The Pass-Through of Sovereign Risk[*]

Luigi Bocola

Northwestern University

and

Federal Reserve Bank of Minneapolis

Abstract

This paper examines the macroeconomic implications of sovereign credit risk in a business cycle model where banks are exposed to domestic government debt. The *news* of a future sovereign default hampers financial intermediation. First, it tightens the funding constraints of banks, reducing their available resources to finance firms (liquidity channel). Second, it generates a precautionary motive for banks to deleverage (risk channel). I estimate the model using Italian data, finding that i) sovereign credit risk was highly recessionary, and that ii) the risk channel was sizable. I then use the model to evaluate the effects of subsidized long term loans to banks, calibrated to the ECB's Longer Term Refinancing Operations. The presence of strong precautionary motives at the policy enactment implies that bank lending to firms is not very sensitive to these credit market interventions.

Keywords: Sovereign Debt Crises, Occasionally Binding Leverage Constraints, Risk, Credit Policies.

JEL: E32, E44, G01, G21

1 Introduction

At the end of 2009, holdings of domestic government debt by banks in European peripheral countries - Greece, Italy, Portugal and Spain - were equivalent to 93% of banks' total equity. At the same time, domestic financial intermediaries in these economies were providing roughly two-thirds of external financing of local firms. It is therefore not surprising that many empirical studies have documented a severe disruption of financial intermediation and a substantial increase in the borrowing costs of firms during the sovereign debt crisis.[1] One proposed explanation of these findings is that the exposure to distressed government bonds hurts the ability of banks to raise funds in financial markets, leading to a pass-through of their increased financing costs into the lending rates paid by firms.[2] This view was at the core of policy discussions in Europe and was a motive for major interventions by the European Central Bank (ECB).

I argue, however, that this view is incomplete. A sovereign default can in fact be the trigger of a severe recession and have adverse effects on the performance of firms. Consequently, as an economy approaches a sovereign default, banks may start perceiving firms to be more "risky", and they may demand higher returns when lending to them as a fair compensation for holding this additional risk. If this mechanism is quantitatively important, policies that address the heightened liquidity problems of banks but do not reduce the increased riskiness of firms may prove ineffective in encouraging bank lending.

I formalize this mechanism in a quantitative model with financial intermediation and sovereign default risk. In the model, the *news* that the government may default in the future has adverse effects on the funding ability of exposed banks (*liquidity channel*) and it raises the risks associated with lending to the productive sector (*risk channel*). I structurally estimate the model on Italian data with Bayesian methods. I find that the risk channel is indeed quantitatively important: it explains up to 45% of the impact of the sovereign debt crisis on the borrowing costs of firms. I then use the estimated model to assess the consequences of credit market interventions adopted by the ECB in mitigating the implications of increased sovereign default risk.

My framework builds on a business cycle model with financial intermediation, in the tradition of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011, 2013). In the model,

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[1]See, for example, the evidence in Klein and Stellner (2013) and Bedendo and Colla (2013) using corporate bond data, the analysis of Bofondi et al. (2013) using Italian firm level data and Neri (2013) and Neri and Ropele (2013) for evidence using aggregate time series. See also ECB (2011).

[2]The report by CGFS (2011) discusses the transmission channels through which sovereign risk affected bank funding during the European debt crisis. For example, banks in the Euro area extensively use government bonds as collateral, and the decline in the value of these securities during the sovereign debt crisis reduced their ability to access wholesale liquidity. See also Albertazzi et al. (2012).

banks collect savings from households and use these funds, along with their own wealth (net worth), to buy long-term government bonds and to lend to firms. This intermediation is important because firms need external finance to buy capital goods. The model has three main ingredients. First, an agency problem between households and banks generates a constraint in the borrowing ability of the latter. This constraint on bank leverage binds only occasionally and typically when bank net worth is low. Second, financial intermediation is risky: bank net worth varies over time mainly because banks finance long-term risky assets with short-term riskless debt. Third, the probability that the government will default on its debt in the future is time-varying and follows a reduced form rule.

To understand the key mechanisms of the model, consider a scenario in which agents' expectations of a future sovereign default rise. The anticipation of a future "haircut" on government bonds depresses their market value, lowering the net worth of exposed banks. This tightens their leverage constraint and it has adverse consequences for financial intermediation: banks' ability to collect funds from households and their lending to the productive sector decline, leading to a contraction of capital accumulation. This is a conventional liquidity channel in the literature.

In addition, the expectation of a future sovereign default has adverse effects on the willingness of banks to hold firms' claims, even when banks are *currently* not facing funding constraints. The sovereign default, in fact, endogenously triggers a deep recession characterized by a severe decline in the payouts of firms. Thus, when the likelihood of this event increases, banks have a precautionary motive to deleverage in order to avoid these losses. More specifically, if the sovereign default happens in the future, the constraint on bank leverage will bind because of the large balance sheet losses associated with their position in government bonds. Banks are thus forced to liquidate their holdings of firms' claims. The associated decline in their market value leads to a further deterioration in bank net worth, feeding a vicious loop. *Ex-ante*, forward-looking banks have a precautionary motive to reduce their holdings of firms' assets because they perceive these claims to be more "risky"- i.e. they pay out little precisely when banks are facing funding constraints and are mostly in need of wealth. I refer to this second mechanism, often overlooked in the literature, as the risk channel.

I measure the quantitative importance of liquidity and risk by estimating the structural parameters of the model with Italian data from 1999:Q1 to 2011:Q4. The major challenge is to separate these two propagation mechanisms since they have qualitatively similar implications for indicators of financial stress commonly used in the literature (e.g., credit spreads). I demonstrate, however, that the Lagrange multiplier on the leverage constraint

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of banks is a function of observable variables, specifically of the TED spread and of the financial leverage of banks. I construct a time series for the Lagrange multiplier and I use it in estimation, along with GDP growth, to measure the cyclical behavior of the leverage constraint. In addition, I use credit default swap spreads on Italian government bonds and data on holdings of domestic government debt by Italian banks to measure the time-varying nature of sovereign risk and the exposure of banks to this risk.[3] The structural estimation is complicated by the fact that the model features time-variation in risk premia and occasionally binding financial constraints. I develop an algorithm for its global solution based on projections and sparse collocation, and I combine it with the particle filter to evaluate the likelihood function.

Having established the good fit of the model using posterior predictive analysis, I use it to answer two applied questions. First, I quantify the effects of the sovereign debt crisis on the financing premia of firms and on output, and I assess the relative importance of the two propagation mechanisms. I estimate that the sovereign debt crisis in Italy was responsible for a rise in the financing premia of firms that reached 93 basis points in 2011:Q4, with the risk channel explaining up to 45% of the overall effects. This increase in the financing costs of firms was associated with a severe decline in real economic activity: output losses of the sovereign debt crisis in Italy cumulate to 1.75% by the end of 2011.

Second, I evaluate the effects of a major credit market intervention adopted by the ECB in the first quarter of 2012, the Longer Term Refinancing Operations (LTROs). I model the policy as a subsidized long-term loan offered to banks, and implement it conditioning on the state of the Italian economy in 2011:Q4. I find that the effects of LTROs on credit to firms and output are small when we average over the 2012:Q1-2014:Q4 period. This is due to the fact that precautionary motives were sizable when the policy was enacted. Banks thus have little incentives to increase their exposure to firms and they mainly use LTROs to cheaply substitute liabilities they have with the private sector.

The general lesson from the policy evaluation is that the success of credit market interventions crucially depends on the economic environment in which they are implemented, more specifically on the relative importance of liquidity and risk. Liquidity constraints, by definition, prevent banks from undertaking profitable investment. Policies that relax these constraints have sizable effects on bank lending to firms and capital accumulation. The risk channel, instead, is an indication that firms are perceived to be a "bad asset" in the future, and banks incentives to hold their claims are less responsive to this type

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[3]A credit default swap (CDS) is a derivative used to hedge the credit risk of an underlying reference asset. CDS spreads on government securities are typically used in the literature as a proxy of sovereign risk, see Pan and Singleton (2008).

of policies. The nonlinear model studied in this paper allows one to measure these two mechanisms in real time, a decomposition that provides useful information for predicting the macroeconomic effects of credit market interventions.

1.1 Related literature

This research is related to several strands of the literature. I contribute to recent work studying the macroeconomic implications of shocks to the balance sheet of financial intermediaries. I build on the framework developed in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011, 2013), where the limited enforcement of debt contracts leads to constraints on intermediaries' leverage. Differently from these papers, I study how changes in the expectation of this constraint being binding in the future influence the choices of intermediaries to hold risky assets today. Technically, I capture these effects because I analyze the model using nonlinear methods. My analysis uncovers two important phenomena. First, intermediaries develop a precautionary motive to deleverage whenever they expect the constraint to bind in the future. Related effects have been emphasized by Brunnermeier and Sannikov (2013), He and Krishnamurthy (2012a) and Bianchi and Mendoza (2012). The novel insight is that in my set-up these precautionary motives are induced by an increase in the default probability of an *unproductive* asset, the government bond. Intermediaries have lower incentives to hold productive assets because of contagion effects- i.e. their balance sheet endogenously generate a correlation in the payoffs of productive and unproductive assets. Second, I show that credit market interventions are highly state and size dependent in this environment.[4]

This paper is also related to the research on sovereign debt crises. Many empirical analyses have documented a strong link between sovereign and private sector spreads, both in emerging economies and more recently in southern European countries.[5] Several authors have recognized the importance of this relationship in different settings, see Neumeyer and Perri (2005) and Uribe and Yue (2006) in the context of business cycles in emerging markets, and Corsetti et al. (2013) for the implications of the sovereign risk pass-through for fiscal multipliers. However, in these and related papers, the reasons underlying the connection between sovereign spreads and private sector interest rate are left unmodeled.

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[4]There are a number of papers that study the effects of credit market interventions in related environments. See, Curdia and Woodford (2010), Del Negro et al. (2012) and Bianchi and Bigio (2013).

[5]For emerging market economies, Durbin and Ng (2005) and Borensztein et al. (2006) provide an empirical analysis of the "sovereign ceiling", the practice of agencies to rate corporations below their sovereign. Cavallo and Valenzuela (2007) document the effects of sovereign spreads on corporate bonds spreads. See footnote 1 and 2 for evidence on southern European economies.

Part of the contribution of this paper is to microfound this link in a fully specified dynamic equilibrium model.

In doing so, my paper relates to recent studies analyzing the relation between sovereign defaults and domestic banking sector. Motivated by robust empirical evidence, Gennaioli et al. (2013b) and Sosa Padilla (2013) study the effects of sovereign defaults on domestic banks, and the impact that the associated output losses have on the government's incentives to default.[6] My research is complementary to theirs: I take sovereign default risk in reduced form, but I explicitly model the behavior of private credit markets when sovereign risk increases. Differently from the above papers, the mere *anticipation* of a future sovereign default is recessionary because of its effects on the ability and the incentives of banks to intermediate productive assets.[7] Anticipation effects have been studied in related environments by Acharya et al. (2013), Broner et al. (2014) and Mallucci (2013). However, their analysis abstract from the effects that a sovereign default has on the perceived riskiness of firms, the key novel mechanism of this paper.

Methodologically, I draw from the literature on the Bayesian estimation of dynamic equilibrium economies (Del Negro and Schorfheide, 2011), more specifically of models where nonlinearities feature prominently (Fernández-Villaverde and Rubio-Ramírez, 2007). The model decision rules are derived numerically using a projection algorithm and the Smolyak sparse grid (Krueger and Kubler, 2003).[8] The likelihood function is evaluated using the auxiliary particle filter of Pitt and Shephard (1999). To my knowledge, this is the first paper that estimates a model with occasionally binding financial constraints using global methods and nonlinear filters. However, there are other papers that use related techniques in different applications (see Gust et al., 2013; Bi and Traum, 2012).

Finally, the shock to sovereign default probabilities considered in this paper is a form of time-varying volatility. As such, my research is related to the literature that studies how different types of volatility shocks influence real economic activity (Bloom, 2009; Fernández-Villaverde et al., 2011). Rietz (1988) and Barro (2006) emphasize the role of large macroeconomic disasters in accounting for asset prices and Gourio (2012) studies how changes in the probability of these events affect risk premia and capital accumulation. The

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[6]Kumhof and Tanner (2005) and Gennaioli et al. (2013a) document that banks are highly exposed to domestic government debt in a large set of countries. Reinhart and Rogoff (2011) and Borensztein and Panizza (2009) show that sovereign defaults typically occur in proximity of banking crises.

[7]In an empirical study, Levy-Yeyati and Panizza (2011) point out that anticipation effects are key to understand the unfolding of sovereign debt crises. See also Aguiar et al. (2009) and Dovis (2013) for models where anticipation effects arise because of debt overhang problems.

[8]Christiano and Fisher (2000) is an early paper documenting the performance of projections in models with occasionally binding constraints. See Fernández-Villaverde et al. (2012) for an application of the Smolyak sparse grid in a model where the zero bound constraint on nominal rates binds occasionally.

sovereign default studied in this paper can be seen as a potential source of macroeconomic disasters.[9]

Layout. The paper is organized as follows. Section 2 presents the model, while Section 3 discusses its main mechanisms using two simplified examples. Section 4 presents the estimation and an analysis of the model's fit. Section 5 presents key properties of the estimated model that are useful to interpret the two quantitative experiments, which are reported in Section 6. Section 7 concludes.

2 Model

I consider a neoclassical growth model enriched with a financial sector as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). I introduce in this setting long term government bonds to which financial intermediaries are exposed. These securities pay in every state of nature unless the government defaults. The probability of this latter event varies over time according to a reduced form stochastic process.

The model economy is populated by households, final good producers, capital good producers and a government. Each household is composed of two types of members: *workers* and *bankers*. Workers supply labor to final good firms in a competitive factor market, and their wages are made available to the household. Bankers manage the savings of other households and use these funds to buy government bonds and claims on firms. The perfectly competitive non-financial corporate sector produces a final good using a constant returns to scale technology that aggregates capital and labor. Firms rent labor from households and buy capital from perfectly competitive capital good producers. Their capital expenses are financed by bankers. Finally, the government issues bonds and taxes households in order to finance government spending, and can default on its debt. The actions of the government are determined via fiscal rules.

The key friction in this environment is the limited enforcement of debt contracts between households and bankers: bankers can walk away with the assets of their franchise, and households can recover only a fraction of their savings when this event occurs. This friction gives rise to constraints on the leverage of banks, with bank net worth being the key determinant of their borrowing capacity. When these incentive constraints bind, or are expected to bind in the future, bankers' ability to intermediate is hampered and credit to the productive sector declines. This has adverse consequences for capital accumulation.

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[9] Arellano et al. (2012), Gilchrist et al. (2013) and Christiano et al. (2013) study the real effects of a different form of time-varying volatility in models with financial frictions.

In this environment, sovereign credit risk has adverse effects on real economic activity because it alters both the *ability* and the *incentives* of bankers to lend to firms.

In the remainder of this section I describe the agents' decision problems, derive the conditions characterizing a competitive equilibrium, and sketch the algorithm used for the numerical solution of the model. In Section 3, I discuss the key mechanisms of interest. I denote by **S** the vector collecting the current value for the state variables and by **S**′ the future state of the economy.

2.1 Agents and their decision problems

2.1.1 Households

A household is composed of a fraction f of workers and a fraction $1 - f$ of bankers. There is perfect consumption insurance between its members. I denote by $\Pi(\mathbf{S})$ the net profits that the household receives from holdings of economic activities, and by $W(\mathbf{S})$ the wage that workers receive from supplying labor to final good firms. The household values consumption c and dislikes labor l according to the flow utility $u(c, l)$, and he discounts the future at the rate β. The household makes contingent plans for consumption, labor supply and savings b' in order to maximize lifetime utility. Savings are managed by financial intermediaries that are run by bankers belonging to other households, and they earn the risk free return $R(\mathbf{S})$. Taking prices as given, a household solves

$$v_h(b; \mathbf{S}) = \max_{b' \geq 0, c \geq 0, l \in [0,1]} \left\{ u(c, l) + \beta \mathbb{E}_{\mathbf{S}}[v_h(b'; \mathbf{S}')] \right\},$$

$$c + \frac{1}{R(\mathbf{S})} b' \leq W(\mathbf{S})l + \Pi(\mathbf{S}) + b - \tau(\mathbf{S}),$$

$$\mathbf{S}' = \mathbf{\Gamma}(\mathbf{S}).$$

$\tau(\mathbf{S})$ denotes the level of lump sum taxes while $\mathbf{\Gamma}(.)$ describes the law of motion for the aggregate state variables. Optimality is governed, at an interior solution, by the intra-temporal and inter-temporal Euler equations

$$u_l(c, l) = u_c(c, l)W(\mathbf{S}), \tag{1}$$

$$\mathbb{E}_{\mathbf{S}}[\Lambda(\mathbf{S}', \mathbf{S})R(\mathbf{S})] = 1, \tag{2}$$

where $\Lambda(\mathbf{S}', \mathbf{S}) = \beta \frac{u_c(c', l')}{u_c(c, l)}$ is the household's marginal rate of substitution.

For the empirical analysis I will use preferences that are consistent with balanced

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growth, $u(c,l) = \log(c) - \chi\frac{l^{1+\nu^{-1}}}{1+\nu^{-1}}$, where ν parameterizes the Frisch elasticity of labor supply.

2.1.2 Bankers

A banker uses his accumulated net worth, n, and households' savings, b', to buy government bonds and claims on firms.[10] Let a_j be asset j held by a banker and let $Q_j(\mathbf{S})$ and $R_j(\mathbf{S'},\mathbf{S})$ be, respectively, the price of asset j and its realized returns next period on a unit of numeraire good invested in asset j. The banker's balance sheet equates total assets to total liabilities:

$$\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j = n + \frac{b'}{R(\mathbf{S})}, \tag{3}$$

where subscript B refers to government bonds and K to firms' claims. The banker makes optimal portfolio choices in order to maximize the present discounted value of dividends paid to his own household. The payment of dividends depends on the turnover of a banker. At any point in time there is a probability $1 - \psi$ that a banker becomes a worker in the next period. When this happens, the banker transfers his accumulated net worth to the household.[11] A banker that continues running the business does not pay dividends, and he keeps on accumulating net worth. Therefore, the present discounted value of dividends equals the present discounted value of the banker's terminal wealth. Net worth next period is given by the difference between realized returns on assets and the payments promised to households.

$$n' = \sum_{j=\{B,K\}} R_j(\mathbf{S'},\mathbf{S})Q_j(\mathbf{S})a_j - b'. \tag{4}$$

Note that bad realizations of $R_j(\mathbf{S'},\mathbf{S})$ lead to reductions in the net worth n' of the banker. Because of a financial friction, this variation in net worth can affect a banker's borrowing ability. More specifically, I assume limited enforcement of contracts between households and bankers. At any point in time, a banker can walk away with a fraction λ of the project and transfer it to his own household. If he does so, households can force him into bankruptcy and recover a fraction $(1 - \lambda)$ of the banks' assets. This friction defines an incentive constraint for the banker: the value of running his franchise must be higher than its outside option, $\lambda[\sum_j Q_j(\mathbf{S})a_j]$.

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[10]A worker who becomes a banker this period obtains start-up funds from his households. These transfers will be specified at the end of the section.

[11]When a banker exits, a worker replaces him so that their relative proportion does not change over time.

Taking prices as given, the banker solves the following program

$$v_b(n; \mathbf{S}) = \max_{a_B, a_K, b} \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi)n' + \psi v_b(n'; \mathbf{S}') \right] \right\},$$

$$n' = \sum_{j=\{B,K\}} R_j(\mathbf{S}', \mathbf{S}) Q_j(\mathbf{S}) a_j - b',$$

$$\sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j \leq n + \frac{b'}{R(\mathbf{S})},$$

$$\lambda \left[\sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j \right] \leq v_b(n; \mathbf{S}),$$

$$\mathbf{S}' = \mathbf{\Gamma}(\mathbf{S}).$$

Result 1 further characterizes this decision problem.

Result 1. *A solution to the banker's dynamic program is*

$$v_b(n; \mathbf{S}) = \alpha(\mathbf{S})n,$$

where $\alpha(\mathbf{S})$ solves

$$\alpha(\mathbf{S}) = \frac{\mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S}')]R(\mathbf{S})\}}{1 - \mu(\mathbf{S})}, \tag{5}$$

and the Lagrange multiplier on the incentive constraint satisfies

$$\mu(\mathbf{S}) = max \left\{ 1 - \left[\frac{\mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S}')]R(\mathbf{S})\}N}{\lambda[Q_K(\mathbf{S})A_K + Q_B(\mathbf{S})A_B]} \right], 0 \right\}, \tag{6}$$

where N, A_B and A_K are, respectively, aggregate bankers' net worth and aggregate bankers' holdings of government bonds and firms assets.

Proof. See Appendix A. □

This result clarifies why bank net worth plays a role in the model. Indeed, because of the linearity of the value function, we can rewrite the incentive constraint as

$$\frac{\sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j}{n} \leq \frac{\alpha(\mathbf{S})}{\lambda}, \tag{7}$$

implying that the leverage of a banker cannot exceed the time-varying threshold $\frac{\alpha(\mathbf{S})}{\lambda}$. Bank net worth is thus a key variable regulating financial intermediation in the model: when net worth is low, the leverage constraint is more likely to bind and this limits the borrowing ability of a banker and, ultimately, his ability to intermediate.

The implications of this constraint for assets' accumulation can be better understood by looking at the Euler equation for risky asset j

$$\mathbb{E}_{\mathbf{S}}\left[\hat{\Lambda}(\mathbf{S}',\mathbf{S})R_j(\mathbf{S}',\mathbf{S})\right] = \mathbb{E}_{\mathbf{S}}\left[\hat{\Lambda}(\mathbf{S}',\mathbf{S})R(\mathbf{S})\right] + \lambda\mu(\mathbf{S}), \tag{8}$$

where $\hat{\Lambda}(\mathbf{S}',\mathbf{S})$ is the economy's pricing kernel, defined as

$$\hat{\Lambda}(\mathbf{S}',\mathbf{S}) = \Lambda(\mathbf{S}',\mathbf{S})[(1-\psi) + \psi\alpha(\mathbf{S}')]. \tag{9}$$

There are two main distinctions between this Euler equation and the one that would arise in a purely neoclassical setting. First, the leverage constraint limits the ability of banks to arbitrage away differences between expected discounted returns on asset j and the risk free rate: this can be seen from equation (8), as the Lagrange multiplier generates a wedge between these two returns. Second, the pricing kernel in equation (9) is not only a function of consumption growth as in canonical neoclassical models, but also of bank leverage. Indeed, as stated in equation (7), financial leverage is proportional to $\alpha(\mathbf{S})$ when $\mu(\mathbf{S}) > 0$. Adrian et al. (2013) provides empirical evidence in support of leverage-based pricing kernels for the U.S. economy and He and Krishnamurthy (2012b) discuss their asset pricing implications in endowment economies. If the leverage constraint never binds, i.e. $\mu(\mathbf{S}) = 0 \; \forall \; \mathbf{S}$, equation (8) collapses to the neoclassical benchmark.[12]

Result 1 also implies that heterogeneity in bankers' net worth and in their asset holdings does not affect aggregate dynamics: this feature of the model makes its numerical analysis tractable. For future reference, it is convenient to derive an expression for the law of motion of aggregate net worth

$$N'(\mathbf{S}',\mathbf{S}) = \psi\left\{\sum_{j=\{B,K\}}[R_j(\mathbf{S}',\mathbf{S}) - R(\mathbf{S})]Q_j(\mathbf{S})A_j + R(\mathbf{S})N\right\} + \omega\sum_{j=\{B,K\}}Q_j(\mathbf{S}')A_j. \tag{10}$$

Aggregate net worth next period equals the sum of the net worth accumulated by bankers who do not switch occupations today and the transfers that households make to newly born bankers. These transfers are assumed to be a fraction ω of the assets intermediated in the previous period, evaluated at current prices.

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[12]Using equation (2), we can see that a solution to equation (5) is $\alpha(\mathbf{S}) = 1 \; \forall \; \mathbf{S}$ whenever $\mu(\mathbf{S}) = 0 \; \forall \; \mathbf{S}$.

2.1.3 Capital good producers

The capital good producers build new capital goods using the technology $\Phi\left(\frac{i}{K}\right)K$, where K is the aggregate capital stock in the economy and i the inputs used in production. They buy inputs in the final good market, and sell capital goods to final good firms at competitive prices. Taking the price of new capital $Q_i(\mathbf{S})$ as given, the decision problem of a capital good producer is

$$\max_{i \geq 0}\left[Q_i(\mathbf{S})\Phi\left(\frac{i}{K}\right)K - i\right].$$

Anticipating the market clearing condition, the price of new capital goods is

$$Q_i(\mathbf{S}) = \frac{1}{\Phi'\left(\frac{I(\mathbf{S})}{K}\right)}, \tag{11}$$

where $I(\mathbf{S})$ is equilibrium aggregate investment.

For the empirical analysis, I specify the production function for capital goods as $\Phi(x) = a_1 x^{1-\xi} + a_2$, where ξ parametrizes the elasticity of Tobin's q with respect to the investment-capital ratio.

2.1.4 Final good producers

Final output y is produced by perfectly competitive firms that operate a constant returns to scale technology

$$y = k^\alpha (e^z l)^{1-\alpha}, \tag{12}$$

where k is the stock of capital goods, l stands for labor services, and z is a neutral technology shock that follows an AR(1) process in growth

$$\Delta z' = \gamma(1-\rho_z) + \rho_z \Delta z + \sigma_z \varepsilon_z', \qquad \varepsilon_z' \sim \mathcal{N}(0,1). \tag{13}$$

Labor is rented in competitive factor markets at $W(\mathbf{S})$. Capital goods depreciate every period at the rate δ. Anticipating the labor market clearing condition, profit maximization implies that equilibrium wages and profits per unit of capital are

$$W(\mathbf{S}) = (1-\alpha)\frac{Y(\mathbf{S})}{L(\mathbf{S})}, \qquad Z(\mathbf{S}) = \alpha\frac{Y(\mathbf{S})}{K}, \tag{14}$$

where $Y(\mathbf{S})$ and $L(\mathbf{S})$ are equilibrium aggregate output and labor.

Firms need external financing to purchase new capital goods. At the beginning of the period, they issue claims to bankers in exchange for funds.[13] For each claim a_K bankers pay $Q_K(\mathbf{S})$ to firms.[14] In exchange, bankers receive the realized return on a unit of the capital stock in the next period:

$$R_K(\mathbf{S}', \mathbf{S}) = \frac{(1-\delta)Q_K(\mathbf{S}') + Z(\mathbf{S}')}{Q_K(\mathbf{S})}. \tag{15}$$

Realized returns to capital move over time because of two factors: variation in firms' profits and variation in the market value of firms' claims. These movements in $R_K(\mathbf{S}', \mathbf{S})$ induce variation in aggregate net worth, as equation (10) suggests.

2.1.5 The government

In every period, the government engages in public spending. Public spending as a fraction of GDP evolves as follows

$$\log(g)' = (1 - \rho_g)\log(g^*) + \rho_g\log(g) + \sigma_g\varepsilon_g', \qquad \varepsilon_g' \sim \mathcal{N}(0,1). \tag{16}$$

The government finances public spending by levying lump sum taxes on households and by issuing long-term government bonds to financial intermediaries. Long term debt is introduced as in Chatterjee and Eyigungor (2013). In every period a fraction π of bonds matures, and the government pays back the principal to investors. The remaining fraction $(1 - \pi)$ does not mature: the government pays the coupon ι, and investors retain the right to obtain the principal in the future.[15] I introduce risk of sovereign default by assuming that the government can default in every period and write off a fraction $D \in [0,1]$ of its outstanding debt. The parameter D can be interpreted as the "haircut" that the government imposes on bondholders in a default. Denoting by $Q_B(\mathbf{S})$ the pricing function for government securities, tomorrow's realized returns on a dollar invested in government bonds are

$$R_B(\mathbf{S}', \mathbf{S}) = \left[1 - d'D\right]\left[\frac{\pi + (1-\pi)\left[\iota + Q_B(\mathbf{S}')\right]}{Q_B(\mathbf{S})}\right], \tag{17}$$

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[13]While these claims are perfectly state contingent and therefore correspond to equity holdings, I interpret them more broadly as privately issued paper such as bank loans.

[14]No arbitrage implies that the price of a unit of new capital equals in equilibrium the price of a claim issued by firms, $Q_i(\mathbf{S}) = Q_K(\mathbf{S})$.

[15]The average duration of bonds is therefore $\frac{1}{\pi}$ periods.

where d' is an indicator variable equal to 1 if the government defaults next period. Realized returns on government bonds vary over time because of two sources. First, when the government defaults, it imposes a haircut on bondholders. Second, $R_B(\mathbf{S}', \mathbf{S})$ is sensitive to variation in the price of government securities: a decline in $Q_B(\mathbf{S}')$, for example, lowers the resale value of government bonds and reduces the returns on holding government debt. This second effect is present even when the government does not default ($d' = 0$), and to the extent that government debt has a maturity longer than one period ($\pi < 1$).

Denoting by B' the stock of public debt, the budget constraint of the government is given by

$$Q_B(\mathbf{S}) \left[\underbrace{B' - (1-\pi)B[1-dD]}_{\text{Newly issued bonds}} \right] = \underbrace{[\pi + (1-\pi)\iota] B[1-dD]}_{\text{Payments of principals and coupons}} + \underbrace{gY(\mathbf{S}) - \tau(\mathbf{S})}_{\text{Primary deficit}}. \quad (18)$$

Taxes respond to past debt according to the law of motion

$$\frac{\tau(\mathbf{S})}{Y(\mathbf{S})} = t^* + \gamma_\tau \frac{B}{Y(\mathbf{S})},$$

where $\gamma_\tau > 0$.[16]

In order to close the model, we need to specify how sovereign risk evolves over time. As was the case for tax policy, sovereign credit risk follows a fiscal rule. In every period the government is hit by a shock ε_d that follows a standard logistic distribution. The government defaults on its outstanding debt if ε_d is sufficiently large. In particular, d' follows

$$d' = \begin{cases} 1 & \text{if} \quad \varepsilon_d' - \Psi(\mathbf{S}; \theta_2) \geq 0 \\ 0 & \text{otherwise,} \end{cases} \quad (19)$$

where θ_2 is a vector of parameters. Given equation (19), we can write the conditional probability of a future sovereign default as follows

$$p^d(\mathbf{S}) \equiv \text{Prob}(d' = 1 | \mathbf{S}) = \frac{\exp\{\Psi(\mathbf{S}; \theta_2)\}}{1 + \exp\{\Psi(\mathbf{S}; \theta_2)\}}. \quad (20)$$

This formulation is intended to capture, in a flexible way, the considerations introduced by the research on the determinants of sovereign risk. In the literature on strategic debt default (Eaton and Gersovitz, 1981; Arellano, 2008), the government's lack of commitment

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[16]This formulation guarantees that the government does not run a Ponzi scheme and that its intertemporal budget constraint is satisfied in every state of nature. See Bohn (1995) and Canzoneri et al. (2001).

to repay his debt implies that default probabilities increase whenever the economy is approaching states of the world where the government's benefits to renege on his debt are larger than their costs. In the literature on fiscal limits (Uribe, 2006; Bi, 2012), sovereign risk moves whenever the current fiscal stance is forecasted to be incompatible with the solvency of the government. These approaches predict that sovereign credit risk responds to the current state of the world. By appropriately choosing the function $\Psi(\mathbf{S};\theta_2)$, one could incorporate in the analysis key drivers of sovereign credit risk identified in the literature: for example, $\Psi(\mathbf{S};\theta_2)$ could depend on the stock of debt, on total factor productivity or the net worth of financial intermediaries. This allows us to keep the framework parimonious, and to focus on the economic mechanisms through which variation in $p^d(\mathbf{S})$ influences real economic activity, rather than those governing its determinants.

For the empirical application studied in the paper, though, I will consider the simple specification $\Psi(\mathbf{S};\theta_2) = s$, with s being an AR(1) process

$$s' = (1 - \rho_s)\log(s^*) + \rho_s s + \sigma_s \varepsilon_s', \qquad \varepsilon_s' \sim \mathcal{N}(0,1). \tag{21}$$

This choice is motivated by two main considerations. First, there is substantial empirical evidence that a large share of the variation in Italian sovereign spreads during the European crisis was driven by factors orthogonal to domestic fundamentals. For example, Bahaj (2013) documents that Italian sovereign spreads were very sensitive to political and social events in Cyprus, Greece, Ireland, Portugal and Spain during the European crisis: using a narrative approach, he finds that foreign events can account for up to 50% of the variation in Italian spreads.[17] These empirical findings are consistent with the view, shared by economists and policymakers, that self-fulfilling beliefs (Conesa and Kehoe, 2012; Lorenzoni and Werning, 2013) and contagion through common creditors (Arellano and Bai, 2013) were key drivers of sovereign risk during the European crisis. The s-shock is intended to capture these considerations in a reduced form way. Second, exogenous variations in the probability of a future sovereign default allow us to clearly isolate, within the model, the economic mechanisms driving the propagation of sovereign credit risk.

Importantly, as discussed in Section 4, the exogeneity of sovereign credit risk will not be used as a restriction when estimating the structural model. Thus, the identification of key model parameters will not rely on this assumption.

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[17] Di Cesare et al. (2013) show that economic fundamentals have limited predictive power in explaining the behavior of Italian sovereign spreads, especially during the 2011-2012 period. Zoli (2013) points out that foreign and domestic political event accounts for large part of the variation in Italian CDS spreads. See also Giordano et al. (2013) for an empirical analysis of contagion effects in European sovereign bonds markets.

2.2 Market clearing

Letting $f(.)$ be the density of net worth across bankers, we can express the market clearing conditions as follows[18]

1. Credit market: $\int a_K(n; \mathbf{S}) f(n) dn = K'(\mathbf{S})$.
2. Government bonds market: $\int a_B(n; \mathbf{S}) f(n) dn = B'(\mathbf{S})$.
3. Market for households' savings: $\int b'(n; \mathbf{S}) f(n) dn = b'(\mathbf{S})$.
4. Market for final goods: $Y(\mathbf{S})(1 - g) = C(\mathbf{S}) + I(\mathbf{S})$.

2.3 Equilibrium conditions and numerical solution

Since the non-stationary technology process induces a stochastic trend in several endogenous variables, it is convenient to express the model in terms of detrended variables. For a given variable x, I define its detrended version as $\tilde{x} = \frac{x}{z}$.[19] The state variables of the model are $\mathbf{S} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s, d]$. As I detail below, the variable $\tilde{P}$ keeps track of aggregate bank net worth. The control variables $\{\tilde{C}(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), Q_B(\mathbf{S})\}$ solve the residual equations (2), (5) and (8) (the last one for both assets).

The endogenous state variables $[\tilde{K}, \tilde{B}, \tilde{P}]$ evolve as follows:

$$\tilde{K}'(\mathbf{S}) = \left\{ (1 - \delta)\tilde{K} + \Phi\left[e^{\Delta z}\left(\frac{\tilde{Y}(\mathbf{S})(1 - g) - \tilde{C}(\mathbf{S})}{\tilde{K}}\right)\right]\tilde{K}\right\} e^{-\Delta z}, \tag{22}$$

$$\tilde{B}'(\mathbf{S}) = \frac{[1 - dD]\{\pi + (1 - \pi)[\iota + Q_B(\mathbf{S})]\}\tilde{B}e^{-\Delta z} + \tilde{Y}(\mathbf{S})\left[g - \left(t^* + \gamma_\tau \frac{B}{\tilde{Y}(\mathbf{S})}\right)\right]}{Q_B(\mathbf{S})}, \tag{23}$$

$$\tilde{P}'(\mathbf{S}) = R(\mathbf{S})[Q_K(\mathbf{S})\tilde{K}'(\mathbf{S}) + Q_B(\mathbf{S})\tilde{B}'(\mathbf{S}) - \tilde{N}(\mathbf{S})]. \tag{24}$$

The state variable $\tilde{P}$ measures the detrended *cum* interest promised payements of bankers to households at the beginning of the period, and it allows us to keep track of the evolution of aggregate bankers' net worth (See Appendix B). Finally, the exogenous state variables

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[18]Note that we have anticipated earlier the market clearing condition for the labor market and for the capital good market.

[19]The endogenous state variables of the model are detrended using the level of technology last period.

$[\Delta z, \log(g), s]$ follow, respectively, (13), (16) and (21), while d follows

$$d' = \begin{cases} 1 & \text{with probability} \quad \frac{\exp\{s\}}{1+\exp\{s\}} \\ 0 & \text{with probability} \quad 1 - \frac{\exp\{s\}}{1+\exp\{s\}}. \end{cases} \tag{25}$$

I use numerical methods to solve for the model decision rules. The algorithm for the global numerical solution of the model relies on projection methods (Judd, 1992; Heer and Maussner, 2009). In particular, let $x(\mathbf{S})$ be the function describing the behavior of control variable x. I approximate $x(\mathbf{S})$ using two sets of coefficients, $\{\gamma^x_{d=0}, \gamma^x_{d=1}\}$. The law of motion for x is then described by

$$x(d, \tilde{\mathbf{S}}) = (1-d)\gamma^{x\prime}_0 \mathbf{T}(\tilde{\mathbf{S}}) + d\gamma^{x\prime}_1 \mathbf{T}(\tilde{\mathbf{S}}),$$

where $\tilde{\mathbf{S}} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s]$ is the vector of state variables that excludes d, and $\mathbf{T}(.)$ is a vector collecting Chebyshev's polynomials. The coefficients $\{\gamma^x_{d=0}, \gamma^x_{d=1}\}_x$ are such that the residual equations are satisfied for a set of collocation points $(d^i, \tilde{\mathbf{S}}^i) \in \{0,1\} \times \tilde{\mathcal{S}}$. I choose $\tilde{\mathcal{S}}$ and the set of polynomial $\mathbf{T}(.)$ using the Smolyak collocation approach. Krueger and Kubler (2003) and Krueger et al. (2010) provides a detailed description of the methodology. When evaluating the residual equations at the collocation points, I evaluate expectations by "precomputing integrals" as in Judd et al. (2011). Finally, I adopt Newton's method to find the coefficients $\{\gamma^x_{d=0}, \gamma^x_{d=1}\}_x$ satisfying the residual equations. Appendix B provides a detailed description of the algorithm and discusses the accuracy of the numerical solution.

3 Two Simple Examples Illustrating the Mechanisms

It is now useful to describe the mechanisms that tie sovereign risk to the funding costs of firms and to real economic activity. An increase in the probability of a future sovereign default lowers capital accumulation via two distinct channels. First, it makes more difficult for banks to raise funds from households, thus hampering their ability to intermediate productive assets. Second, it signals that the economy is more likely to be in a bad state of the world tomorrow: precautionary motives, then, reduce the willingness of bankers to intermediate the firms' claims.

I illustrate these effects using two stylized versions of the model. First, I discuss the macroeconomic implications of a decline in the current net worth of bankers and a tightening of their leverage constraint (Section 3.1). Second, I show that bad *news* about future net

worth leads bankers to act more cautiously and reduce their holdings of productive assets today. I also discuss how sovereign credit risk triggers these two propagation mechanisms in the full model. Section 3.3 explains the policy relevance of these mechanisms.

3.1 A decline in current net worth

I consider a deterministic economy with full depreciation ($\delta = 0$), no capital adjustment costs ($\xi = 0$) and no government. Moreover, I assume that the transfers to newly born bankers equal a fraction ω of current output, $N = \omega Y$ and that bankers live only one period ($\psi = 0$).

As in the neoclassical model with full depreciation and log utility, the saving rate is constant in this economy. Specializing equation (6) to this particular parameterization, we obtain an expression for the Lagrange multiplier on the incentives constraint

$$\mu = \frac{\lambda \sigma - \omega}{\lambda \sigma},$$

where σ is the saving rate. Using equation (8), we can solve for σ:

$$\sigma = \min \left\{ \frac{\alpha \beta + \omega}{1 + \lambda}, \alpha \beta \right\}.$$

I assume that the leverage constraints is currently binding ($\lambda \beta \alpha > \omega$), and I analyze the implications of an unexpected transitory decline in the transfers to bankers.[20] While analytical solutions for this example can be easily derived, I illustrate the transition to steady state using a numerical example.

The top left panel of Figure 1 plots the equilibrium in the credit market prior to the decline in ω. The supply of funds is derived from the bankers' optimization problem: if the leverage constraint was not binding, bankers would be willing to lend at the risk free rate R since this economy is non-stochastic. The supply of funds to firms is inelastic at $K' = \frac{\alpha}{\lambda} N$, the point at which the leverage constraint binds. The demand for credit is downward sloping and equal to the expected marginal product of capital, $\alpha K'^{\alpha-1} \mathbb{E}[L'^{\alpha}]$. Since the leverage constraint binds, expected returns to capital equal $R(1 + \lambda \mu)$.

The unanticipated decline in ω tightens the leverage constraint, and the inelastic part of the supply schedule shifts leftward. The right panels of the figure describe the adjust-

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[20]More specifically, I assume that at time $t = 1$ the transfer to bankers ω declines, then goes back to its previous level at $t = 2$, and no further changes occur at future dates. Agents do not expect such a change, but are perfectly informed about the path of the transfer from period $t = 1$ ownward and they make rational choices based on this path.

Figure 1: **A decline in current net worth**
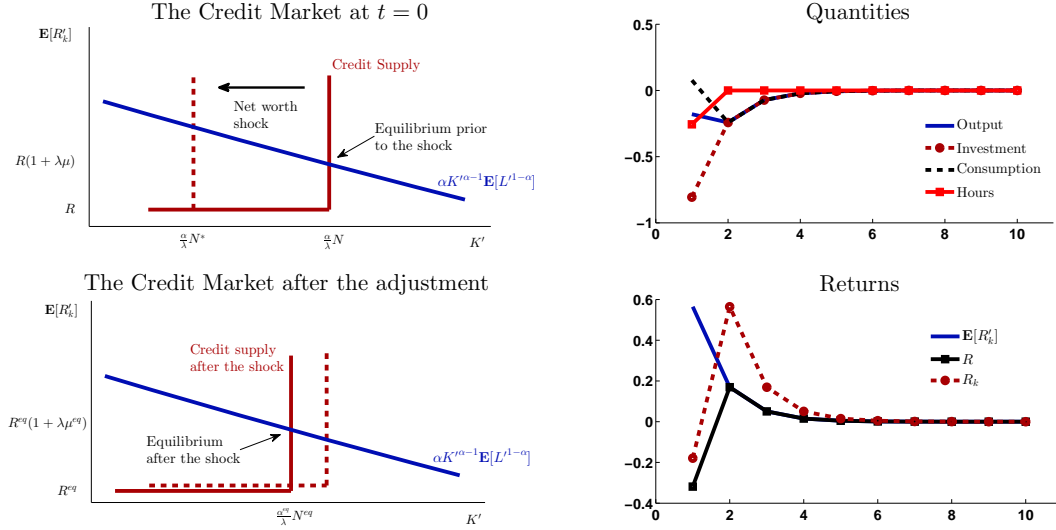
The Credit Market at $t = 0$

Quantities

The Credit Market after the adjustment

Returns

Notes: The figure reports the transitional dynamics induced by a transitory and unexpected 5% decline in net worth. The parametrization adopted is $[\alpha = 0.33, \nu = \infty, \beta = 0.995, \lambda = 0.44, \omega = 0.10]$. The right panels report variables expressed as percentage deviations from their steady state.

ments for quantities and prices. The tightening of credit has adverse effects on capital accumulation. Consumption increases because agency costs makes savings in banks less attractive to households. The risk free rate, as a consequence, declines. Aggregate hours fall because the low returns to savings make working unattractive. The decline in hours leads to a drop in output.

There are three important points to note about this example. First, the leverage constraint generates negative comovement between bankers' marginal value of wealth and the realized returns on holding capital. On the one hand, a unit of wealth is very valuable for bankers when they are facing tight funding constraints, as this unit would allow them to make profits by arbitraging away differences between $\mathbb{E}[R'_K]$ and R. On the other hand, firms' profits decline when bankers face funding constraints, because of the low supply of credit to firms. As we will see in the subsequent analysis, this negative comovement between R_K and α conditional on a tightening of the leverage constraint is the key mechanism generating endogenous risk in the model. Second, variation in bank net worth is amplified in the full model because of endogenous response in Tobin's q. This occurs if there are frictions in the production of capital goods, $\xi > 0$. Brunnermeier et al. (2013) provide a detailed discussion of these amplification effects in models with financial frictions. Third, consumption and output move in opposite directions conditional on a tightening of the leverage constraint. This "comovement problem" arises frequently in neoclassical settings, see Barro and King (1984) for a general formulation, and Hall (2011) and Bigio

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(2012) for specific analysis in models with financial frictions.[21]

An increase in the probability of a future sovereign default in the model of Section 2 triggers a decline in bank net worth and a tightening of their leverage constraint. An increase in s, in fact, implies a decline in the market value of government bonds because investors anticipate a future haircut. Thus, realized returns on government bonds decline. From equation (17) we can see that this effect is stronger the longer the maturity of bonds.[22] Low realized returns on government bonds have a negative impact on bank net worth as we can see from equation (10). Thus, an s-shock can activate the process of Figure 1 through its adverse effects on bank net worth. I will refer to this mechanism as the *liquidity* channel.

3.2   Bad *news* about future net worth

Besides affecting the current constraint of financial intermediaries, an s-shock signals that the constraint is more likely to bind in the future. This affects bankers' incentives to intermediate risky assets. It is helpful at this stage to derive an equilibrium relation describing the pricing of assets in the economy of Section 2. From equation (8) and (5) we find that expected returns to asset j equal

$$\mathbb{E}_{\mathbf{S}}[R_j(\mathbf{S}',\mathbf{S}) - R(\mathbf{S})] = \frac{\lambda \mu(\mathbf{S})}{\mathbb{E}_{\mathbf{S}}[\Lambda(\mathbf{S}',\mathbf{S})]} - \frac{\text{cov}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}',\mathbf{S}), R_j(\mathbf{S}',\mathbf{S})]}{\mathbb{E}_{\mathbf{S}}[\Lambda(\mathbf{S}',\mathbf{S})]}. \tag{26}$$

Equation (26) defines the cross-section of assets' returns. Expected excess returns to capital typically carry a risk premium represented by $\text{cov}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}',\mathbf{S}), R_K(\mathbf{S}',\mathbf{S})]$. As emphasized by Ayiagari and Gertler (1999), this component reacts to changes in the expectation of how tight the leverage constraint will be in the future.

This can be illustrated with a simple modification of the previous set-up. I now allow ψ to be greater than 0. Moreover, I assume that there are two regimes in the economy:

1. *Normal times*: transfers are fixed at their steady state, and the constraint on bank leverage is not binding.

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[21]One way to restore comovement would be to allow the demand for labor to be directly affected by the tightenss of bank leverage constraint. This could be done, for example, by introducing working capital constraint as in Mendoza (2010) and using preferences that mute the wealth effect on labor. While this extention is straightforward to pursue in the current set up, I focus on a benchmark real model for comparability with previous research.

[22]The parameter π also has an indirect effect on the elasticity of R_B to the s-shock, since bond prices are more responsive to this shock when the maturity is longer.

2. *Financial crises*: bankers are hit by the transitory decline in transfers described in the previous section.

I assume that the economy is currently in normal times, and I denote by p the probability that it falls into a financial crisis next period. Once in a financial crisis, the economy experiences the temporary decline in ω described in the previous section. I assume that $p = 0$ at $t = 0$. In period $t = 1$, p unexpectedly increase to 0.1. In period $t = 2$, p returns to 0 and no further changes are anticipated. The agents are surprised by the initial increase in p, but they are aware of its future path from $t = 1$ onward, and they make rational choices based on this path. Figure 2 describes how the credit market and equilibrium quantities are affected by this increase in p.

Figure 2: **Bad *news* about future net worth**
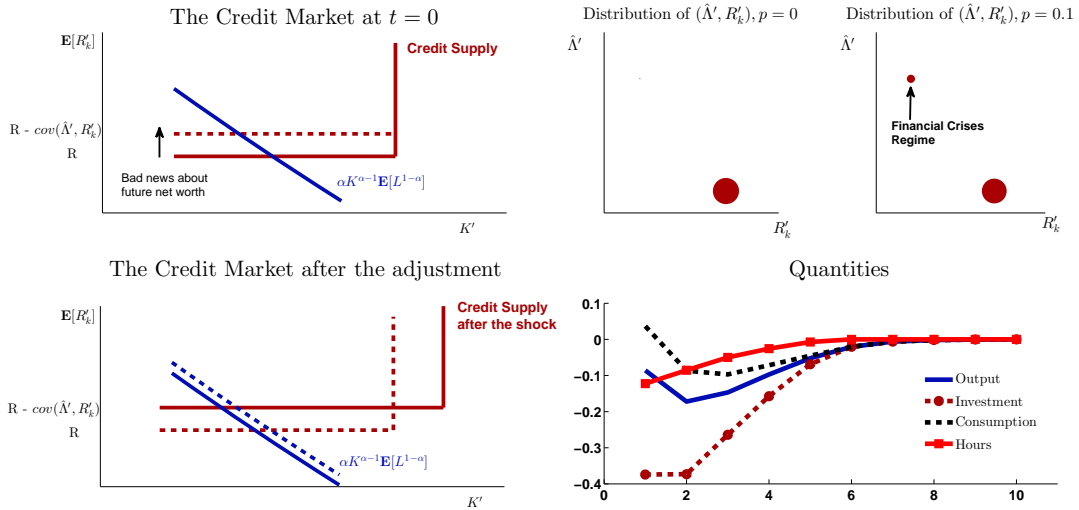
Notes: The figure reports the transitional dynamics induced by a transitory and unexpected increase in p from 0 to 0.10. The parametrization adopted is $[\alpha = 0.33, \nu = \infty, \beta = 0.995, \lambda = 0.44, \omega = 0.10, \psi = 0.95]$. The bottom right panel reports variables expressed as percentage deviations from their steady state.

The increase in p shifts the elastic component of the credit supply schedule upward because of a decline in $\text{cov}(\hat{\Lambda}', R_k')$. The top right panels of the figure explain where this change in the covariance originates. The first panel plots the joint distribution for $(\hat{\Lambda}', R_k')$ conditional on being in normal times when $p = 0$. This is a point distribution: the pricing kernel equals β while realized returns to capital are equal to β^{-1}. When p increases to 0.1, bankers assign a higher probability of falling into a financial crisis next period. As discussed in the previous section, realized returns to capital are low in this state while bankers' marginal valuation of wealth is high. Therefore, as p increases, bankers perceive firms' claims to be more risky: if the crisis occurs next period, these claims will pay out

little precisely when bankers are most in need of wealth. For this reason, banks have a precautionary motive to take out these claims from their balance sheet when the economy is approaching a financial crisis. The bottom right panel describes the macroeconomic consequences of this increased precaution of bankers.

A sovereign default in the model of Section 2 resembles the financial crisis regime discussed here: banks suffer large losses on their government bond holdings. Claims on firms pay off badly in this state because of low firm profits and of a decline in their market value. These low payouts are highly discounted by bankers because they are already facing large balance sheet losses, and their marginal value of wealth is high. When the likelihood of this event increases, banks have a precautionary incentive to reduce their holdings of firms' claims because the economy is approaching a state where these assets are not particularly valuable. This deleveraging leads to a decline in capital accumulation. I refer to this second mechanism through which sovereign credit risk propagates to the real economy as the *risk channel*.

3.3 Policy relevance

While these two propagation mechanisms have similar implications for quantities and prices, they carry substantially different information. This can be seen by comparing the credit markets in Figure 1 and Figure 2. In Figure 1, excess returns on firms' claims increase because banks cannot rise enough funds to undertake profitable investment opportunities: if they had an additional unit of wealth, they would invest it in firms' claims. In Figure 2, instead, excess returns reflect fair compensation for increased risk: bank leverage constraint are not binding, and there are no unexploited profitable opportunities.

This distinction has important implications for the evaluation of credit policies in the model. For example, it is reasonable to expect that an injection of equity to the banking sector may be more effective in stimulating banks' lending when these latter are facing tight constraints on their leverage, while their aggregate implications may be muted when precautionary motives are strong. We will see in Section 5 that this intuition holds in the model. First though, I move to the empirical analysis.

4   Empirical Analysis

The model is estimated using Italian quarterly data (1999:Q1-2011:Q4). This section proceeds in three steps. Section 4.1 describes the data used in estimation. Section 4.2 illus-

trates the estimation strategy. I place a prior on parameters and conduct Bayesian inference. Because of the high computational costs involved in solving the model repeatedly, I adopt a two-step procedure. In the first step, I estimate a version of the model without sovereign default risk on the 1999:Q1-2009:Q4 subsample. In the second step, I estimate the parameters for the $\{s_t\}$ shock using a time series for sovereign default probabilities for the Italian economy. Section 4.3 presents basic diagnostics regarding model fit.

4.1 Data

While the previous section has described qualitatively the mechanisms of interest, their quantitative relevance rests on the numerical value of the model parameters, in particular those governing the stochastic properties of sovereign risk, the exposure of financial intermediaries to this risk and the macroeconomic implications of the financial friction considered. I now describe the data series that I use to inform these model parameters.

First, I ensure that the time-varying nature of sovereign risk in the model is realistic. For this purpose, I use CDS spreads on Italian government securities with a five-year maturity. This time series is available at daily frequencies starting in January 2001 from Markit. See Appendix C.1 for further details.

Second, I measure the exposure of banks to this risk. I collect data on holdings of domestic government debt by the five largest Italian banks using the 2011 stress test of the European Banking Authority.[23] As detailed in Appendix C.2, these data include holdings of domestic government securities, loans to central government and local authorities and other provisions, and these items are classified in terms of their maturity. Moreover, I correct for the possibility that banks insured against the risk of an Italian default by netting out their positions in derivative contracts. I match this information with end of 2010 consolidated balance sheet data obtained from Bankscope, which allows me to measure how exposed were these five banks to the Italian government as a fraction of their total assets.

Third, I construct a model consistent indicator of agency costs. More specifically, I use a subset of the model's equilibrium conditions to relate the Lagrange multiplier associated to the leverage constraint of banks to a set of observable variables. As detailed in in Appendix C.3, this Lagrange multiplier can be expressed as a function of financial leverage, lev_t, and of the spread between a risk free security that is traded *only* by bankers and the

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[23]The five banks are: Unicredit, Intesa-San Paolo, MPS, BPI and UBI. Their total assets at the end of 2010 accounted for 82% of the total assets of domestic banking groups in Italy.

risk free rate, $\frac{R_t^f - R_t}{R_t^f}$

$$\mu_t = \frac{\left[\frac{R_t^f - R_t}{R_t}\right] \text{lev}_t}{1 + \left[\frac{R_t^f - R_t}{R_t}\right] \text{lev}_t}. \tag{27}$$

Intuitively, the leverage constraint prevents bankers from exploiting profitable arbitrage opportunities. Therefore, whenever binding, the constraint generates a wedge between expected excess returns on *any* asset intermediated by bankers, and their cost of funds. Differently from risky assets, which carry a compensation for risk, spreads on riskless securities will purely reflect the inability of intermediaries to borrow and, as such, they are mostly informative about the tightness of the constraint.

Figure 3: **Lagrange multiplier and GDP growth: 1999:Q1-2012:Q4**
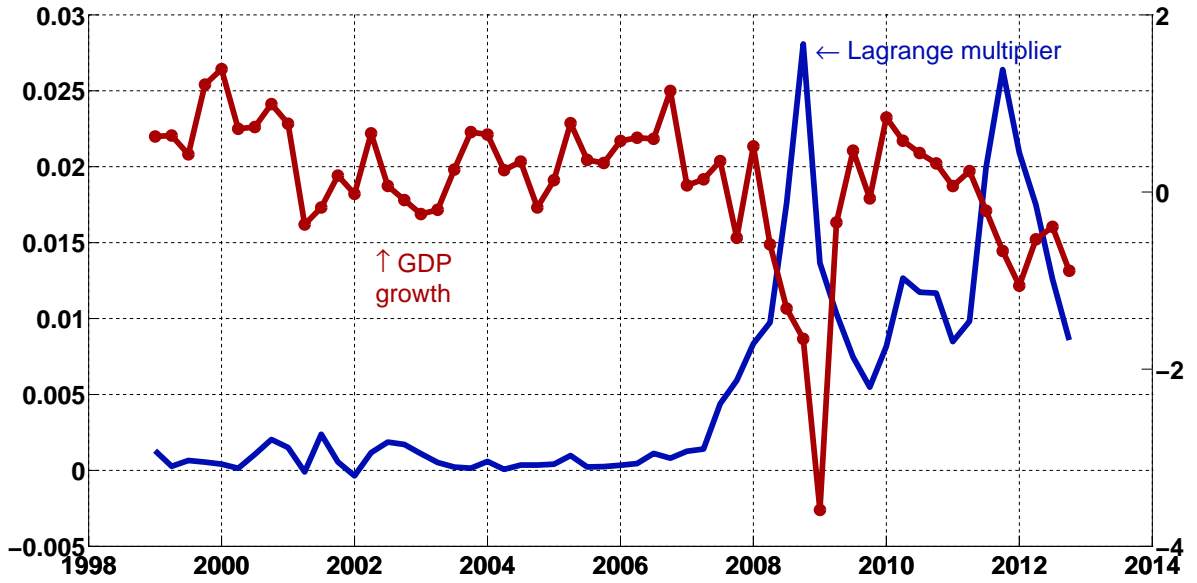
Notes: The Lagrange multiplier on banks' leverage constraint is the solid line (left axis). The circled line is GDP growth (right axis). Appendix C provides information on data sources.

I use equation (27) to generate a time series for μ_t. I measure R_t^f with the prime rate on interbank loans for Italian banks belonging to the EURIBOR panel.[24] The risk free rate R_t is matched with the yields on German government securities. The leverage of financial intermediaries is measured using the Italian flow of funds. Appendix C.3 describes in detail the steps involved in measuring μ_t. Figure 3 reports this time series along with

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[24]This is the natural rate to consider in the current set-up. Indeed, we can interpret the model of Section 2 as having a frictionless interbank market of the type considered in Gertler and Kiyotaki (2010) without altering its equilibrium conditions. In this set-up, R_t^f would be the rate at which bankers trade claims between themselves, or the interbank rate.

GDP growth. Two main facts stand out from a visual inspection of the figure. First, the Lagrange multiplier is countercyclical, rising substantially in periods in which GDP growth is markedly below average. Second, it is very close to 0 until 2007:Q2. Thus, the constraint seems to bind only occasionally in sample.

Clearly, these three sets of data are not informative for all model parameters. Thus, I complement them with time series for the labor income share, the investment-output ratio, the government spending-output ratio and hours worked. Appendix C.4 provides detailed definitions and data sources.

4.2 Estimation strategy

I denote by $\theta \in \Theta$ the vector of model parameters. It is convenient to organize the discussion around the following partition, $\theta = [\theta_1, \theta_2]$

$$\theta_1 = \left[\mu^{bg}, \psi, \xi, \sigma_z, \rho_z, \gamma, \pi, g^*, \rho_g, \sigma_g, \gamma_t, \nu, \alpha, \frac{i^{bg}}{y^{bg}}, l^{bg}, lev^{bg}, R^{bg}, exp^{bg}, q_b^{bg}, adj^{bg} \right], \quad \theta_2 = [D, s^*, \rho_s, \sigma_s].$$

Conceptually, we can think of θ_1 as indexing a restricted version of the model without sovereign risk, while θ_2 collects the parameters determining the sovereign default process.[25]

While a nonlinear analysis of the model is necessary to capture time variation in risk premia and the fact that the leverage constraint binds only occasionally, it complicates inference substantially since repeated numerical solutions of the model are computationally very costly. I therefore estimate θ using a two-step procedure. In the first step, I infer θ_1 by estimating the model without sovereign risk on the 1999:Q1-2009:Q4 subsample. This restricted version of the model has fewer state variables and it is easier to analyze numerically.[26] Moreover, focusing on the restricted model over the 1999:Q1-2009:Q4 subsample should not alter substantially the inference over θ_1. During this period, in fact, CDS spreads on 5 years Italian government bonds averaged only 27 basis points, and the decision rules of the restricted model closely approximate those of the full model if we condition on small sovereign default probabilities. In the second step, and conditional on the first step parameters, I estimate θ_2 using a retrieved time series of sovereign default

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[25]Notice that I have reparametrized $[\lambda, \omega, \delta, \chi, \iota, \tau^*, a_1, a_2]$ with balanced growth values for, respectively, the Lagrange multiplier on leverage constraints (μ^{bg}), the leverage ratio (lev^{bg}), the investment-output ratio $\left(\frac{i^{bg}}{y^{bg}} \right)$, worked hours ($l^{bg}$), the price of government securities (q_b^{bg}), the ratio of government securities held by bankers to their total assets (exp^{bg}) and the size of capital adjustment costs (adj^{bg}).

[26]The full model, in fact, requires the solution of a system of 3112 nonlinear equations. The restricted model, instead, is characterized by 964 equations.

probabilities.

Note that this two-step approach does not use the correlation between GDP growth and CDS spreads when estimating θ. This can be an attractive feature relative to a full information approach. Because sovereign risk is assumed to be exogenous, in fact, the model would interpret this moment as purely reflecting the causal effect of sovereign risk on real economic activity and a full information approach would use it to inform parameters values. Therefore, the two-step procedure adopted here is more robust to potential misspecifications of the sovereign default process described in Section 2.1.5.

4.2.1 Estimating the model without sovereign risk

The model without sovereign risk has five state variables $\mathbf{S}_t = [\hat{K}_t, \hat{P}_t, \hat{B}_t, \Delta z_t, g_t]$. The parameters are

$$
\theta_1 = \left[\underbrace{\mu^{bg}, \psi, \xi, \sigma_z, \rho_z, \gamma}_{\tilde{\theta}_1}, \underbrace{\pi, g^*, \rho_g, \sigma_g, \gamma_t, \nu, \alpha, \frac{i^{bg}}{y^{bg}}, l^{bg}, lev^{bg}, R^{bg}, exp^{bg}, q_b^{bg}, adj^{bg}}_{\theta_1^*} \right].
$$

I construct the likelihood function of the model using time series for GDP growth and the Lagrange multiplier on the bankers' leverage constraint described earlier. As explained in Section 3, the cyclical behavior of the model's financial friction is important to assess the impact of sovereign risk on the real economy: a likelihood-based approach guarantees a high degree of consistency between the model implied behavior for these variables and their data counterparts.

This choice has some limitations. First, I am discarding potentially important information as one could incorporate the components of the Lagrange multiplier into the likelihood function: the TED spread and the leverage of banks. There are, however, some good reasons for doing so. First, tracking financial leverage over time would provide little additional information about the financial friction since μ_t is mostly driven by the TED spread over the 1999:Q1-2009:Q4 subsample, see Figure A-3 in Appendix C.3. Second, this version of the model is too restrictive to track the time series behavior of financial leverage in the earlier part of the sample. This happens because technology shocks and government spending shocks do not generate enough variation in asset prices when the constraint is far from binding unless the production function of capital goods has strong curvature. By forcing the model to track financial leverage, one would obtain large estimates for ξ, which would have counterfactual business cycle implications and they would

tend to increase the relative importance of the risk channel.[27]

Second, certain model parameters are only weakly affected by the information in the likelihood function and their identification is problematic. For this reason, and prior to conduct full information inference, I determine a subset of θ_1, θ_1^*, prior to the estimation. Table 1 reports the numerical values for these parameters. I set $[\frac{i^{bg}}{y^{bg}}, lev^{bg}, l^{bg}, R^{bg}]$ to the sample average of their empirical counterparts while α is determined using the sample average of the labor income share. I use Table A-1 in Appendix C.2 to determine $[exp^{bg}, \pi]$: holdings of government securities account for 7.6% of banks' total assets in the model, and the average maturity of those bonds is set to 18 months. I select $[g^*, \rho_g, \sigma_g]$ from the estimation of an AR(1) on the spending-output ratio over the 1999:Q1-2009:Q4 period. The remaining parameters in θ^* are determined through normalizations or previous research. I set the Frisch elasticity of labor supply to 2 and γ_τ to 1. The former is in the high range of the estimates obtained using U.S. data (Rios-Rull et al., 2012), but it is not an uncommon value in the profession for the analysis of Real Business Cycle models. The choice of γ_τ has limited effects on aggregate dynamics since taxes are lump sum in the model. Following common practice in the profession, I set adjustment costs to zero in a balanced growth path while I normalize q_b^{bg} to 1.

Table 1: **Parameters determined with external information**

	Parameters				**Source**
$\frac{i^{bg}}{y^{bg}}$	lev^{bg}	l^{bg}	R^{bg}	α	OECD, EU-KLEMS,
0.20	4.34	0.32	1.0029	0.30	ECB, BoI
exp^{bg}	π				EBA,
0.076	0.056				Bankscope
e^{g^*}	ρ_g	σ_g			OECD
0.21	0.97	0.009			
q_b^{bg}	adj^{bg}	ν	γ_τ		Normalizations,
1	0	2	1		Previous Research

Notes: See Appendix C for information on data sources.

I next turn to the estimation of $\tilde{\theta}_1 = [\mu^{bg}, \psi, \xi, \gamma, \rho_z, \sigma_z]$. Let $\mathbf{Y}_t = [\text{GDP Growth}_t, \mu_t]'$,

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[27] A large value of ξ makes Tobin's Q more sensitive to variation in capital demand. Thus, bankers make more losses on their holdings of firms' claims when a sovereign default occurs. Ex-ante, they will demand higher compensation for holding firms' assets when the likelihood of a future sovereign default increases.

and let $\mathbf{Y}^\tau = [\mathbf{Y}_1, \ldots, \mathbf{Y}_\tau]'$. The model defines the nonlinear state space system

$$
\begin{aligned}
\mathbf{Y}_t &= f_{\tilde{\theta}_1}(\mathbf{S}_t) + \eta_t & \eta_t &\sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}) \\
\mathbf{S}_t &= g_{\tilde{\theta}_1}(\mathbf{S}_{t-1}, \varepsilon_t) & \varepsilon_t &\sim \mathcal{N}(\mathbf{0}, \mathbf{I}),
\end{aligned}
$$

where η_t is a vector of measurement errors and ε_t are the structural shocks.[28] I approximate the likelihood function of this nonlinear state space model using sequential importance sampling (Fernández-Villaverde and Rubio-Ramírez, 2007).[29] The posterior distribution of model parameters is

$$
p(\tilde{\theta}_1 | \mathbf{Y}^T) = \frac{\mathcal{L}(\tilde{\theta}_1 | \mathbf{Y}^T) p(\tilde{\theta}_1)}{p(\mathbf{Y}^T)},
$$

where $p(\tilde{\theta}_1)$ is the prior, $\mathcal{L}(\tilde{\theta}_1 | \mathbf{Y}^T)$ the likelihood function and $p(\mathbf{Y}^T)$ the marginal data density. I characterize the posterior density of $\tilde{\theta}_1$ using the Random Walk Metropolis Hastings developed in Schorfheide (2000) with an adaptive variance-covariance matrix for the proposal density. Appendix D provides a description of the estimation algorithm. Table 2 reports the prior along with posterior statistics for $\tilde{\theta}_1$.

Table 2: **Prior and Posterior distribution of $\tilde{\theta}_1$**

Parameter	Prior	Para 1	Para 2	Posterior Mean	90% Credible Set
$\mu^{bg} \times 100$	Uniform	0	∞	0.37	[0.11,0.66]
ψ	Uniform	0	1	0.97	[0.92,0.98]
ξ	Beta	0.5	0.25	0.43	[0.19,0.86]
$\gamma \times 400$	Normal	1.25	0.5	0.98	[0.17,1.80]
ρ_z	Beta	0.3	0.25	0.51	[0.20,0.74]
$\sigma_z \times 100$	Inverse Gamma	0.75	2	0.55	[0.44,0.74]

Notes: Para 1 and Para 2 list the mean and standard deviation for Beta and Normal distribution; and s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma | \nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2 / 2\sigma^2}$. The prior on γ is truncated at 0. Posterior statistics are computed using 10000 draws from the posterior distribution of model's parameters. The table reports equal tail probability 90% credible sets.

---

[28]The functions $g_{\tilde{\theta}_1}(.)$ and $f_{\tilde{\theta}_1}(.)$ are implicitly defined by the numerical solution of the model that does not feature sovereign credit risk. Since θ_1^* is fixed, I omit from the notation the dependence of decision rules on these parameters.

[29]I use the auxiliary particle filter of Pitt and Shephard (1999) which, in this application, substantially improves the efficiency of the likelihood evaluation. See Aruoba and Schorfheide (2013) for a recent application to economics. I consider a diagonal matrix $\mathbf{\Sigma}$ where the nonzero elements are equal to 25% of the sample variance of $\{\mathbf{Y}_t\}$. Appendix D provides a description of the evaluation of the model's likelihood function.

The prior on the TFP process is centered using presample evidence while I center ξ to 0.5, a conventional value in the literature. Priors on these three parameters are fairly diffuse. I choose uniform priors over μ^{bg} and ψ, implying that the shape of the posterior is determined by the shape of the likelihood. Regarding posterior estimates, the Lagrange multiplier is estimated to be close to 0 in a deterministic balanced growth path. This suggests that agency costs are fairly small on average in the model. This is not surprising given the time series behavior of μ_t in Figure 3. Capital adjustment costs and the TFP process are in the range of what is typically obtained in the literature when using U.S. data.

4.2.2 Estimating sovereign risk

I next turn to the estimation of $\theta_2 = [D, s^*, \rho_s, \sigma_s]$. The empirical strategy consists of i) constructing a time series for the probabilities of a sovereign default and ii) using this time series, along with equation (20) and (21), to estimate θ_2.

I accomplish the first task by exploiting the pricing equation of the model. Using equation (8) and equation (5), we can define the *risk neutral* measure as:[30]

$$\hat{p}(\mathbf{S}'|\mathbf{S}) = \frac{R^f(\mathbf{S})p(\mathbf{S}'|\mathbf{S})\hat{\Lambda}(\mathbf{S}',\mathbf{S})}{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})] + \lambda\mu(\mathbf{S})}.$$

After integrating the above expression over states $\mathbf{S}'$ associated with a sovereign default next period, I obtain an expression for the actual probability of a sovereign default, p_t^d. This time series is related to its risk neutral counterpart, $\hat{p}_t^d$, as follows

$$p_t^d = \hat{p}_t^d \frac{\alpha_t(1 - \mu_t) + \lambda\mu_t}{R_t^f \mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]}. \tag{28}$$

Equation (28) is important because it allows us to measure actual probabilities of sovereign default using empirical counterparts of its right hand side.

First, we can obtain a time series for $\{\hat{p}_t^d\}$ using CDS spread on Italian government securities, up to a normalization of the parameter D. More specifically, I compute risk neutral probabilities by scaling quarterly CDS spreads with the recovery rate $(1 - D)$. The haircut D is fixed at 0.55, consistent with the Greek debt restructuring of 2012 (Zettelmeyer et al., 2013).

---

[30]Note that $\hat{p}(\mathbf{S}'|\mathbf{S})$ is nonnegative and it integrates to 1. To see the last property, note that the return on a risk free security traded by bankers can be written as $R^f(\mathbf{S}) = \frac{\alpha(\mathbf{S})[1-\mu(\mathbf{S})]+\lambda\mu(\mathbf{S})}{\mathbb{E}_\mathbf{S}[\hat{\Lambda}(\mathbf{S}',\mathbf{S})]}$ using equation (8) and equation (5).

Second, I construct a time series for $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$. This is a difficult task because of the absence of a sovereign default in the sample. I indirectly use the model's restrictions to conduct this extrapolation. In particular, I approximate this object as follows

$$\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1] \approx \mathbb{E}_t[\hat{\Lambda}_{t+1}] + \kappa \mathrm{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}, \tag{29}$$

where $\kappa > 0$ is a hyperparameter. The idea of equation (29) is that the marginal value of wealth for bankers is above average in the event of a sovereign default because they are more likely to face funding constraints: κ parameterizes the number of standard deviations by which $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$ is greater than $\mathbb{E}_t[\hat{\Lambda}_{t+1}]$.

I use the model restrictions to generate the terms $\{\mathbb{E}_t[\hat{\Lambda}_{t+1}], \mathrm{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}\}$. From equation (7) and (9), we can write $\hat{\Lambda}_t$ as a function of observables and of model parameters estimated in the first step

$$\hat{\Lambda}_t = \beta e^{-\Delta \log(c_t)}[(1 - \psi) + \psi \lambda \mathrm{lev}_t], \tag{30}$$

where Δc_t is the growth rate of real personal consumption expenditure and lev_t is the leverage of financial intermediaries. I first estimate a first order Bayesian Vector Autoregressive model on consumption growth and financial leverage, and I generate conditional forecasts of $[\Delta c_{t+1}, \mathrm{lev}_{t+1}]$. I then use these conditional forecasts, the posterior mean of $[\beta, \psi, \lambda]$ and equation (30) to generate $\{\mathbb{E}_t[\hat{\Lambda}_{t+1}], \mathrm{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}\}$. Finally, I use these estimates and equation (29) to approximate $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$. The hyperparameter κ is selected with the help of the structural model. I consider a set of values $\kappa^i \in \{1, 3, 5\}$ and I select the value that minimizes, in model simulated data, average root mean square errors for the approximation of $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$. This gives a value of $\kappa = 3$.

Third, I combine the retrieved time series for $\{\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]\}$ with lev_t, μ_t and R_t^f to generate the risk correction. I make use of the fact that the marginal value of wealth for bankers is proportional to financial leverage when the constraint binds, and measure the risk correction as follows:

$$\frac{\lambda \mathrm{lev}_t(1 - \mu_t) + \lambda \mu_t}{R_t^f\{\mathbb{E}_t[\hat{\Lambda}_{t+1}] + \kappa \mathrm{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}\}}. \tag{31}$$

Figure 4 plots $\{p_t^d\}$ along with its decomposition of equation (28) for the different values of κ. The top left panel reports the risk neutral probabilities, the bottom-left panel plots the risk correction and the right panel reports the time series for actual sovereign default probabilities. The estimates imply that roughly 30% of actual sovereign default

probabilities in the sample is due to risk premia.

Figure 4: **Sovereign default probabilities**
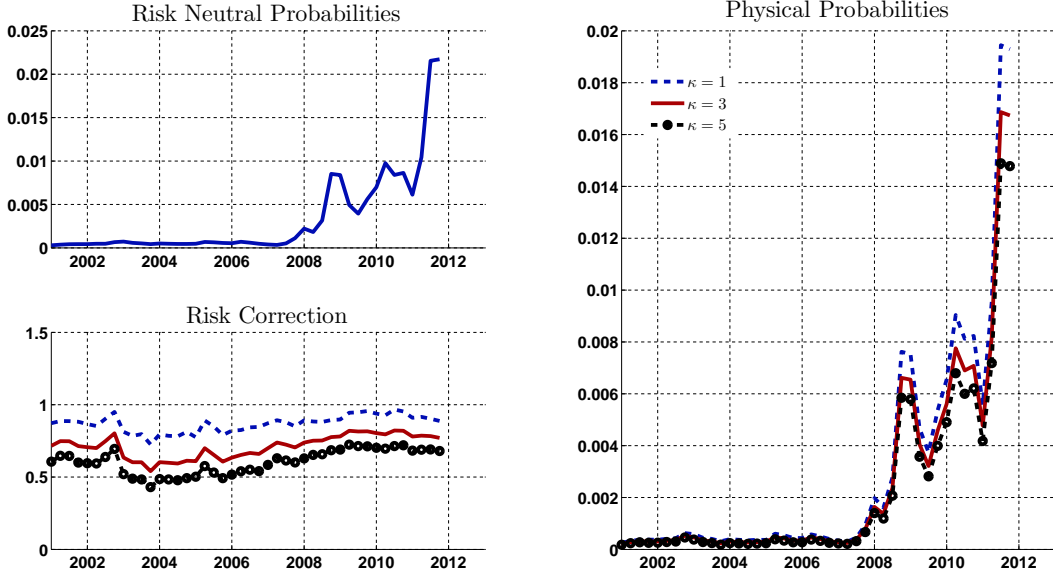
Notes: The top left panel reports risk neutral probabilities of a sovereign default. The bottom right panel reports the risk correction, defined in equation (31). The right panel reports actual probabilities of a sovereign default, defined in equation (28).

I then use $\{p_t^d\}$ to estimate the parameters of the sovereign risk shock s_t. Indeed, the two are related in the model as follows

$$\log\left(\frac{p_t^d}{1 - p_t^d}\right) = s_t, \qquad s_t = (1 - \rho_s)s^* + \rho_s s_{t-1} + \sigma_s \varepsilon_{s,t}, \qquad (32)$$

where $\varepsilon_{s,t}$ is a standard normal random variable. I use the Kalman filter to evaluate the likelihood function of this linear state space model. Table 3 reports prior and posterior statistics for $[s^*, \rho_s, \sigma_s]$. As I do not have presample information, I consider fairly uninformative priors. Posterior statistics are computed from a canonical Random Walk Metropolis Hastings algorithm.

4.3 Model fit

I now verify whether model simulated trajectories for the Lagrange multiplier, GDP growth and sovereign default probabilities resemble those observed in the data. This is accomplished through posterior predictive checks.[31] I use simulation techniques to ob-

---

[31]See Geweke (2005) for a general discussion of predictive checks in Bayesian analysis and Aruoba et al. (2013) for a recent application to the evaluation of estimated nonlinear Dynamic Stochastic General

Table 3: **Prior and Posterior distribution of** $[s^*, \rho_s, \sigma_s]$

Parameter	Prior	Para 1	Para 2	Posterior Mean	90% Credible Set
s^*	Normal	-7	5	-6.67	[-9.17,-4.11]
ρ_s	Beta	0.5	0.3	0.97	[0.93,0.99]
σ_s	Inverse Gamma	0.75	4	0.41	[0.35,0.49]

Notes: Para 1 and Para 2 are, respectively, the mean and standard deviation for Beta and Normal distribution; and s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. Posterior statistics are computed using 10000 draws from the posterior distribution of model parameters. The table reports equal tail probability 90% credible sets.

tain model implied distributions for a set of sample statistics. I then ask how the same statistics, computed from actual data, compare with the model implied distributions.

First, I examine the performance of the model regarding GDP growth and the multiplier. I summarize their joint behavior using the following sample statistics: mean, standard deviation, first order autocorrelation, skewness, kurtosis and their correlation. These are collected in $\mathcal{S}$. The model implied densities for $\mathcal{S}$ are generated using the following algorithm

Posterior predictive densities: Let θ^i denote the i'th draw from the posterior density of the model's parameter. For $i = 1$ to M

1. Conditional on θ^i simulate a realization for GDP growth and the Lagrange multiplier of length T=100.[32] Let $\mathbf{Y}^T(i)$ denote this realization.

2. Based on the simulated trajectories $\mathbf{Y}^T(i)$, compute a set of sample statistics $\mathcal{S}^i$. □

Given the draws $\{\mathcal{S}^i\}$, I use percentiles to describe the predictive density $p(\mathcal{S}(.)|\mathbf{Y}^T)$. Figure 5 shows the 5th and 95th percentile of the model implied density (the box) along with its median (the bar) and their sample counterpart (the dot).

The model generates trajectories for the Lagrange multiplier and GDP growth whose moments are in line with those observed in the data. The main discrepancy with the data is in the excess kurtosis for the GDP growth trajectory: the model is too restrictive to replicate this feature of the data. In addition, the model captures part of the left skewness of GDP growth. This derives from two properties: the amplification of the leverage constraint and the fact that it binds in recessions. In fact, GDP growth is more sensitive to structural shocks when the leverage constraint binds. Since these constraints are only

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Equilibrium models.

[32]These simulations are generated from the restricted model (no sovereign risk). Simulations are initialized at the ergodic mean of the state vector.

Figure 5: **Posterior predictive checks: Lagrange multiplier and GDP Growth**
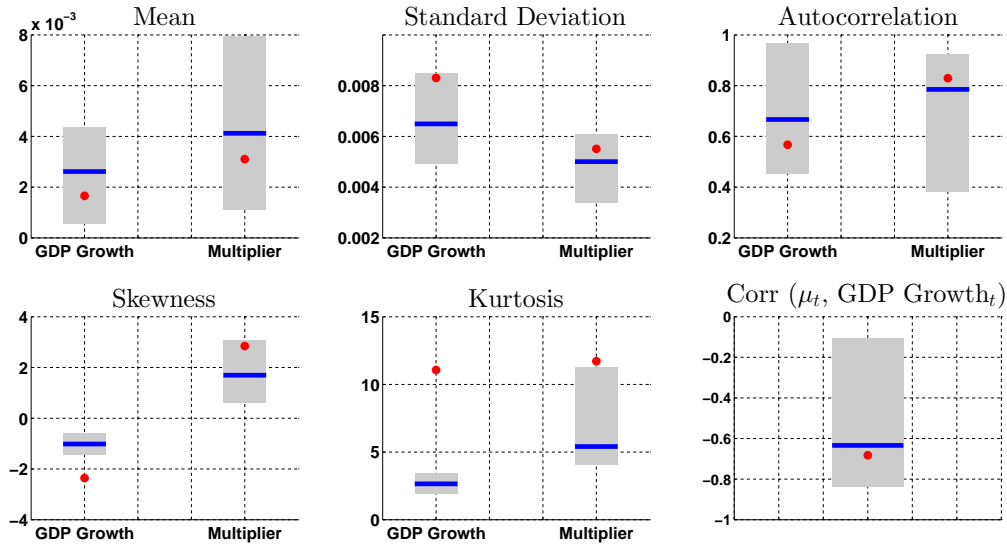
Notes: Dots correspond to the value of the statistic computed from actual data. Solid horizontal lines indicate medians of posterior predictive distribution for the sample statistic and, the boxes indicate the equal tail 90% credible set associated with the posterior predictive distribution.

occasionally binding, this amplification generates asymmetry in the unconditional distribution for GDP growth. Left skewness is then the result of GDP growth and the Lagrange multiplier being negatively correlated.

Second, I ask whether the behavior of sovereign default probabilities in the model is in line with that observed in the data. The posterior predictive checks are reported in Table 4. We can verify that the logistic model used captures key features of the empirical distribution of sovereign default probabilities.

Table 4: **Posterior predictive checks: sovereign default probabilities**

Statistic	Data	Posterior Median	90% Credible Set
Median	0.04	0.13	[0.01,3.07]
Mean	0.25	0.24	[0.03,6.20]
Standard Deviation	0.40	0.24	[0.03,6.58]
Autocorrelation	0.91	0.86	[0.73,0.93]
Skewness	2.18	1.88	[0.85,3.52]
Kurtosis	7.69	6.55	[3.01,17.8]

Notes: Based on 1000 draws from the posterior distribution of $[s^*, \rho_s, \sigma_s]$. For each draw, I simulate the $\{s_t\}$ process for 100 periods. Statistics are computed on each of these 1000 samples. The table reports the posterior median and equal tail probability 90% credible set for the posterior predictive distributions.

Overall, the results in this section suggest that: i) the cyclical behavior of the leverage constraint in the estimated model is empirically reasonable; and that ii) agents in the model have beliefs about the time-varying nature of sovereign credit risk that closely track what was observed in the data.

5 Model Analysis

This section analyzes some properties of the estimated model that are important for the interpretation of the main experiments of this paper, which will be presented in Section 6. There are three key points emerging from this analysis. First, a sovereign default triggers a deep decline in real economic activity. This happens because the haircut on government bonds tightens the funding constraints of banks and leads to a fall in aggregate investment (Section 5.1). Second, an increase in the probability of a future sovereign default leads to an increase in the financing premia of firms by i) tightening the funding constraints of bankers (*liquidity channel*), and by ii) increasing the required risk premia demanded by bankers to hold firms' assets (*risk channel*). This increase in the financing premia of firms is associated with a decline in capital accumulation and output (Section 5.2). Third, the aggregate effects of equity injections into the banking sector are highly state dependent. These interventions are more successful in stimulating real economic activity when bankers are facing severe funding constraints. Conversely, these policies have substantially weaker effects when bankers perceive firms to be very risky (Section 5.3).

Since the aim of this section is purely illustrative, the model's parameters are fixed at their posterior mean.

5.1 A sovereign default

Figure 6 shows the behavior of key model's variables around a typical sovereign default. I apply event study techniques to the simulated time series and report their average path around the default. The window covers 10 quarters before and after the event.

At $t = 0$ the government imposes a haircut on bondholders. As a consequence, bank net worth declines and the leverage constraint tightens, thus forcing them to reduce their holdings of firms' assets. This has adverse effects on aggregate investment and output: they are, respectively, 20% and 2% below their trend at $t = 0$. From $t = 1$ onward, bank net worth recovers because excess returns are above average. This loosens the leverage constraint, and the economy slowly returns to its balanced growth path.

Figure 6: **A sovereign default**
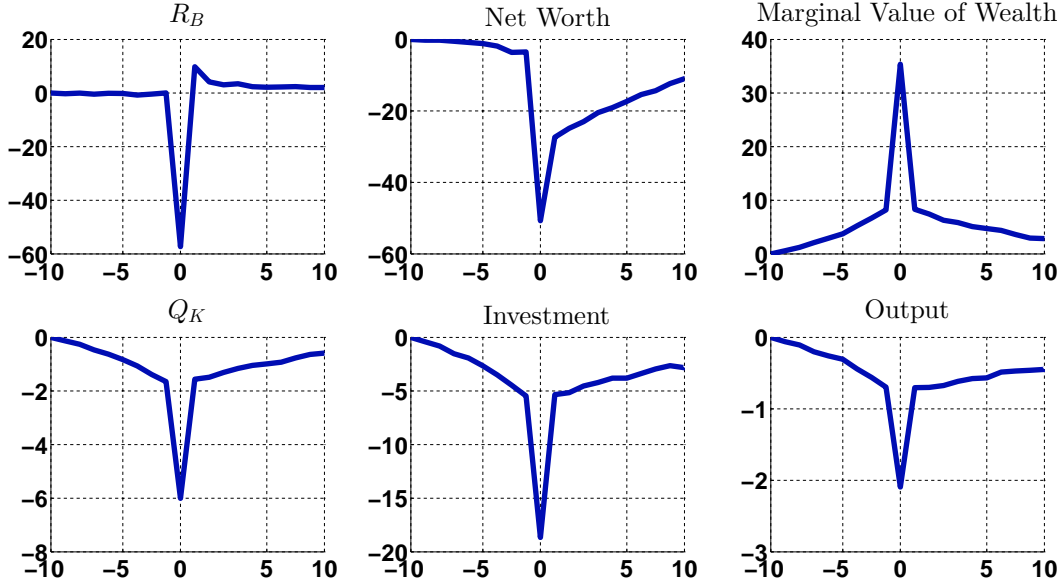
It is important to stress two important facts about a sovereign default in the model. First, the behavior of asset prices substantially amplifies the impact of the sovereign default on real economic activity. The tightening of the leverage constraint forces banks to restrict lending to firms. The associated decline in capital demand puts downward pressure on asset prices because of Tobin's Q and this decline in the market value of firms' claims further depresses bank net worth. As we can see from the figure, the market value of firms is 6% below trend at $t = 0$, while bank net worth declines by roughly twice the size of the haircut imposed by the government. Second, as the top-right panel of the figure shows, the marginal value of wealth for bankers is high during a sovereign default because bankers are facing tight funding constraints. It is also interesting to note that a sovereign default is preceded by a deep slowdown in real economic activity, which conforms with historical evidence on these episodes, see Levy-Yeyati and Panizza (2011). This observation is typically rationalized in the literature via a selection argument: equilibrium models of sovereign defaults predict that incentives for the government to renege on debt are high in bad economic times, see Arellano (2008) and Mendoza and Yue (2012) for example. In the model analyzed here, though, the "V" shape behavior of output around a default event occurs purely because of *anticipation* effects: increases in the likelihood of a

future sovereign default are, in fact, recessionary. The next section explains why.

5.2 An increase in the probability of a future sovereign default

From equation (26) we obtain a decomposition of expected excess returns to capital into two pieces: a liquidity premium and a risk premium.

$$\underbrace{\frac{\mathbb{E}_t[R_{K,t+1} - R_t]}{R_t}}_{\text{EER}_t} = \underbrace{\frac{\lambda \mu_t}{\alpha_t[1 - \mu_t]}}_{\text{Liquidity premium}_t} - \underbrace{\frac{\text{cov}_t[\hat{\Lambda}_{t+1}, R_{K,t+1}]}{\alpha_t[1 - \mu_t]}}_{\text{Risk premium}_t}. \tag{33}$$

According to equation (33), expected excess returns can be high because of two distinct sources. First, banks may face tight funding constraints and this restricts the flow of funds to firms (liquidity premium). Second, banks may require a premium for lending to firm because this intermediation is risky (risk premium). Both of these premia are sensitive to news about a future sovereign default.

Figure 7: **IRFs to an s-shock: financial variables**
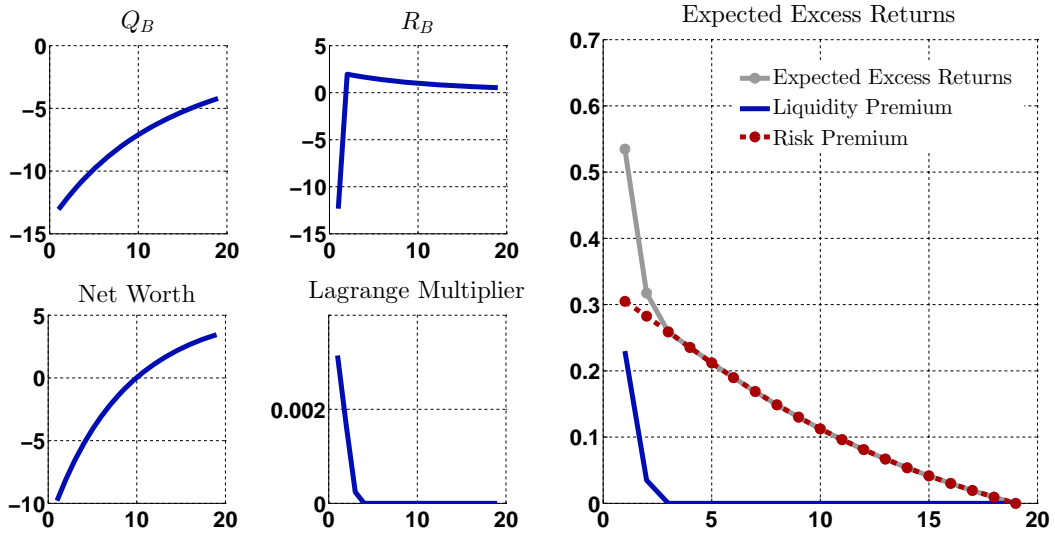
Notes: IRFs are computed via simulations initialized at the ergodic mean of the state vector. Q_B and Net Worth (linearly detrended) are expressed as percentage deviations from their ergodic mean value. Returns are reported in annualized basis points.

Figure 7 plots Impulse Response Functions (IRFs) to an s-shock when the economy is at the ergodic mean. The initial impulse in s is such that the probability of a future sovereign default goes from 0.13% to 2.5%. This represents roughly a 3 standard deviations shock. The figure shows that this shock tightens the leverage constraint of banks. The price of

government bonds declines by 13%, leading to a reduction in their realized returns of roughly the same magnitude. The net worth of banks declines by 10%. Because of this decline in net worth, the leverage constraint binds, and banks have fewer resources to lend to firms: spreads over firms' claims increase by 25 basis points as a result of this liquidity premium. The Figure shows that the risk premium component is also sensitive to the *s*-shock: this risk channel contributes to a rise in expected excess returns of 30 basis points on impact.

### Figure 8: **The *s*-shock and risk premia**
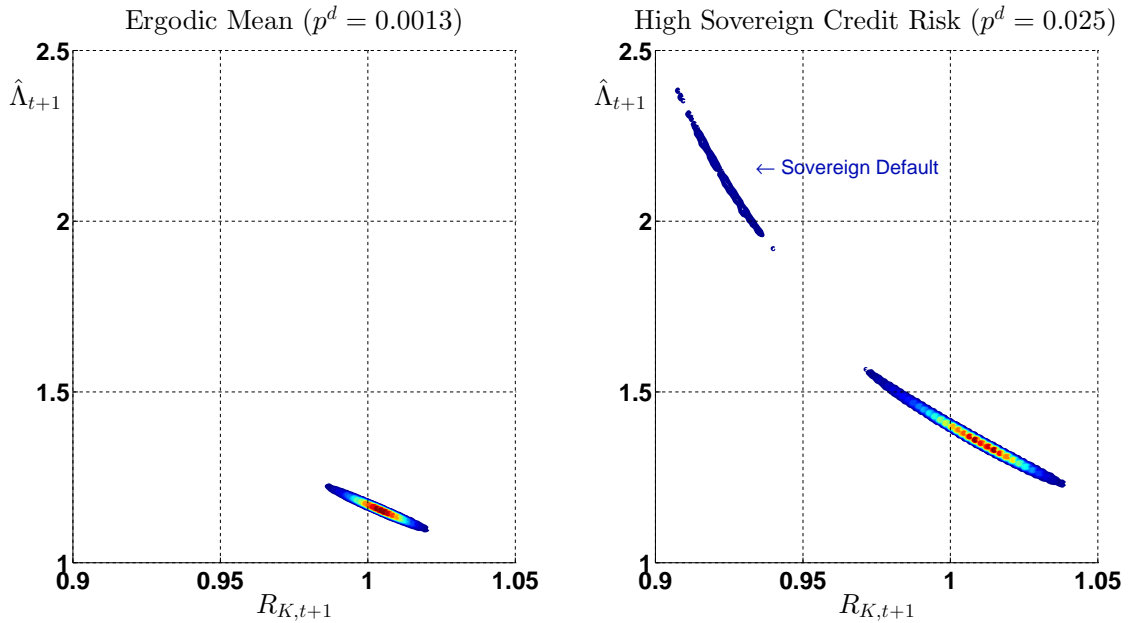
Notes: The left panel reports the joint density of $\{\hat{\Lambda}_{t+1}, R_{K,t+1}\}$ at the ergodic mean. This is constructed as follows. Simulate $M = 15000$ realizations for $\{\hat{\Lambda}, R_K\}$. Each simulation is initialized at the ergodic mean of the state vector and it has length $T = 2$. The contour lines are generated from a nonparametric density smoother applied to $\{\hat{\Lambda}_2^m, R_{K,2}^m\}_{m=1}^M$. The right panel reports the same information, at a different point in the state space. The procedure to construct the figure is the same as above, but the simulations are initialized as follows: i) *s*-shock is set so that $p_t^d = 0.025$; ii) the other state variable are set at their ergodic mean.

Figure 8 provides an explanation of why bankers demand high compensation for holdings firms' claims when the economy approaches a sovereign default. The figure reports the joint probability density function (contour lines) for $\hat{\Lambda}_{t+1}$ and $R_{K,t+1}$ using different conditioning sets. The left panel reports it when the state vector is at its ergodic mean. The right panel reports the same object with the only exception that the probability of a sovereign default next period equals 2.5%. We can see from the left panel of the figure a clear negative association between realized returns to capital and the pricing kernel, suggesting that the model generates a non-trivial compensation for risk at the ergodic mean. As the economy approaches a sovereign default (right panel), these variables be-

come more negatively associated. This motivates an increase in the compensation for holding claims on firms in their balance sheet. Intuitively, capital is a "bad" asset to hold during a sovereign default because the decline in its market value has adverse effects on bank net worth, and these balance sheet losses are very costly since banks' marginal value of wealth is high. This makes the s-shock a priced risk factor for firms' claims.

The rise in expected excess returns after an s-shock is associated with a decline in capital accumulation. Figure 9 reports the response of aggregate investment and output to the s-shock. The increase in the probability of a sovereign default leads to a decline in output and aggregate investment of, respectively, 0.65% and 4.5%. The economic mechanisms through which this happens are those described in Section 3.

Figure 9: **IRFs to an s-shock: quantities**
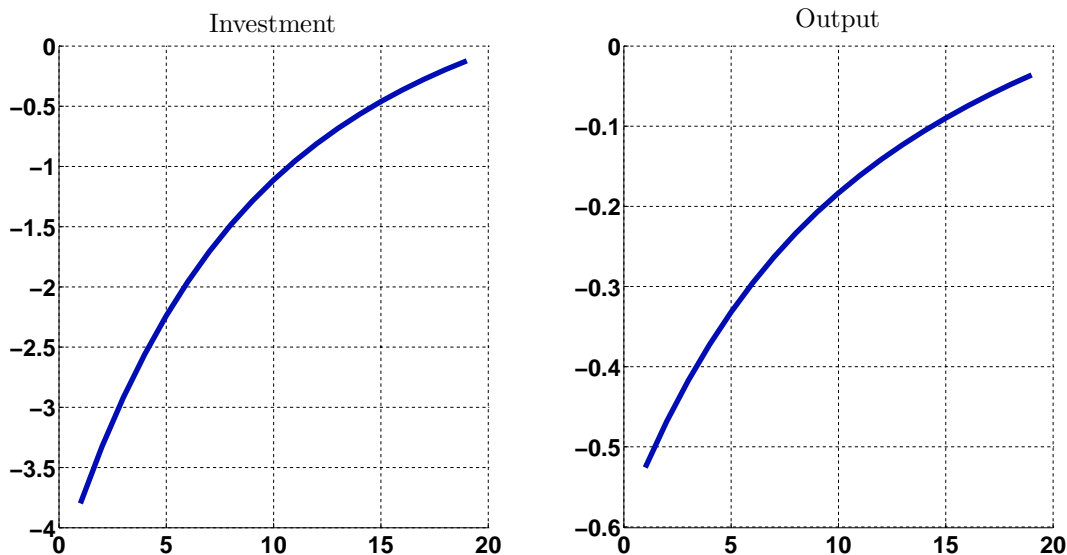
Notes: IRFs are computed via simulations on linearly detrended data initialized at the ergodic mean of the state vector. The variables are expressed as percentage deviations from their ergodic mean.

5.3 An injection of equity into banks

The distinction between liquidity and risk is crucial for the evaluation of credit policies within this model. I illustrate this point by studying the effects of an equity injection into the banking sector. I assume that at $t = 1$ the government transfers resources from households to banks using lump sum taxes. This policy is not anticipated by agents, and no further policy interventions are expected in the future. This intervention has the effect of changing the liability structure of banks, raising their net worth relative to their debt.

To make the experiment realistic, I implement the policy when the economy is in a

"financial recession". I define this as a state in which output growth is 1.5 standard deviations below average while expected excess returns are 1.5 standard deviations above average. I denote by $\{\mathbf{S}_i^*\}$ a set of states variables that is consistent with this definition of financial recession.[33] For each element of $\{\mathbf{S}_i^*\}$, I compute the expected path for selected endogenous variables under the policy and without the intervention. The policy effects are reported as percentage differences between these two paths. In order to interpret the results, I define

$$\delta_i = \frac{\text{Risk premium}_i}{\text{EER}_i}, \tag{34}$$

where the risk premium and *EER* are defined in equation (33). $\delta_i \in [0,1]$ can be thought as an indicator of how "risky" banks perceive firms to be in state $\mathbf{S}_i^*$. In fact, when $\delta_i = 0$, expected excess returns exclusively reflects agency costs while $\delta_i = 1$ means that they entirely reflect a compensation for firms' risk. Figure 10 plots two sets of results. The solid line reports the effect of the policy on outcomes when we condition on $\delta_i \leq 0.25$, while the dotted line conditions on $\delta_i \geq 0.75$.

Figure 10: **An injection of equity into banks**
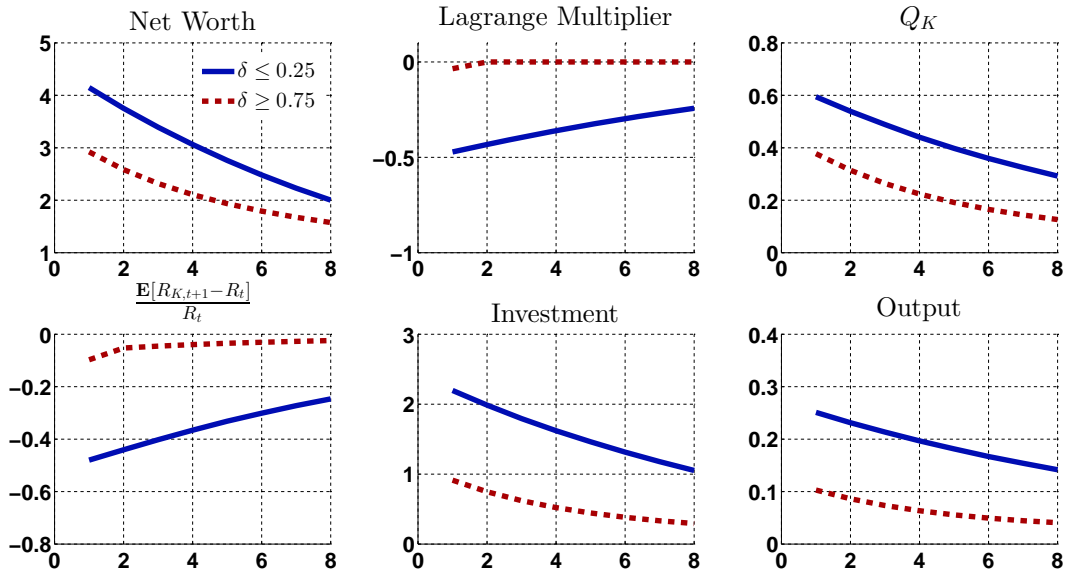
Notes: The figure is constructed as follows: i) simulte the model for $T = 20000$ periods; ii) select state variables such that output growth is 1.5 standard deviations below average and expected excess returns 1.5 standard deviations above average; iii) for each of these states as initial condition, compute the expected path under the equity injection and in absence of the policy; iv) take the difference between these expected paths. The figure reports the effects of the policy on outcome variables when conditioning on different values of δ, see equation (34). Net Worth, Output, and Investment are linearly detrended in simulations. Returns are reported in basis points. The other variables as percentage changes.

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[33]Operationally, this set is constructed by simulating time series of length $T = 20000$ from the model and selecting $\{\mathbf{S}_i^*\}$ so that output growth and expected excess returns satisfy the threshold restrictions.

The figure shows that equity injections are particularly effective in stimulating real economic activity when agency costs are large ($\delta \leq 0.25$). The policy relaxes the constraint on bank leverage and leads to an increase in capital accumulation. The same policy has substantially weaker effects when implemented in regions of the state space where firms' risk is high ($\delta \geq 0.75$). The red dotted line reports this case. We can observe that the response of investment and output to the equity injection is 2.5 times smaller with respect to the previous case.

This state-dependence in the effects of equity injections has an intuitive explanation. Expected excess returns in the model can be high because of two reasons: liquidity premia (low δ-regions) and risk premia (high δ-regions). Liquidity premia indicates that banks cannot undertake profitable investment opportunities because of funding constraints. Therefore, policies that relax these constraints stimulate capital accumulation because they facilitate the flow of funds from households to firms. A large value of δ, instead, indicates that the observed high excess returns reflect mainly compensation for risk: banks incentives to demand firms' claims are less responsive to equity injections because these latter have only indirect effects on this risk.[34]

6 Measurement and Policy Evaluation

I now turn to the two main quantitative experiments of this paper. In Section 6.1, I measure the effect of sovereign credit risk on the financing premia of firms and output, and I assess the contribution of the liquidity and risk channel. Section 6.2 proposes a quantitative assessment of the Longer Term Refinancing Operations (LTROs) implemented by the European Central Bank (ECB) in the first quarter of 2012. As we saw in the earlier section, credit market interventions are state and size dependent in this model due to its highly nonlinear nature. Therefore, an integral part of the policy evaluation is to specify the "initial conditions". I do so by estimating the state of the Italian economy in 2011:Q4 using the particle filter. The evaluation of LTROs is conducted from an *ex-ante* perspective.

6.1 Sovereign risk, firms' financing premia and output

What were the effects of sovereign credit risk on the financing premia of firms and on real economic activity in Italy? What was the relative strength of liquidity and risk in

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[34]By strengthening bank net worth, the policy provides a buffer when a sovereign default hits the economy. This dampens the effects of a sovereign default on realized returns to capital and lowers risk premia ex-ante. The size of the equity injection is thus an important determinant of the policy effects.

driving this propagation? In order to answer these questions, I conduct a counterfactual experiment. First, I extract the historical sequence of shocks for the Italian economy via the particle filter. Second, I feed the model with counterfactual trajectories for these shocks: these are equivalent to the estimated ones, with the exception that $\varepsilon_{s,t}$ is set to 0 for the entire sample. I then compare the actual and counterfactual path for a set of the model's endogenous variables. Their difference reflects the effects of heightened sovereign credit risk on the variables of interest. More specifically, I use the following algorithm[35]

Counterfactual experiment: Let θ^i denote the i'th draw from the posterior distribution of the model's parameter. For $i = 1$ to M

1. Conditional on θ^i, apply the particle filter to $\{\mathbf{Y}_t = [\text{GDP Growth}_t, \mu_t, p_t^d]\}_{t=2001:Q1}^{2011:Q4}$ and construct the densities $\{p(\mathbf{S}_t|\mathbf{Y}^t, \theta^i)\}_{t=2001:Q1}^{2011:Q4}$.

2. Sample N realizations of the state vector from $\{p(\mathbf{S}_t|\mathbf{Y}^t, \theta^i)\}_{t=2001:Q1}^{2011:Q4}$.

3. Feed the model with each realization, $n \in N$, and generate a path for a set of outcome variables, $\{\mathbf{x}_t(i,n)\}_t$.

4. For each realization n, replace the sovereign risk shock with its unconditional mean. Feed the model with this counterfactual realization of the state vector and collect the implied outcome variables of interest in $\{\mathbf{x}_t^c(i,n)\}_t$.

5. The effect of sovereign credit risk for the outcome variable x is measured as $x_t^{\text{eff}}(i,n) = x_t(i,n) - x_t^c(i,n)$. □

I first analyze the effect of sovereign risk on the financing costs of firms and on output. I then decompose these effects into the component due to the liquidity and that due to risk.

The top-left panel of Figure 11 reports the filtered and counterfactual trajectories for GDP growth while the bottom-left panel reports the effects of sovereign risk on expected excess returns. The rise in sovereign risk in Italy over the 2010:Q1-2011:Q4 period led to an increase in the financing costs of firms and a decline in output growth. The model predicts that expected excess returns increased by 34 basis points on average over this period, with a peak of 93 basis points in the last quarter of 2011. Cumulative output losses due to the sovereign debt crisis were 1.75% during the 2010-2011 period.

The bottom panels of Figure 11 reports also the decomposition of expected excess returns into the liquidity and risk premium components. The model shows that the risk

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[35]Regarding the specifics of this experiment, I select the parameters' draws by subsampling, picking 1 of every 100. Thus, $M = 100$. In the filtering stage, I set measurement errors to 0.5% of the sample variance of $\{\mathbf{Y}_t\}_t$, and I use 500,000 particles.

Figure 11: **Sovereign risk, firms' financing premia and output**
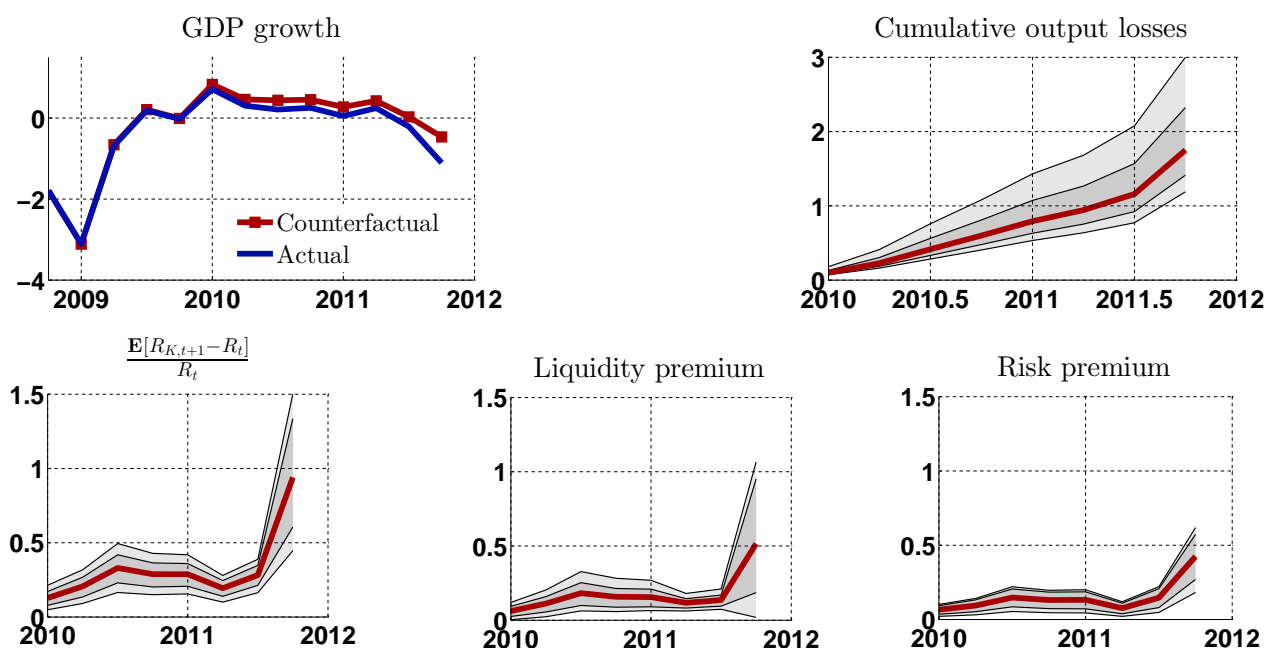
Notes: The solid line in the top-left panel reports the posterior mean for the filtered GDP growth series, and the dotted line reports the posterior mean of its counterfactual. The solid lines in the remaining panels represent the posterior mean of the corresponding statistic. The dark and light shaded area represents, respectively, a 60% and 90% equal tail probability credible sets for the variables of interest.

channel played quantitatively a first order role in the propagation of sovereign credit risk in Italy, and its relevance grew over time: at the end of 2011:Q4, risk premia accounts for 45% of the effects of sovereign risk on the financing premia of firms.

6.2 Longer term refinancing operations

The ECB undertook several interventions in response to the euro-area sovereign debt crisis. Some of these policies were targeted toward easing the tensions in the bond market of distressed governments. The Security Markets Program (SMP) and the Outright Monetary Transactions (OMTs) fall within this category.[36] Other interventions, instead, had the objective of loosening the funding constraints of banks exposed to distressed government debt. The LTROs launched by the ECB in December 2011 and February 2012 were the

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[36]In May 2010, the ECB started the SMP. Under the SMP, the ECB could intervene by buying, on secondary markets, the securities that it normally accepts as collateral. This program was extensively used for sustaining the price of government securities of southern European countries. The program was replaced by OMTs in August 2012. This latter program had two main differences compared with SMP: i) OMTs are *ex-ante* unlimited; ii) their approval is subject to a conditionality program from the requiring country.

most important in this class. Relative to canonical open market operations in Europe,[37] these interventions featured a long maturity (36 months), a fixed-interest rate and special rules for the collateral that could be used by banks. Moreover, the two LTROs were the largest refinancing operations in the history of the ECB, as more than 1 trillion euros were lent to banks through these interventions.

A full assessment of this policy is beyond the scope of this paper. LTROs, in fact, are not sterilized and the real model considered here misses this aspect. Moreover, the policy may have indirectly reduced sovereign credit risk (Coimbra, 2014), and the analysis in this paper does not capture this effect either. However, we can use the model to ask whether the provision of liquidity, by itself, stimulated bank lending to firms. I model LTROs as a nonstationary version of the discount window lending considered in Gertler and Kiyotaki (2010). The government gives banks the option at $t = 1$ of borrowing resources up to a threshold $\overline{m}$. These resources are financed through lump sum taxes. The loans have a fixed interest rate R_m. Banks repay the loan (principal plus interest) at a future date T and no interests are payed between t and T.[38] Within the logic of the model, this intervention has the effect of relaxing the leverage constraint of banks, and it has a positive effect on their net worth. These two points are explained in Appendix E, along with a description of the numerical algorithm used to implement the policy. The evaluation of LTROs is conducted using the following algorithm

Evaluating LTROs: Let θ^i denote the i'th draw from the posterior distribution of the model's parameter. For $i = 1$ to M

1. Conditional on θ^i, sample N realizations of the state vector from $p(\mathbf{S}_{2011:Q4}|\mathbf{Y}^{2011:Q4}, \theta^i)$.

2. For each $\{\mathbf{S}^n_{2011:Q4}\}_n$, simulate the model forward J times with and without the policy intervention.

3. For each outcome variable x, compute the difference between these two paths. Collect these paths in $\mathbf{x}^{\text{eff}}_t(i, n, j)$. $\square$

The density $p(\mathbf{S}_{2011:Q4}|\mathbf{Y}^{2011:Q4}, \theta^i)$ is computed using the particle filter. The vector of variables $\{\mathbf{x}^{\text{eff}}_t(i, n, j)\}_t$ denotes the effect of the policy on variable x. The results of this

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[37]Open market operations in the euro-area are conducted through refinancing operations. These are similar to repurchase agreements: banks put acceptable collateral with the ECB and receive cash loans. Prior to 2008, there were two major types of refinancing operations: main refinancing operations (loans of a weekly maturity) and LTROs, with a three month maturity.

[38]The government has perfect monitoring of banks, so that these liabilities do not count for their leverage constraint. If this was not the case, the loans would perfectly crowd out households' deposits by construction: see Gertler and Kiyotaki (2010).

experiment can be interpreted as an *ex-ante* evaluation of the policy, since I am conditioning on retrospective estimates for the state vector in the 2011:Q4 period. In order to make the experiment more realistic, I calibrate the policy to the actual ECB intervention. I set $R_m = 1.00$, $T = 12$ and $\overline{m} = 0.1\hat{Y}^{ss}$.

Figure 12: **Ex-ante assessment of LTROs**
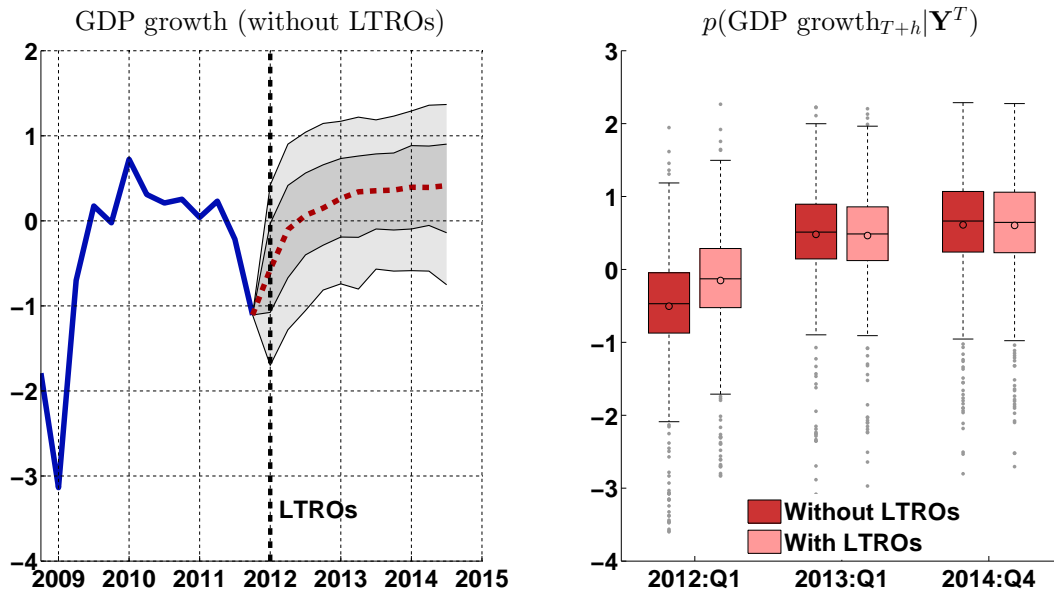
Notes: The solid line in the left panel is the conditional mean forecasts of the GDP growth time series from 2012:Q1 to 2014:Q4. The Dark and light shaded area represents, respectively, a 60% and 90% equal tail probability credible sets. The right panels report the predictive densities for GDP growth with and without LTROs (box plots).

As a benchmark, I first discuss the forecasted path for GDP growth in absence of the policy. The left panel of Figure 12 reports the posterior median of the model's forecast for GDP growth in absence of the policy along with its 60% and 90% credible set. The model predicts a "risky" recovery for GDP growth from the 2011:Q4 point of view. While on average GDP growth returns to its trend by 2013, we can see a long left tail in these forecasts, especially in the early part of 2012.

The right panel of Figure 12 shows how LTROs influence these forecasts. I use box plots to describe the predictive densities $p(\text{GDP Growth}_{T+h}|\mathbf{Y}^T)$ with and without LTROs for $h = \{1, 4, 12\}$. The box stands for the interquartile range, while the line within the box is the median while the circle represents the mean. The refinancing operations have a positive effect on GDP growth in the first quarter of 2012. Indeed, the median forecast for GDP growth under the policy is above its unconditional one. More strikingly, the policy removes most of the downside risk: the left tail of the predictive density for GDP growth in 2012:Q1 almost disappears. This happens because the policy increases the maturity of

banks' liabilities, and makes their balance sheet less sensitive to adverse shocks. As time goes on and the repayment date approaches, though, GDP growth forecasts under LTROs become fairly similar to those in absence of this policy. At the scheduled repayment date, the predictive density for GDP growth is actually more left-skewed relative to that obtained in absence of the policy.

In order to better understand the effects of LTROs, I report in Table 5 posterior statistics of its effects on the level of output, expected excess returns and their decomposition into liquidity and risk premia. On impact, the refinancing operations lower expected excess returns and increases the level of output. The reduction in the firms' financing costs is mainly due to lower liquidity premia: the policy, in fact, loosens the funding constraints of intermediaries. However, the risk premium component is barely affected. Notice that these initial positive effects are reversed over time. One year after its implementation, the model places some probability of LTROs increasing the financing costs of firms and reducing the level of output. This result is driven by the behavior of the model at the repayment stage. At that stage, banks need to repay the loans they took on, and adverse net worth shocks at that date are very costly for them. The anticipation of the repayment stage makes banks more cautious *ex-ante* and leads them to demand higher compensation for bearing risk. This counteracts the initial positive effects of the policy. On average, LTROs lead to a reduction of private sector spreads of 20 basis points and an increase in the level of output of 0.20%. Thus, these operations seems quite ineffective in stimulating bank lending to firms once we condition on empirically reasonable regions of the state space in 2011:Q4.

This result does not imply that refinancing operations are a bad policy instrument. Rather, that their effects depend on the economic environment in which they are implemented. In order to see this last point, I implement LTROs in a different region of the state space, drawn from the density $p(\mathbf{S}_{2008:Q3}|\mathbf{Y}^{2008:Q3}, \theta^i)$. In contrast to the 2011:Q4 period, the model interprets the financial distress of 2008:Q3 as driven mainly by banks liquidity problems. Results are reported in the bottom panel of Table 5. Two main differences stand out compared to the previous analysis. First, the policy has a substantially stronger effect on the financing premia of firms and on output when implemented in 2008:Q3. Expected excess returns decline by 78 basis points on impact, while the level of output increases by 0.45%. Second, the downside risk at the repayment stage is substantially reduced. This can be seen by comparing the credible sets for output at the repayment stage for the two cases.

The reasons underlying this state dependence are related to the discussion of equity injections in Section 5.3. In 2008:Q3, agency costs are estimated to be high. This indicates

Table 5: **Effects of LTROs: 2008:Q3 vs. 2011:Q4**

	Impact	One Year After	Repayment	Average
Initial Condition drawn from $p(\mathbf{S}_{2011:Q4}|\mathbf{Y}^{2011:Q4})$				
Output	0.35	0.23	0.13	0.20
	[0.29,0.41]	[0.04,0.29]	[0.00,0.19]	[0.03,0.25]
EER	-0.56	-0.20	-0.08	-0.20
	[-0.75,-0.32]	[-0.30,0.17]	[-0.90,0.18]	[-0.34,-0.03]
Liquidity Premium	-0.55	-0.19	-0.13	-0.18
	[-0.74,-0.31]	[-0.30,0.15]	[-0.85,0.16]	[-0.34,-0.01]
Risk Premium	-0.01	-0.01	0.06	0.01
	[-0.02,0.01]	[-0.07,0.02]	[0.00,0.11]	[-0.04,0.01]
Initial Condition drawn from $p(\mathbf{S}_{2008:Q3}|\mathbf{Y}^{2008:Q3})$				
Output	0.45	0.34	0.23	0.30
	[0.39,0.50]	[0.23,0.50]	[0.18,0.26]	[0.20,0.35]
EER	-0.78	-0.40	-0.13	-0.29
	[-1.10,-0.45]	[-0.55,-0.12]	[-0.26,0.03]	[-0.47,0.01]
Liquidity Premium	-0.76	-0.39	-0.14	-0.28
	[-1.06,-0.42]	[-0.57,-0.08]	[-0.33,0.00]	[-0.45,-0.01]
Risk Premium	-0.00	-0.00	0.03	-0.00
	[-0.01,0.01]	[-0.01,0.01]	[-0.00,0.10]	[-0.00,0.00]

Notes: Posterior statistics on the effects of LTROs on output and expected excess returns on impact (period 1), one year after (period 5), at the repayment stage (period 12) and on average over the period. The top-panel initializes the state vector at $p(\mathbf{S}_{2011:Q4}|\mathbf{Y}^{2011:Q4}, \theta^i)$, the bottom panel at $p(\mathbf{S}_{2008:Q3}|\mathbf{Y}^{2008:Q3}, \theta^i)$.

that there are profitable investment opportunities in the economy and banks use the funds from the LTROs to lend to firms. General equilibrium, then, generates a positive loop: the market value of firms' claims is positively influenced by higher demand for capital, and this strengthens bank net worth. As a result, banks arrive at the repayment stage with a buffer that makes their balance sheet less sensitive to adverse shocks. In 2011:Q4, instead, agency costs are smaller. Rather than lending to firms, banks use part of the LTROs funds to substitute the liabilities they have with the household's sector. As a result, the general equilibrium effects described above are muted.

7 Conclusion

In this paper I have conducted a quantitative analysis of the transmission of sovereign credit risk to the borrowing costs of firms and real economic activity. I studied a model where banks are exposed to risky government debt and they are the main source of finance for firms. *News* about a future sovereign default hampers financial intermediation because of two mechanisms. First, by reducing the market value of government securities, higher sovereign credit risk reduces the net worth of banks and hampers their funding ability: their increased financing costs pass-through into the borrowing rates of firms (liquidity channel). Second, an increase in the probability of a future sovereign default raises the risks associated with lending to firms: if the default occurs in the future, in fact, claims on the productive sector will pay out little and banks will have to absorb these losses. I referred to this second mechanism as the risk channel. The structural estimation of the model on Italian data suggests that the sovereign debt crisis significantly increased the financing premia of firms, with the risk channel explaining up to 45% of these effects. Moreover, the rise in the probability of a sovereign default had severe adverse consequences for the Italian economy: cumulative output losses were 1.75% at the end of 2011. In counterfactual experiments, I use the estimated model to evaluate the policy response adopted by the ECB, with particular emphasis on the LTROs of the first quarter of 2012. The model estimates that this intervention, by itself, has limited effects on bank lending to firms. This happens because firms were perceived to be risky at the time these policies were implemented, and banks preferred to use the LTROs funds to cheaply refinance their liabilities. More generally, the analysis shows that the stabilization properties of these interventions are state dependent in the model, and their aggregate effects depend on the relative strength of liquidity and risk.

There are a number of dimensions in which the model could be extended. First, it would be interesting to introduce an optimizing government, and study its optimal default policy. Beside its importance for answering normative questions, understanding how the government's choice to default is affected by the wealth of the financial sector would allow for a more complete evaluation of the policy responses adopted by the ECB. A second extension would be that of considering an open economy. I believe that this dimension will help the empirical identification of the mechanisms discussed in this paper, since they are likely to generate differential implications for international capital flows. While both of these issues are challenging, and require a substantial departure from this framework, they represent exciting opportunities for future work.

Abstracting from the current application, recent research advocates the use of indica-

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tors of credit spreads as observables when estimating quantitative models with financial intermediation. This paper adds to that by underscoring the importance of measuring the *sources* driving the movements in these indicators of financial stress. Understanding whether firms' financing premia during crises are high because of "frictions" in financial markets or because of fair compensation for increased risk is a key information for policy makers. Incorporating the nonlinearities emphasized in this paper in larger scale models used for policy evaluation is technically challenging. Moreover, given the policy relevance of these nonlinearities, there is a need for developing tools for their empirical validation in the data. I plan to address these issues in future work.

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Online Appendix: The Pass-Through of Sovereign Risk

Luigi Bocola

A Derivation of Results 1

Combine equation (3) and (4) to eliminate the demand for households' savings from the decision problem of the banker. The decision problem is then

$$v_b(n; \mathbf{S}) = \max_{a_B, a_K} \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi)n' + \psi v_b(n'; \mathbf{S}') \right] \right\},$$

$$n' = \sum_{j=\{B,K\}} [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] Q_j(\mathbf{S}) a_j + R(\mathbf{S})n,$$

$$\lambda \left[\sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j \right] \leq v_b(n; \mathbf{S}),$$

$$\mathbf{S}' = \mathbf{\Gamma}(\mathbf{S}).$$

Guess that the value function is $v(n, \mathbf{S}) = \alpha(\mathbf{S})n$. Necessary and sufficient conditions for an optimum are

$$\mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi) + \psi \alpha(\mathbf{S}') \right] [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] \right\} = \lambda \mu(\mathbf{S}) \quad j = \{B, K\}, \qquad \text{(A.1)}$$

$$\mu(\mathbf{S}) \left(\alpha(\mathbf{S})n - \lambda \sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j \right) = 0. \qquad \text{(A.2)}$$

Substituting the guess in the dynamic program, and using the law of motion for n', we obtain

$$v_b(n, \mathbf{S}) = \max_{a_B, a_K} \left\{ \sum_{j=\{B,K\}} \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi \alpha(\mathbf{S}')][R_j(\mathbf{S}') - R(\mathbf{S})] \right\} Q_j(\mathbf{S}) a_j \right\} +$$

$$\mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi \alpha(\mathbf{S}')] \right\} R(\mathbf{S})n.$$

Note that the first term on the right hand side of the above equation equals $\mu(\mathbf{S})\alpha(\mathbf{S})n$. Indeed, when the leverage constraint does not bind ($\mu(\mathbf{S}) = 0$), expected discounted returns on assets held by bankers equal the discounted risk free rate by equation (A.1). This implies that the term equals 0. When the constraint binds ($\mu(\mathbf{S}) > 0$), instead, this

term can be written as

$$\lambda \mu(\mathbf{S}) \sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j.$$

Using the condition in (A.2), we can express $\lambda \mu(\mathbf{S}) \sum_{j=\{b,k\}} Q_j(\mathbf{S}) a_j$ as $\mu(\mathbf{S}) \alpha(\mathbf{S}) n$. Thus, the value function under the guess takes the following form:

$$\alpha(\mathbf{S}) n = \mu(\mathbf{S}) \alpha(\mathbf{S}) n + \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi \alpha(\mathbf{S}')] \right\} R(\mathbf{S}) n.$$

Solving for $\alpha(\mathbf{S})$, we obtain

$$\alpha(\mathbf{S}) = \frac{\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi \alpha(\mathbf{S}')] \} R(\mathbf{S})}{1 - \mu(\mathbf{S})}.$$

The guess is verified if $\mu(\mathbf{S}) < 1$. From equation (A.2) we obtain:

$$\mu(\mathbf{S}) = \max \left\{ 1 - \frac{\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi \alpha(\mathbf{S})] \} R(\mathbf{S}) n}{\lambda \left(\sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j \right)}, 0 \right\} < 1.$$

Finally, notice that financial leverage equals across bankers whenever $\mu(\mathbf{S}) > 0$. This implies that $\frac{n}{\lambda \sum_{j=\{B,K\}} Q_j a_j}$ is equal to $\frac{N}{\lambda \left[\sum_{j=\{B,K\}} Q_j A_j \right]}$ when the constraint binds.

B Numerical Solution

B.1 Equilibrium conditions

The states of the model are $\mathbf{S} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s, d]$. The controls $\{\tilde{C}(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), Q_B(\mathbf{S})\}$ solve the residual equations

$$E_{\mathbf{S}} \left[\beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} R(\mathbf{S}) \right] - 1 = 0, \tag{A.3}$$

$$\mathbb{E}_{\mathbf{S}} \left\{ \beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} [(1 - \psi) + \psi\alpha(\mathbf{S}')] \left[\frac{(1-\delta)Q_K(\mathbf{S}') + \alpha\frac{\tilde{Y}(\mathbf{S}')}{\tilde{K}'(\mathbf{S})}e^{\Delta z'}}{Q_K(\mathbf{S})} \right] \right\} - \lambda\mu(\mathbf{S}) = 0, \tag{A.4}$$

$$\mathbb{E}_{\mathbf{S}} \left\{ \beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} [(1 - \psi) + \psi\alpha(\mathbf{S}')] [1 - d'D] \left[\frac{\pi + (1-\pi)[\iota + Q_b(\mathbf{S}')]}{Q_B(\mathbf{S})} \right] \right\} - \lambda\mu(\mathbf{S}) = 0, \tag{A.5}$$

$$\alpha(\mathbf{S}) - \frac{(1 - \psi) + \psi R(\mathbf{S}) \mathbb{E}_{\mathbf{S}} \left[\beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} \alpha(\mathbf{S}') \right]}{1 - \mu(\mathbf{S})} = 0, \tag{A.6}$$

where $Q_K(\mathbf{S})$ is the market value of the capital stock and the multiplier $\mu(\mathbf{S})$ is given by

$$\mu(\mathbf{S}) = \max \left\{ 1 - \left[\frac{\mathbb{E}_{\mathbf{S}} \left\{ \beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} [(1 - \psi) + \psi\alpha(\mathbf{S}')] R(\mathbf{S}) \right\} \tilde{N}(\mathbf{S})}{\lambda [Q_K(\mathbf{S})\tilde{K}'(\mathbf{S}) + Q_B(\mathbf{S})\tilde{B}(\mathbf{S})]} \right], 0 \right\}. \tag{A.7}$$

The endogenous state variables $[\tilde{K}, \tilde{B}, \tilde{P}]$ evolve as follows

$$\tilde{K}'(\mathbf{S}) = \left\{ (1-\delta)\tilde{K} + \Phi \left[e^{\Delta z} \left(\frac{\tilde{Y}(\mathbf{S})(1-g) - \tilde{C}(\mathbf{S})}{\tilde{K}} \right) \right] \tilde{K} \right\} e^{-\Delta z}, \tag{A.8}$$

$$\tilde{B}'(\mathbf{S}) = \frac{[1 - dD]\{\pi + (1-\pi)[\iota + Q_B(\mathbf{S})]\}\tilde{B}e^{-\Delta z} + \tilde{Y}(\mathbf{S})\left[g - \left(t^* + \gamma_\tau \frac{B}{\tilde{Y}(\mathbf{S})}\right)\right]}{Q_B(\mathbf{S})}, \tag{A.9}$$

$$\tilde{P}'(\mathbf{S}) = R(\mathbf{S})[Q_k(\mathbf{S})\tilde{K}'(\mathbf{S}) + Q_b(\mathbf{S})\tilde{B}'(\mathbf{S}) - \tilde{N}(\mathbf{S})]. \tag{A.10}$$

The state variable $\tilde{P}$ measures the detrended *cum* interest deposits that bankers pay to households at the beginning of the period and it is sufficient to keep track of the evolution of aggregate bankers' net worth. Indeed, the aggregate net worth of banks can be

expressed as

$$\tilde{N}(\mathbf{S}) = \psi \left\{ \left[Q_K(\mathbf{S}) + \alpha \frac{\tilde{Y}(\mathbf{S})}{\tilde{K}} e^{\Delta z} \right] \tilde{K} + [1 - dD] \left[\pi + (1 - \pi) [\iota + Q_B(\mathbf{S})] \right] \tilde{B} - \tilde{P} \right\} \tag{A.11}$$

$$+ \omega [Q_K(\mathbf{S}) \tilde{K} + Q_B(\mathbf{S}) \tilde{B}].$$

Using the intratemporal Euler equation of the household, we can express detrended output as

$$\tilde{Y}(\mathbf{S}) = \left[\chi^{-1} \frac{(\tilde{K} e^{-\Delta z})^{\alpha}}{\tilde{C}(\mathbf{S})} \right]^{\frac{1-\alpha}{\alpha+\nu-1}} (\tilde{K} e^{-\Delta z})^{\alpha}. \tag{A.12}$$

The exogenous state variables $[\Delta z, \log(g), s]$ evolve as follows

$$\Delta z' = (1 - \rho_z) \gamma + \rho_z \Delta z + \sigma_z \varepsilon_z', \tag{A.13}$$

$$\log(g') = (1 - \rho_g) g^* + \rho_g \log(g) + \sigma_g \varepsilon_g', \tag{A.14}$$

$$s' = (1 - \rho_s) s^* + \rho_s s + \sigma_s \varepsilon_s', \tag{A.15}$$

while d follows

$$d' = \begin{cases} 1 & \text{with probability} \quad \frac{\exp\{s\}}{1 + \exp\{s\}} \\ 0 & \text{with probability} \quad 1 - \frac{\exp\{s\}}{1 + \exp\{s\}}. \end{cases} \tag{A.16}$$

It will be convenient to express detrended state and control variables as log-deviations from their deterministic steady state. I denote this transformation for variable x as $\hat{x}$.

B.2   Algorithm for numerical solution

I approximate the control variables of the model using piece-wise smooth functions, parametrized by $\gamma = \{\gamma_{d=0}^x, \gamma_{d=1}^x\}_{x=\{\hat{C}, \hat{\alpha}, \hat{Q}_b, \hat{R}\}}$. The law of motion for a control variable x is described by

$$x(d, \hat{\mathbf{S}}) = (1 - d) \gamma_{d=0}^x{}' \mathbf{T}(\hat{\mathbf{S}}) + d \gamma_{d=1}^x{}' \mathbf{T}(\hat{\mathbf{S}}), \tag{A.17}$$

where $\hat{\mathbf{S}} = [\hat{K}, \hat{P}, \hat{B}, \Delta \hat{z}, \hat{g}, \hat{s}]$ and $\mathbf{T}(.)$ is a vector collecting Chebyshev's polynomials.[39] Define $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}\})$ to be a 4×1 vector collecting the left hand side of the residual equations (A.3)-(A.6) for the candidate solution γ^c evaluated at $\{d, \hat{\mathbf{S}}\}$. The numerical solution of the model consists in choosing γ^c so that $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}\}) = 0$ for a set of collocation points $\{d, \hat{\mathbf{S}}^i\} \in \{0, 1\} \times \mathcal{S}$.

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[39]The shocks are expressed as deviation from their mean.

The choice of collocation points and of the associated Chebyshev's polynomials follows Krueger et al. (2010). The rule for computing conditional expectations when evaluating $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}\})$ follows Judd et al. (2011). To give an example of this latter, suppose we wish to compute $\mathbb{E}_{d, \mathbf{S}^i}[y(d', \hat{\mathbf{S}}')]$, where y is an integrand of interest.[40] Given a candidate solution γ^c, we can compute y at every collocation point using the model's equilibrium conditions. Next, we can construct an implied policy function for y, $\{\gamma_{d=0}^y, \gamma_{d=1}^y\}$, via a Chebyshev's regression. Using the law of total probability, the conditional expectation of interest can be expressed as

$$
\mathbb{E}_{d, \mathbf{S}^i}[y(d', \hat{\mathbf{S}}')] = (1 - \text{Prob}\{d' = 1|\hat{\mathbf{S}}^i\})\mathbb{E}_{\hat{\mathbf{S}}^i}[\gamma_{d=0}^y{}' \mathbf{T}(\hat{\mathbf{S}}')] +
$$

(A.18)

$$
\text{Prob}\{d' = 1|\hat{\mathbf{S}}^i\}\mathbb{E}_{\hat{\mathbf{S}}^i}[\gamma_{d=1}^y{}' \mathbf{T}(\hat{\mathbf{S}}')],
$$

where $\text{Prob}\{d' = 1|\hat{\mathbf{S}}^i\} = \frac{e^{s^i}}{1+e^{s^i}}$. Judd et al. (2011) propose a simple procedure to evaluate integrals of the form $\mathbb{E}_{\hat{\mathbf{S}}^i}[\gamma_{d=1}^y{}' \mathbf{T}(\hat{\mathbf{S}}')]$. In proposition 1 of their paper, they show that, under weak conditions, the expectation of a polynomial can be calculated via a linear transformation $\mathcal{I}$ of the coefficient vector γ_d^y, where $\mathcal{I}$ depends exclusively on the deep parameters of the model. The authors provide general formulas for the transformation $\mathcal{I}$.

The algorithm for the numerical solution of the model goes as follows

Step 0.A: Defining the grid and the polynomials. Set upper and lower bounds on the state variables $\hat{\mathbf{S}} = [\hat{K}, \hat{P}, \hat{B}, \Delta z, g, s]$. Given these bounds, construct a μ-level Smolyak grid and the associated Chebyshev's polynomials $\mathbf{T}(.)$ following Krueger et al. (2010).

Step 0.B: Precomputing integrals. Compute $\mathcal{I}$ using Judd et al. (2011) formulas.

Step 1: Equilibrium conditions at the candidate solution. Start with a guess for the model's policy functions γ^c. For each $(d, \hat{\mathbf{S}}^i)$, use γ^c and equation (A.17) to compute the value of control variables $\{\hat{C}(d, \hat{\mathbf{S}}^i), \hat{\alpha}(d, \hat{\mathbf{S}}^i), \hat{Q}_b(d, \hat{\mathbf{S}}^i), \hat{R}(d, \hat{\mathbf{S}}^i)\}$. Given the control variables, solve for the endogenous state variables next period using the model's equilibrium conditions. Given the value of control and state variables, compute the value of every integrand in equations (A.3)-(A.6) at $(d, \hat{\mathbf{S}}^i)$. Collect these integrands in the matrix $\mathbf{y}$.

Step 2: Evaluate conditional expectations. For each $d = \{0, 1\}$, run a Chebyshev regression for the integrand in $\mathbf{y}$, and denote by γ_d^y the implied policy function for

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[40]For example, y could be $\frac{e^{-\Delta z'}}{C(\mathbf{S}')}$ in equation (A.3).

an element $y \in \mathbf{y}$. Conditional expectations are calculated using equation (A.19) and the matrix $\mathcal{I}$.

Step 3: Evaluate residual equations. Given conditional expectations, compute the multiplier using equation (A.7). Evaluate the residual equations $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}^i\})$ at every collocation point $(d, \hat{\mathbf{S}}^i)$. The dimension of the vector of residuals equals 4 times the cardinality of the state space. Denote by r the Euclidean norm for this vector.

Step 4: Iteration. If $r \leq 10^{-20}$, stop the algorithm. Else, update the guess and repeat Step 1-4. $\square$

The specific for the algorithm are as follows

Choice of bounds. The bounds on $[\Delta\hat{z}, \hat{g}]$ are +/- 3 standard deviations from their mean. The bounds on $\hat{s}$ are larger and set to $[-5, +5]$. The bounds on the endogenous state variables $\hat{\mathbf{S}} = [\hat{K}, \hat{P}, \hat{B}]$ are set to +/- 4.5 their standard deviations in the model without sovereign risk. The standard deviation is calculated by simulating a third order perturbation of the model without sovereign risk. A desirable extention of the algorithm is to select different bounds depending on whether the economy is in a default state or not.

Smolyak grid. For tractability, I choose $\mu = 3$ for the Smolyak grid. This implies that I have 389 distinct points in $\mathcal{S}$.

Precomputation of integrals. I use Gaussian numerical quadrature for computing the matrix $\mathcal{I}$.

Iterative algorithm. I find the zeros of the residual equation using a variant of the Newton algorithm. To speed up computations, numerical derivatives are computed in parallel.

I denote the model solution by γ^*.

B.3 Accuracy of numerical solution

I check the accuracy of the numerical solution by computing the errors of the residual equations (Judd, 1992). More specifically, I proceed as follows. First, I simulate the model forward for 5000 periods. This gives a simulation for the state variable of the model $\{d_t, \hat{\mathbf{S}}_t\}_{t=1}^{5000}$. Second, for each pair $(d_t, \hat{\mathbf{S}}_t)$, I calculate the errors of the residual equations $\mathcal{R}(\gamma^*, \{d_t, \hat{\mathbf{S}}_t\})$. As an example, let's consider equation (A.4). Then, the residual error at $(d_t, \hat{\mathbf{S}}_t)$ for this equation is defined as

$$1 - \frac{\mathbb{E}_{\mathbf{d_t}, \hat{\mathbf{S}}_t}\left\{\beta \frac{\tilde{C}(d_t, \hat{\mathbf{S}}_t)}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'}[(1-\psi)+\psi\alpha(\mathbf{S}')]\left[\frac{(1-\delta)Q_K(\mathbf{S}')+\alpha \frac{\tilde{Y}(\mathbf{S}')}{\tilde{K}'(d_t, \hat{\mathbf{S}}_t)} e^{\Delta z'}}{Q_K(d_t, \hat{\mathbf{S}}_t)}\right]\right\}}{\lambda\mu(d_t, \hat{\mathbf{S}}_t)},$$

where the model's policy functions are used to generate the value for endogenous variables at $d_t, \hat{\mathbf{S}}_t$. Note that, by construction, the residual errors are zero at the collocation points. This residual equation errors provide a measure of how large are the discrepancy between the decision rule derived from the numerical algorithm and those implied by the model's equilibrium condition in other points of the state space. Following standard practice, I report the decimal log of the absolute value of these residual errors. Figure A-1 below reports the density (histogram) of those errors.

Figure A-1: **Residual equations errors**
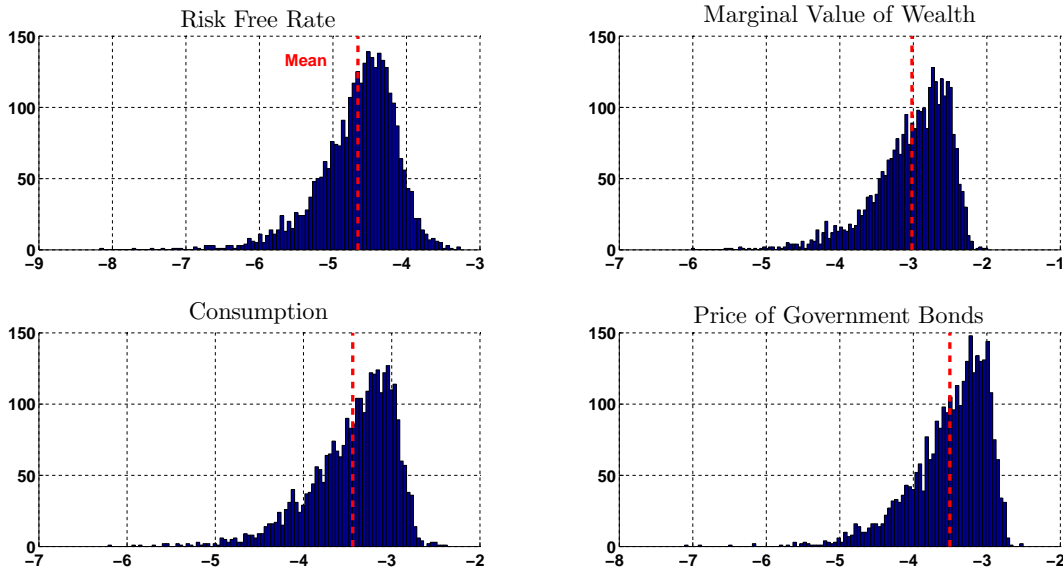
Notes: The histograms reports the residual equations errors in decimal log basis. The dotted line marks the mean residual equation error.

On average, residual equation errors are in the order of -4.65 for the risk free rate, -3.5

for consumption and the price of government securities and -3 for the marginal value of wealth. These numbers are comparable to values reported in the literature for models of similar complexity, and they are still very reasonable. Figure A-2 reports residual equation errors for a sequence of states $\{\mathbf{S}_t\}_{t=2004:Q1}^{2011:Q4}$ extracted using the particle filter (See Section 4). The figure shows that residual equation errors are reasonable in empirically relevant region of the state space.

Figure A-2: **Residual equations errors: empirically relevant region**
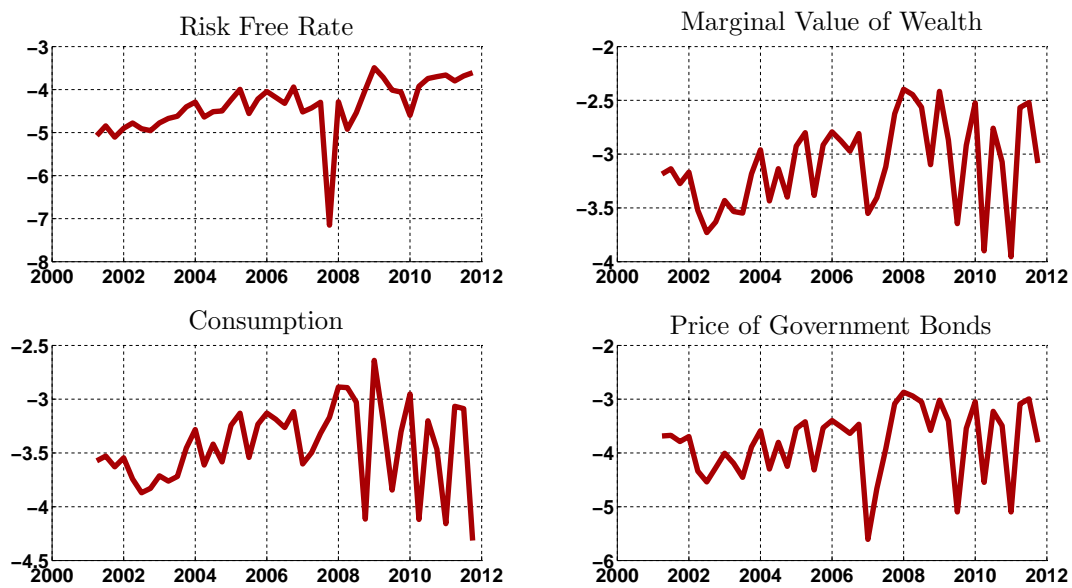
C   Data source

C.1   Credit default swap (CDS) spread

Daily CDS spreads on 5 years Italian governmet securities (RED code: 4AB951). The restructuring clause of the contract is CR (complete restructuring). The spread is denominated in basis points and paid quarterly. The source is Markit, accessed from the Wharton Research Data Services.

C.2   Banks' exposure to the Italian government

The European Banking Authority (EBA) published information on holdings of government debt by European banks participating to the 2011 stress test. Five Italian banks were in this pool: Unicredit, Intesa-San Paolo, Monte dei Paschi di Siena (MPS), Banco Popolare (BPI) and Unione di Banche Italiane (UBI). Results of the stress test for each of these five banks are available at http://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2011/results. I measure exposure of each bank to Italian central and local government as net direct long exposure (gross exposures net of cash short position of sovereign debt to other counterparties only where there is maturity matching) plus direct sovereign exposures in derivatives. This information is available by maturity of financial instrument, and it reflects positions as of 31st of December 2010. I match these data on exposure with end of 2010 total financial assets for each of the five institutions. This latter information is obtained using consolidated banking data from Bankscope, accessed from the Wharton Research Data Service. Table A-1 reports these information.

C.3   Construction of the Lagrange multiplier

Result 2. *In equilibrium, the multiplier on the incentive constraint of bankers is a function of financial leverage and of the spread between a risk free security traded by bankers and the risk free rate*

$$\mu_t = \frac{\left[\frac{R_t^f - R_t}{R_t}\right] lev_t}{1 + \left[\frac{R_t^f - R_t}{R_t}\right] lev_t}. \tag{A.19}$$

Table A-1: **Exposure to domestic sovereign by major Italian banks: end of 2010**

	3Mo	1Yr	2Yr	3Yr	5Yr	10Yr	15Yr	Tot.	Tot. Assets
Intesa	17.18	9.31	2.46	4.87	7.71	6.42	10.12	58.08	658.76
Unicredit	17.78	9.85	2.78	6.12	4.24	5.90	1.44	48.11	929.49
MPS	5.61	4.96	3.92	3.57	1.35	3.71	8.91	32.03	240.70
BPI	3.90	1.65	1.15	3.64	0.78	0.39	0.25	11.76	134.17
UBI	1.27	3.56	0.22	0.30	0.54	2.47	1.76	10.11	129.80
Total	45.75	29.32	10.53	18.5	14.61	18.89	22.48	160.01	2092.99

Notes: Data is reported in billions of euros.

Proof. Since the asset is risk free, one has that $\text{cov}_t(\hat{\Lambda}_{t+1}, R^f_{t+1}) = 0$. Therefore, using equation (26) in the main text, one has:

$$\left[\frac{R^f_t - R_t}{R_t} \right] = \frac{\lambda}{\alpha_t} \frac{\mu_t}{1 - \mu_t}.$$

Equation (A.19) follows from the fact that $\frac{\alpha_t}{\lambda}$ equals financial leverage when $\mu_t > 0$. $\quad\square$

Essentially, Result 2 tells us that the agency friction can be interpreted as a markup on financial intermediation. To measure this markup, one needs to focus on returns on assets, traded only by bankers, that have the same risk properties of households' deposits. I use the prime interbank rate to measure R^f_t since the model of Section 2 can be interpreted as having a frictionless interbank market as in Gertler and Kiyotaki (2010). The time series used in the construction of the multiplier are the following

Financial leverage: The definition of financial leverage in the model is banks' equity divided by the market value of total assets. I use quarterly data from the Italian flow of funds (*Conti Finanziari*) to construct these two time series. First, I match banks in the model with Monetary and Financial Institutions (MFIs). This category includes commercial banks, money market funds and the domestic central bank. I use balance sheet information for the Bank of Italy to exclude the latter from this pool. Second, I construct a time series for bank equity as the difference between "total assets" and total debt liabilities. This latter

is defined as "total liabilities" minus "shares and other equities" (liabilities) and "mutual fund shares" (liabilities). Financial leverage is the ratio between equity and total assets. Data can be downloaded at `http://bip.bancaditalia.it/4972unix/`. See Bartiloro et al. (2003) for a description of the Italian flow of funds.

TED spread: The prime interbank rate (EURIBOR 1yrs.), at monthly frequencies, is obtained from `http://www.euribor-ebf.eu/euribor-org/euribor-rates.html`. I consider the average rate offered by all the banks belonging in the Euribor panel at the last day of the month untill 2003:M12. From 2004:M1 onward, I consider the average rate offered by the Italian banks in the panel.[41] I match the model's risk free rate with the yields on German bonds with a 1 year maturity. This is obtained from the time series database of the Deutsche Bundesbank, at `http://www.bundesbank.de/Navigation/EN/Statistics/Time_series_databases/time_series_databases.html`. I construct a quarterly measure of the TED spreads by averaging the computed series over three months.

Figure A-3 plots these three time series.

Figure A-3: **Ted spread, financial leverage and the Lagrange multiplier**
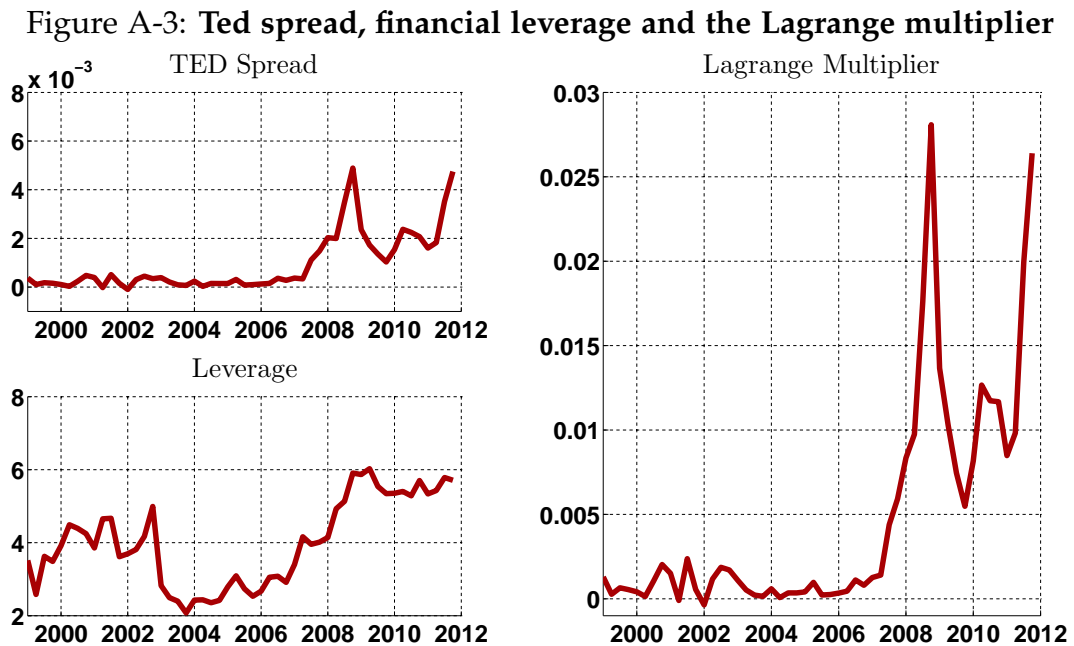
C.4   Other time series

GDP growth: Real GDP growth is the growth rate relative to previous quarter of real gross domestic product (*B1_EG*). Data are quarterly, 1980:Q1-2012:Q4. The source is *OECD*

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[41]Prior to 2004:M1, in fact, there is no information on the single banks belonging to the panel.

Quarterly National Accounts.

Consumption growth: Consumption growth is the growth rate relative to previous quarter of real private final consumption expenditure (*P31S14_S15*). Data are quarterly, 1980:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Spending-Output ratio: Spending-output ratio is general government final consumption expenditure divided by gross domestic product. Both series are seasonally adjusted and in volume estimates. Data are quarterly, 1980:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Investment-Output ratio: Investment-output ratio is gross capital formation divided by gross domestic product. Both series are seasonally adjusted and in volume estimates. Data are quarterly, 1980:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Labor income share: I obtain annual data (1970-2007) for and labor compensation (*LAB*) and value added (*VA*) from EU KLEMS database. Labor share is defined as $\frac{LAB}{VA}$. Data can be downloaded at `http://www.euklems.net/`.

Worked hours: Average numbers of hours worked per year by person engaged. I scale the series by $(24 - 8) \times 7 \times 52$. Data are annual (1970-2007), and obtained from EU KLEMS. Data can be downloaded at `http://www.euklems.net/`.

D Estimating the model without sovereign risk

The model without sovereign risk has five state variables $\mathbf{S}_t = [\hat{K}_t, \hat{P}_t, \hat{B}_t, \Delta z_t, g_t]$. Let $\mathbf{Y}_t$ be a 2×1 vector of observables collecting output growth and the time series for the multiplier on the leverage constraint. The state-space representation is

$$\mathbf{Y}_t = f_{\tilde{\theta}}(\mathbf{S}_t) + \eta_t \qquad \eta_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}) \tag{A.20}$$

$$\mathbf{S}_t = g_{\tilde{\theta}}(\mathbf{S}_{t-1}, \varepsilon_t) \qquad \varepsilon_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}). \tag{A.21}$$

The first equation is the measurement equation, with η_t being a vector of Gaussian measurement errors. The second equation is the transition equation, which represents the law of motion for the model's state variables. The vector ε_t represents the innovation to the structural shocks Δz_t and g_t. The function $f_{\tilde{\theta}}(.)$ and $g_{\tilde{\theta}}(.)$ are generated using the numerical procedure described in Appendix B applied to the model without sovereign risk. I characterize the posterior distribution of $\tilde{\theta}$ using full information Bayesian methods. I denote by $p(\tilde{\theta})$ the prior on $\tilde{\theta}$. In what follows, I provide details on the likelihood evaluation and on the posterior sampler adopted.

D.1 Likelihood evaluation

Let $\mathbf{Y}^t = [\mathbf{Y}_1, \ldots, \mathbf{Y}_t]$, and denote by $p(\mathbf{S}_t | \mathbf{Y}^{t-1}; \theta)$ the conditional distribution of the state vector given observations up to period $t - 1$. The likelihood function for the state-space model of interest can be expressed as

$$\mathcal{L}(\mathbf{Y}^T | \theta) = \prod_{t=1}^{T} p(\mathbf{Y}_t | \mathbf{Y}^{t-1}; \theta) = \prod_{t=1}^{T} \left[\int p(\mathbf{Y}_t | \mathbf{S}_t; \theta) p(\mathbf{S}_t | \mathbf{Y}^{t-1}; \theta) d\mathbf{S}_t \right]. \tag{A.22}$$

While the conditional density of $\mathbf{Y}_t$ given $\mathbf{S}_t$ is known and Gaussian, there is no analytical expression for the density $p(\mathbf{S}_t | \mathbf{Y}^{t-1}, \theta)$. I use the auxiliary particle filter of Pitt and Shephard (1999) to approximate this density via a set of pairs $\{S_t^i, \tilde{\pi}_t^i\}_{i=1}^{N}$. This approximation is then used to estimate the likelihood function.

> **Step 0: Initialization.** Set $t = 1$. Initialize $\{S_0^i, \tilde{\pi}_0^i\}_{i=1}^{N}$ from the model's ergodic distribution and set $\tilde{\pi}_0^i = \frac{1}{N} \, \forall i$.

> **Step 1: Prediction.** For each $i = 1, \ldots, N$, draw $\mathbf{S}_{t|t-1}^i$ values from the proposal density $g(\mathbf{S}_t | \mathbf{Y}^t, \mathbf{S}_{t-1}^i)$.

Step 2: Filtering. Assign to each $\mathbf{S}^i_{t|t-1}$ the particle weight

$$\pi^i_t = \frac{p(\mathbf{Y}_t|\mathbf{S}^i_{t|t-1};\theta)p(\mathbf{S}_t|\mathbf{S}^i_{t|t-1};\theta)}{g(\mathbf{S}_t|\mathbf{Y}^t,\mathbf{S}^i_{t-1})}.$$

Step 3: Sampling. Rescale the particles $\{\pi^i_t\}$ so that they add up to unity, and denote these rescaled values by $\{\tilde{\pi}^i_t\}$. Sample N values for the state vector with replacement from $\{\mathbf{S}^i_{t|t-1}, \tilde{\pi}^i_t\}^N_{i=1}$. Call each draw $\{\mathbf{S}^i_t\}$. If $t < T$, set $t = t+1$ and go to Step 1. Else, stop. $\square$

The likelihood function of the model is then approximated as

$$\mathcal{L}(\mathbf{Y}^T|\theta) \approx \frac{1}{N}\left(\prod_{t=1}^{T}\left[\frac{1}{N}\sum_{i=1}^{N}p(\mathbf{Y}_t|\mathbf{S}^i_{t|t-1};\theta)\right]\right).$$

Regarding the tuning of the filter, I set $N = 20000$. The matrix $\mathbf{\Sigma}$ is diagonal, and the diagonal elements equal 25% of the variance of the observable variables. The choice for the proposal density $g(\mathbf{S}_t|\mathbf{Y}^t,\mathbf{S}^i_{t-1})$ is more involved. I sample the structural innovations ε_t from $\mathcal{N}(m_t,\mathbf{I})$. Then, I use the model's transition equation (A.21) to obtain $\mathbf{S}^i_{t|t-1}$. The center for the proposal distribution for ε_t is generated as follows:

- Let $\overline{\mathbf{S}}_{t-1}$ be the mean for $\{\mathbf{S}^i_{t-1}\}$ over i.
- Set m_t to the solution of this optimization program

$$\mathrm{argmin}_\varepsilon\left\{[\mathbf{Y}_t - f_{\tilde{\theta}}(g_{\tilde{\theta}}(\overline{\mathbf{S}}_{t-1},\varepsilon))]'[\mathbf{Y}_t - f_{\tilde{\theta}}(g_{\tilde{\theta}}(\overline{\mathbf{S}}_{t-1},\varepsilon))] + \varepsilon'\mathbf{\Sigma}^{-1}\varepsilon\right\}.$$

The first part of the objective function pushes ε toward values such that the state vector can rationalize the observation $\mathbf{Y}_t$. The second part of the objective function imposes a penalty for ε that are far away from their high density regions. I verify that this proposal density results in substantial efficiency gains relative to the canonical particle filter, especially when the model tries to fit extreme observations for $\mathbf{Y}_t$.

D.2   Posterior sampler

I characterize the posterior density of $\tilde{\theta}$ using a Random Walk Metropolis Hastings with proposal density given by

$$q(\tilde{\theta}^p|\tilde{\theta}^{m-1}) \sim \mathcal{N}(\tilde{\theta}^{m-1}, c\mathbf{H}).$$

The sequence of draws $\{\tilde{\theta}^m\}$ is generated as follows

1. Initialize the chain at $\tilde{\theta}^1$.

2. For $m = 2, \ldots, M$, draw $\tilde{\theta}^p$ from $q(\tilde{\theta}^p | \tilde{\theta}^{m-1})$. The jump from $\tilde{\theta}^{m-1}$ to $\tilde{\theta}^p$ is accepted ($\tilde{\theta}^m = \tilde{\theta}^p$) with probability $\min\{1, r(\tilde{\theta}^{m-1}, \tilde{\theta}^p | \mathbf{Y}^T)\}$, and rejected otherwise ($\tilde{\theta}^m = \tilde{\theta}^{m-1}$). The probability of accepting the draw is

$$r(\tilde{\theta}^{m-1}, \tilde{\theta}^p | \mathbf{Y}^T) = \frac{\mathcal{L}(\mathbf{Y}^T | \tilde{\theta}^p) p(\tilde{\theta}^p)}{\mathcal{L}(\mathbf{Y}^T | \tilde{\theta}^{m-1}) p(\tilde{\theta}^{m-1})}.$$

First, I run the chain for $M = 10000$ with $\mathbf{H}$ being the identity matrix and $c = 0.001$. The chain is initialized from an estimation of the model using the Method of Simulated Moments.[42] I drop the first 5000 draws, and I use the remaining draws to initialize a second chain and to construct a new candidate density. This second chain is initialized at the mean of the 5000 draws. Moreover, the variance-covariance matrix $\mathbf{H}$ is set to the empirical variance-covariance matrix of these 5000 draws. The parameter c is fine tuned to obtain an acceptance rate of roughly 60%. I run the second chain for $M = 20000$. Posterior statistics are based on the latter 10000 draws.

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[42] The moments used in this step are: i) mean, standard deviation and autocorrelation for GDP growth and the multiplier; ii) correlation between GDP growth and the multiplier.

E Policy Experiments

E.1 Refinancing operations

It is instructive to first consider the stationary problem. The government allows bankers to borrow up to $\overline{m}$ at the fixed interest rate R_m, and this intervention is financed through lump-sum taxation. Moreover, these loans are not subject to limited enforcement problems. The decision problem of the banker becomes

$$v_b(n; \mathbf{S}) = \max_{a_B, a_K, b} \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi)n' + \psi v_b(n'; \mathbf{S}') \right] \right\},$$

$$n' = \sum_{j=\{B,K\}} [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})]Q_j(\mathbf{S})a_j + [R_m - R(\mathbf{S})]m - R(\mathbf{S})b,$$

$$\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j = n + b,$$

$$\lambda \left[\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j - m \right] \leq v_b(n; \mathbf{S}),$$

$$m \in [0, \overline{m}],$$

$$\mathbf{S}' = \mathbf{\Gamma}(\mathbf{S}).$$

Assuming that $m \geq 0$ does not bind,[43] the first order condition with respect to m is

$$\mathbb{E}_s \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi) + \psi \frac{\partial v_b(n'; \mathbf{S})}{\partial n'} \right] \right\} [R(\mathbf{S}) - R_m] + \lambda \mu(\mathbf{S}) = \chi(\mathbf{S})$$

It can be showed, following a similar logic of Result 1, that $v_b(n; \mathbf{S}) = \alpha(\mathbf{S})n + x(\mathbf{S})$, with $x(\mathbf{S}) \geq 0$. The leverage constraint becomes

$$\frac{\sum_j Q_j(\mathbf{S})a_j}{n} \leq \frac{\alpha(\mathbf{S})}{\lambda} + \frac{x(\mathbf{S})}{\lambda} + \overline{m}$$

Notice that refinancing operations have two main direct effects on banks. First, they represent an implicit transfer to banks. Indeed, to the extent that $(R_m < R(\mathbf{S}))$, banks benefit from the policy as their debt is subsidized. This has a positive effect on the net worth of banks relative to what would happen in the no-policy case. Second, the policy relaxes the leverage constraint of banks. This happens because of two distinct reasons: i) the loan from the government does not enter in the computation of the constrained level

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[43]This is not a restriction, as the policy considered involves an R_m substantially below R, meaning that bankers are willing to accept the loan.

of leverage (the $\overline{m}$ component); and ii) the value function of bankers increase as a result of the subsidized loan, this lowering the incentives of the banker to walk away.

E.2 Longer term refinancing operations (LTROs)

The LTROs are a non-stationary version of the refinancing operations discussed above. The government allows banker to borrow up to $\overline{m}$ in period $t = 1$, and they receive the prical and the interest in a later period T. Figure A-4 describes the timing of transfers between government and banks under LTROs.

Figure A-4: **Timing of LTROs**
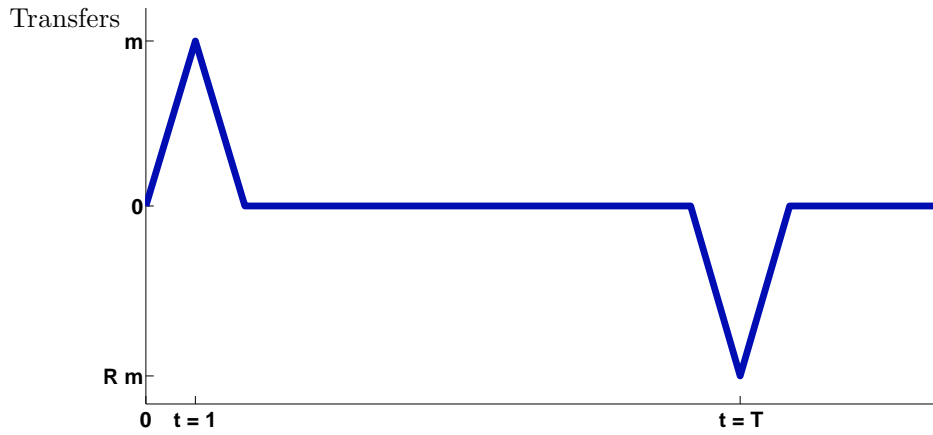
I assume that the policy was unexpected by agents. At time $t = 1$, agents are perfectly informed about the time path of the loans and they believe that the policy will not be implemented in the future. Note that the decision rules under LTROs are time dependent: the dynamics at $t = 1$ will be different from those at $t = T - 1$ as in the latter case we are getting closer to the repayement stage and banks will have a different behavior. In order to solve for the path of model's decision rules, I follow a backward induction procedure. From period $t = T + 1$ onward, the decision rules are those in absence of policy. Thus, at $t = T$, agents use those decision rules to form expectations. By solving the equilibrium conditions under this assumption and the repayment of the loan, we can obtain decision rules for $c_T(\mathbf{S}), R_T(\mathbf{S}), \alpha_T(\mathbf{S}), Q_{B,T}(\mathbf{S})$. At $t = T - 1$ we proceed in the same way, this time using $c_T(\mathbf{S}), R_T(\mathbf{S}), \alpha_T(\mathbf{S}), Q_{B,T}(\mathbf{S})$ to form expectations. More specifically, the policy functions in the transition $\{c_t(\mathbf{S}), R_t(\mathbf{S}), \alpha_t(\mathbf{S}), Q_{b,t}(\mathbf{S})\}_{t=1}^{T}$, are derived as follows:

1. **Period T**: Solve the model using $\{c(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), q(\mathbf{S})\}$ to form expectations. The multiplier is modified as follows

$$\mu_T(\mathbf{S}) = \max\left\{1 - \frac{\mathbb{E}_{\mathbf{S}}\{\Lambda_{T+1}(\mathbf{S}')[(1-\psi) + \psi\alpha_{T+1}(\mathbf{S})]\}R_T(\mathbf{S})(N'-m)}{\lambda\left(Q_{b,T}(\mathbf{S})B'_T + Q_{k,T}(\mathbf{S})K'_T\right)}, 0\right\}$$

Denote the solution by $\{c_T(\mathbf{S}), R_T(\mathbf{S}), \alpha_T(\mathbf{S}), q_T(\mathbf{S})\}$.

2. **Period** $t = T-1, \ldots, 1$: Solve the model using $\{c_{t+1}(\mathbf{S}), R_{t+1}(\mathbf{S}), \alpha_{t+1}(\mathbf{S}), q_{t+1}(\mathbf{S})\}$ to form expectations. The multiplier is modified as follows

$$\mu_t(\mathbf{S}) = \max\left\{\frac{\lambda\left(TA_t(\mathbf{S}) - m\mathbf{1}_{t=1}\right) - \mathbb{E}_{\mathbf{S}}\{\Lambda_{t+1}(\mathbf{S}')[(1-\psi)+\psi\alpha_{t+1}(\mathbf{S})]\}R_t(\mathbf{S})(N'+m\mathbf{1}_{t=1}) - \psi\mathbb{E}_{\mathbf{S}}[\Lambda_{t+1}(\mathbf{S}')\alpha_{t+1}(\mathbf{S}')x_{t+1}(\mathbf{S}')]}{\lambda\left(Q_{b,t}(\mathbf{S})B'_t + Q_{k,t}(\mathbf{S})K'_t\right)}, 0\right\}$$

where x_t follows the recursion $x_t(\mathbf{S}) = \frac{\lambda m\mu_t(\mathbf{S}) + \psi\mathbb{E}_S[\Lambda_{t+1}(\mathbf{S}')\alpha_{t+1}(\mathbf{S}')x_{t+1}(\mathbf{S}')]}{1-\mu_t(\mathbf{S})}$. The initial condition of this recursion is $x_T(\mathbf{S}) = -\alpha_T(\mathbf{S})m$. Store the solution. $\square$