counterfactual?")

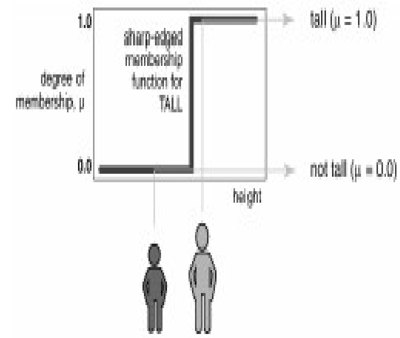
Fuzzy logic and set theory treat the vague, uncertain, problematic data or relations in terms of logic and is well-suited for developing expert systems based on how people see problems and for modeling such systems.

"As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost virtually exclusive characteristics."
Lotfi Zadeh founder of fuzzy thinking:

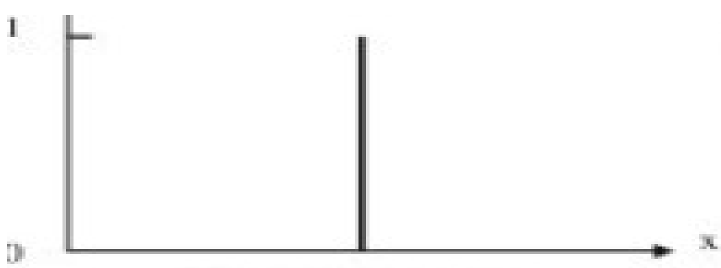
Fuzzy replaces crisp statements that "You are tall" or "not tall" with a **MEMBERSHIP Function** that gives the degree to which you are in the tall set. **A fuzzy set is a function that maps an object into a number between 0 and 1 that indicates degree of membership into a set**.



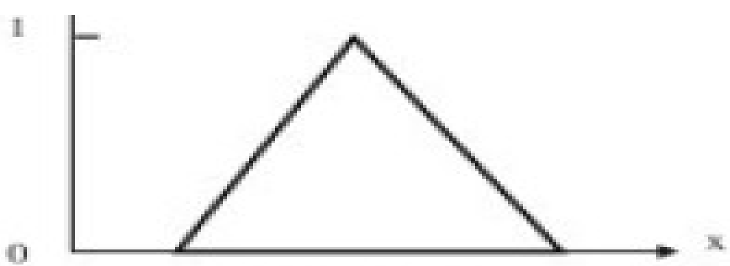




Consider middle-aged people. The membership function maps numeric age into the middle aged set with values between 0 and 1. The set for young and old maps age into other membership functions. The set for tall maps height into the membership function for tall depending on your height. The fuzzy set differs from the "crisp" set of standard symbolic logic. Fuzzy sets can represent numbers. Take 5. The number 5 has a truth value of membership in the #5 set of 1. But where is 4.9 or 5.1? Or 4.99999--> If you are rounding, they are also "5", but not as much "5" as 5. Here is a fuzzy set for 5. The triangle around the central value 5 is a standard membership function. The triangle shape is chosen for functional ease. But other functions also work.



Singleton fuzzy set

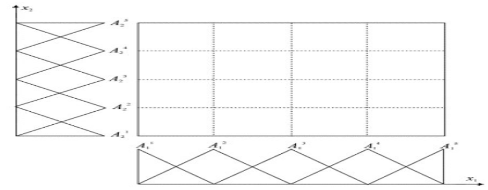


Continuous fuzzy set

Here are two other ways of showing the distinction between fuzzy and crisp sets.

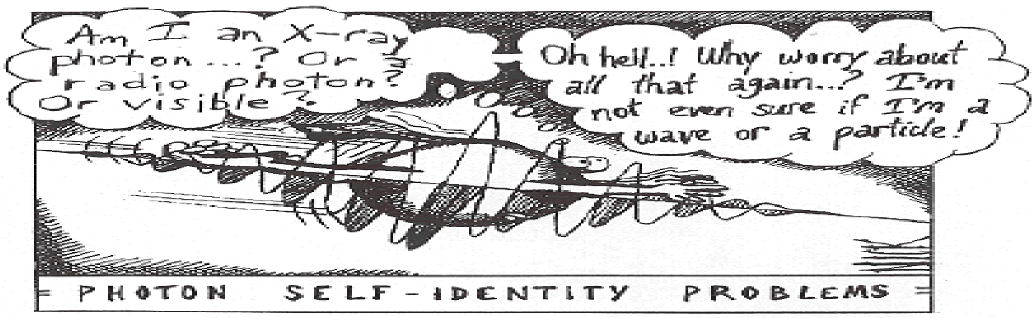
1.Consider the square, with tall and friendly as the characteristics on the axes.

	Tall	Friendly	
Tall and Friendly	YES	YES	(1,1)
Not Tall and Not Friendly	No	No	(0,0)
Not Tall and Friendly	No	Yes	(0,1)
Tall and Not friendly	No	Yes	(1,0)

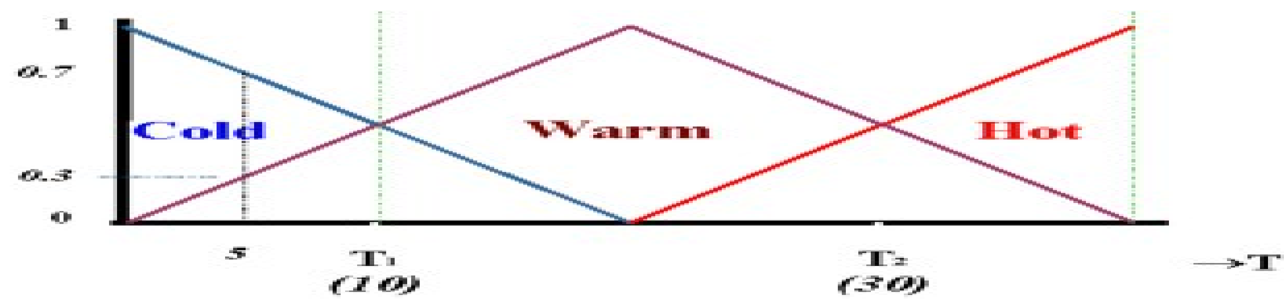


With a crisp set we have four choices, represented by the points at the vertexes and associated truth table. With fuzzy, the entire space represents the features. **A fuzzy set GENERALIZES a crisp set** to create leeway to deal with vague statements. While membership in a fuzzy set has the metric of probabilities, fuzzy membership **is not a probability** Probability restricts points in the space. There are just four points, but we would attach a probability to those points, with the probability summing to one. Fuzzy allows for the entire rectangle of points.

Fuzzy offers interpretation of logical paradoxes about how you must in be in one set or another: A Cretan says all Cretans are liars? Are Cretans liars? Fuzzy declares that the statement that Cretans are liars has a truth value of 0.5 and so too does the opposite statement. No paradox. Hmm Is a photon of light a wave or a particle? Sounds fuzzy.



2.Consider overlapping sets. You are partly in the hot crowd and partly in the warm crowd. The value of fuzzy comes from the fact that partitions or sets overlap and from behavior in TRANSITION phases.



Fuzzy Arithmetic: Say you have number 4 and represent it as (1,4,7) -- with a range of 3 on each side, with truth set values falling as you move away from 4. Now you want add this number/set to 6. But your 6 is sharper (5,6,7).

Addition is: add the central points, to get 10. Add the ranges and divide by 2 (6+2 = 8)/2= 4 so (6,10,14) is the solution. Subtraction, multiplication, division, follow similar procedure. Operate on central number and the range:

Subtract: 4 - 6 = -2 with a range of 4 again so the answer is (-6, -2, 2)
Multiply: 4 x 6 = 24 with range of 4 so the answer is (20, 24, 28)
Divide is 4/6 = .67 so the answer is (-3.33, 0.67, 4.67)

Why triangles to give the range around the number? Ease. Any inverse U shape will give similar results

2.FUZZY SETS

Fuzzy set theory defines operations on sets that are consistent but where you could develop alternative operational definition or fuzzify/defuzzify using different operators. The main operations:

AND -- You are sort of (.6) friendly **AND** only a bit (.3 tall), then you are (.3) sort of friendly AND a bit tall.
The logical AND is the minimum of the truth values.

OR -- You are sort of (.6) friendly **OR** only a bit (.3) tall, then you are (.6) sort of friendly OR a bit tall.
The logical OR is the maximum of the truth values

NOT -- You are NOT sort of friendly = 1 - truth value for sort of friendly -- 0.4.

Unlike in normal set theory, a thing and it's opposite has a positive truth value in fuzzy.
You are NOT sort of friendly AND You are sort of friendly has the truth value 0.4 (AND is the MINIMUM).
With crisp sets A and not A = NULL

You are NOT sort of friendly OR You are sort of friendly has the truth value 0.6 (OR is the MAXIMUM).
With crisp sets A or Not A = 1, the whole set.
If this sounds weird think of the photon identify crisis: Am I a particle or a wave?

Use Fuzzy Logic to make IF-THEN Statements: Take a rule that gives us an IF THEN response to a situation. IF specifies the condition. IF X then Y …. IF NOT X then Z . In a fuzzy world, BOTH rules may operate and you average to decide what to do.

Consider voting on a bill to constrain big banks. Your expert political advisor warns you that bankers will buy attack ads against you if you vote yes while student protesters will disrupt your rallies if you vote no.

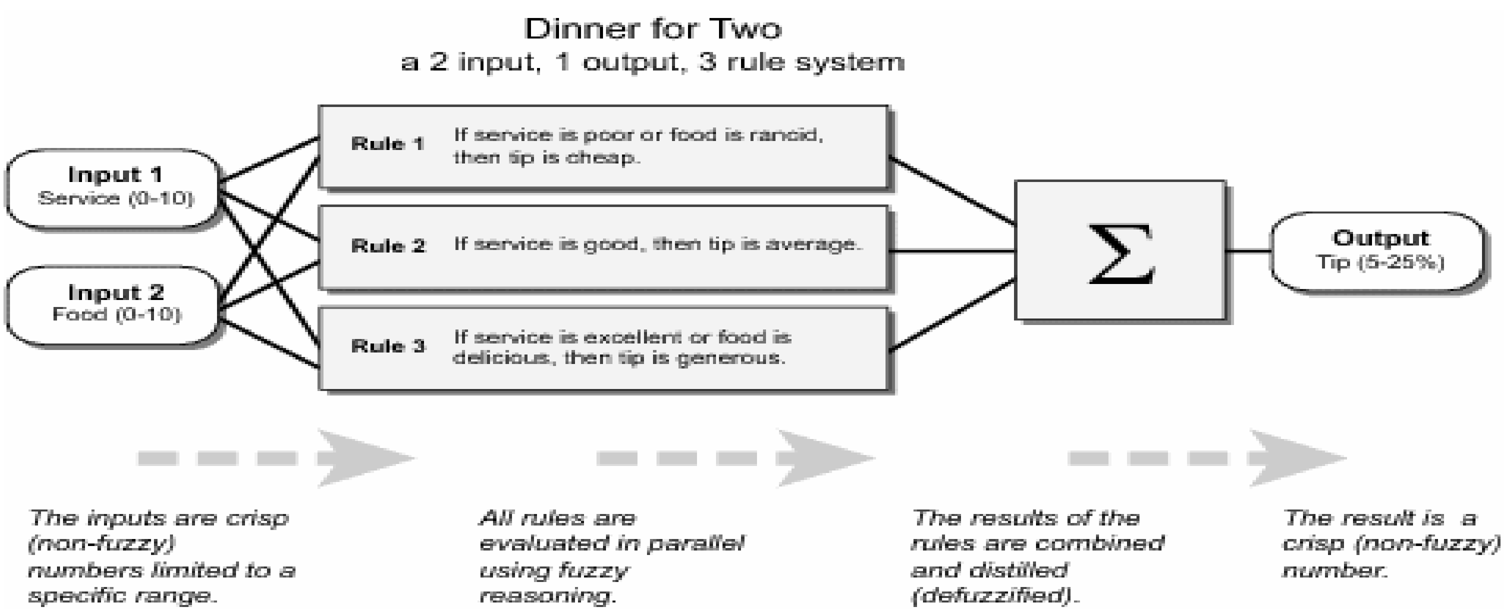
Rule 1: If bankers have truly big bucks, vote no as they want.
2: if they have modest big bucks, promise to fight to weaken bill in committee but vote yes
3: if students are very angry and not in exam period, vote yes
4: if students are somewhat angry and in exam period promise to fight for "appropriate" bill in committee
5: if students not very angry or on summer holiday, say free beer for all and vote no.

Since several rules tell you what to do, you must weigh the outcomes by averaging the rules by their truth values, then choose a response. SEVERAL RULES operate because there is an OVERLAP in the representation. You have to add together the actions from these rules, then **"defuzzify"** to make decision.

Defuzzifying consists of averaging what each rule says to do. There are different ways of averaging but the most popular is to use the truth value to weight the rules.

If there is uncertainty about the if part of the rule statement and the rule being right, you use the fuzzy AND. You have membership function value for the rule being right (it has a truth value of 0.8 in the set of true rules) and you have some confidence in the antecedent IF facts (it has a truth value of 0.6). Should you fire this rule or not? The posterior confidence in the rule will be the fuzzy AND (minimum). So it is 0.6. Then you need some threshold for saying "fire this rule if it falls above 0.5" or do not if it falls below 0.5.

The payoff to fuzzy is that easier to write a fuzzy model with simple If -Then rules than a detailed specification with complicated If-Then rules. You don't specify a zillion situations and ask the expert how he would respond. You get fuzzy statements and the program does the hard work. If you have 10 variables with 3 cases, you have 10^3 possible situations. Fuzzy beats complicated rule matrix Example from Mathworks on giving a tip:



Consider Go. Fuzzy would build an expert model based on detailed analysis of Go player decisions with rules that we could understand. But Alphago won using deep learning neural net program. What does this mean for how we encapsulate knowledge to get best outcomes/predictions? Do you trust AI expert vs fuzzy model of human expert?

Fuzzy expert model has three parts: 1)Translate linguistic variables into fuzzy math;
2)Operate mathematically on these variables;
3)Defuzzify – retranslate into word

Fuzzy quantifies linguistic variables/statements in normal language -- "Often" "Hardly ever" by giving them numeric values. "Hardly ever" --> close to 0 probability. Often --> something close to 80%? "when the road is empty I drive pretty fast in the daytime but a bit slower at night." Etc, Slews of such words. If e**xperts describe TACIT knowledge in linguistic terms, fuzzy quantifies the linguistic te**rms. For the system to work, need agreement in quantification. When expert 1 says *usually* Y produces Z and expert 2 says Y *usually* produces Z, do they mean the same? One way to check is to ask people to scale words from 0 to 100 in terms of how many times out of 100 you expect the event to occur. Here are results from **Bass, 1974:**

TABLE 1

MEANS AND STANDARD DEVIATIONS OF EXPRESSIONS OF FREQUENCY

Expressions of frequency	$\bar{X}$	SD
Always	58.01	3.52
Continually	50.16	3.18
Constantly	49.70	3.31
Frequently if not always	45.24	3.06
Very often	42.45	3.08
A great deal of the time	41.37	3.08
Very frequently	40.02	3.33
A great many times	39.28	3.09
Usually	39.18	3.13
Often	37.64	3.09
Frequently	36.07	3.29
Quite often	35.39	3.93
Rather frequently	34.44	3.18
Commonly	32.97	3.14
Fairly often	32.64	3.30
Fairly many times	30.65	3.00
Sometimes	19.42	2.86
Some of the time	18.01	3.01
To some degree	15.52	2.92
Now and then	15.19	3.04
Occasionally	14.92	3.06
Once in a while	10.22	2.89
Not often	7.78	2.55
Not very often	7.23	2.56
Fairly infrequently	6.99	2.72
Infrequently	6.47	2.61
Rather seldom	6.42	2.66
Very seldom	4.72	2.64
Rarely	4.56	2.23
Very infrequently	4.54	2.47
Seldom if ever	3.69	2.42
Hardly at all	3.47	2.38
Hardly ever	3.34	2.23
Very rarely	2.99	2.11
Almost never	2.63	2.10
Seldom	.33	2.60
None of the time	.17	1.49
Not at all	.15	1.53
Never	.08	1.41

MEANS AND STANDARD DEVIATIONS OF EXPRESSIONS OF AMOUNT

Expressions of amount	$\bar{X}$	SD
All	66.12	4.32
An exhaustive amount of	59.27	4.71
Almost entirely	57.61	5.76
Completely	57.35	5.90
An extraordinary amount of	54.46	4.38
Almost completely	51.38	4.71
An extremely abundant amount of	48.89	4.52
An extreme amount of	48.20	3.59
A great amount of	41.56	2.74
A great deal of	41.36	2.83
Very much	40.59	2.94
A full amount of	40.50	3.79
A lot of	37.10	2.77
Much	35.14	2.64
Quite a bit of	34.24	2.71
A good bit of	32.65	3.13
A considerable amount of	31.44	2.90
Pretty much	30.04	2.87
Fairly much	27.70	2.42
An ample amount of	26.22	2.81
An adequate amount of	24.07	2.60
A moderate amount of	21.80	3.42
Some	18.63	2.92
To some extent	13.42	2.95
To some degree	13.10	2.72
Somewhat	11.75	3.03
A limited amount of	9.57	2.85
A little	7.81	2.50
A small amount of	7.51	2.52
Comparatively little	7.22	2.71
A little bit of	7.20	2.67
Not much	7.02	2.70
A small degree of	5.27	2.50
Very little	5.21	2.45
A slight amount of	5.09	2.58
A meager amount of	4.28	2.66
A scanty amount of	3.68	2.41
A minimum amount of	3.64	2.71
A trifling amount of	3.13	2.59
Scarcely any	2.98	2.20
A trivial amount of	2.85	2.60
An insignificant amount of	2.48	2.11
Hardly any	2.28	2.20
None	.15	1.65

The Fuzzy Expert

To the user, an expert system is a black box. Inside the box are a **Knowledge base** and an **Inference Engine. The knowledge base contains all that the program has captured about expert knowledge. This is the hard part of the system since it requires that you get the "right" expertise into the computer program.**

WAYS TO GET THE KNOWLEDGE BASE

1)Set up scenarios and run experts through them. You describe the scenario. Ask for their decision. Then vary the situation and see if the decision varies. Run the scenario through many experts and find out why they differ. This may teach you that you did not ask the right question or that there is disagreement.

The problem with asking about scenarios is that you can get LOTS of CONFIGURATIONS.

2)Ask experts to tell you about their LAST or most IMPORTANT decision in the area. Perhaps they should RANK factors that led to success or failure

3)Ask classificatory/basic questions. What kinds of union organizing drives are there – worker initiated? union initiated; company initiated (VW in Tennessee)? what are the specific Issues?: wage; working conditions; unfair treatment. You would organize your model around those.

4) Ask for rules of thumb. Codify and check on consistency/posit some relationships between them. You may have to set up THEORY

FUZZY MEETS STONE PILE PROBLEM

A PILE OF STONES MAKES A HEAP, REMOVING A SINGLE STONE DOES NOT DESTROY THE HEAP, but remove them all and the heap is gone. Indeed, as we make the pile smaller. we don't have a heap of stones.

Can FUZZY resolve this problem?

THE PILE IS A HEAP — TRUTH VALUE AT BEGINNING –1.0

IF WE TAKE AWAY A STONE, THE TRUTH VALUE OF BEING A HEAP FALLS – 0.95

But repeat operation as FUZZY AND we get truth value of the MINIMUM – 0.95. If we take away a stone, and the truth value starts at 0.95 and it falls to 0.95 the minimum is 0.95.

What you need is a multiplicative concept – something that acts like a normal probability so that as you keep removing stones the probability that the PILE is a HEAP falls — .95 x .95 x .95 etc.

A fuzzy expert system for earthquake prediction, case study: the Zagros range by Arash Andalib, Mehdi Zare, Farid Atry arXiv:1610.04028v2 (Submitted on 13 Oct 2016 (v1), last revised 17 May 2017

A methodology for the development of a fuzzy expert system (FES) with application to earthquake prediction is presented. The idea is to reproduce the performance of a human expert in earthquake prediction. To do this, at the first step, rules provided by the human expert are used to generate a fuzzy rule base. These rules are then fed into an inference engine to produce a fuzzy inference system (FIS) and to infer the results. In this paper, we have used a Sugeno type fuzzy inference system to build the FES. At the next step, the adaptive network-based fuzzy inference system (ANFIS) is used to refine the FES parameters and improve its performance. The proposed framework is then employed to attain the performance of a human expert used to predict earthquakes in the Zagros area based on the idea of coupled earthquakes. While the prediction results are promising in parts of the testing set, the general performance indicates that prediction methodology based on coupled earthquakes needs more investigation and more complicated reasoning procedure to yield satisfactory predictions.

A FUZZY EXAMPLE: A Fuzzy Expert System for Heart Disease Diagnosis Ali.Adeli, Mehdi.Neshat Proceedings of the International MultiConference of Engineers and Computer Scientists 2010 Vol I, IMECS 2010, March 17 - 19, 2010,

Abstract— The aim of this study is to design a Fuzzy Expert System for heart disease diagnosis. The designed system based on the V.A. Medical Center, Long Beach and Cleveland Clinic Foundation data base. The system has 13 input fields and one output field. Input fields are chest pain type, blood pressure, cholesterol, resting blood sugar, maximum heart rate, resting electrocardiography (ECG), exercise, old peak (ST depression induced by exercise relative to rest), thallium scan, sex and age. The output field refers to the presence of heart disease in the patient. It is integer valued from 0 (no presence) to 4 (distinguish presence (values 1, 2, 3, 4)). This system uses Mamdani inference method. The results obtained from designed system are compared with the data in upon database and observed results of designed system are correct in 94%. The system designed in Matlab software. The system can be viewed as an alternative for existing methods to distinguish heart disease presence.

Fuzzy logic—a personal perspective Lotfi A.Zadeh Fuzzy Sets and Systems Volume 281, 15 December 2015, 4-20

This paper marks the 50th anniversary of the publication of my first paper on fuzzy sets, “Fuzzy sets,” Information and Control, 1965. What is of historical interest is that initially—and for some time thereafter—my paper was an object of indifference, skepticism and derision. A prominent school of thought claimed that fuzzy set theory is probability theory in disguise. Positive comments were few and far between. In contrast, my ideas were welcomed with open arms in Japan. In the seventies and eighties of last century, fuzzy set theory and fuzzy logic began to gain acceptance in Europe and, more particularly, in Eastern Europe and the Soviet Union. In part, many negative reactions to my papers reflected the fact that the word “fuzzy” has pejorative connotations. In large measure, science is based on the classical, Aristotelian, bivalent logic. Binarization—drawing a sharply defined boundary between two classes—is a deeply entrenched Cartesian tradition. What is not widely recognized is that this tradition has outlived its usefulness. One of the principal contributions of fuzzy logic is providing a basis for a progression from binarization to graduation, from binarism to pluralism, from black and white to shades of gray. Graduation involves association of a class which has unsharp (fuzzy) boundaries with degrees/grades of membership. Classes with unsharp boundaries are pervasive in human cognition. Most words in natural language are labels of such classes. This paper is a concise exposition of what I consider to be my principal contributions to the development of fuzzy set theory and fuzzy logic. Among the contributions which are discussed are: introduction of the concept of a fuzzy set, FL-generalization, the concept of a linguistic variable, information granulation, precisiation of meaning, generalized theory of uncertainty (GTU), the concept of a restriction, restriction-centered theory of truth and meaning, the information principle, and similarity-based definitions of possibility and probability.

Isothermal brain predicting Mcculloch-Pitts logic implicating Walter freeman ionic diffusion and Lotfi Zadeh fuzzy logic that might be useful for coding autonomous vehicles to be coexisting with the pedestrians