Counting molecules

Bacterial Chemotaxis

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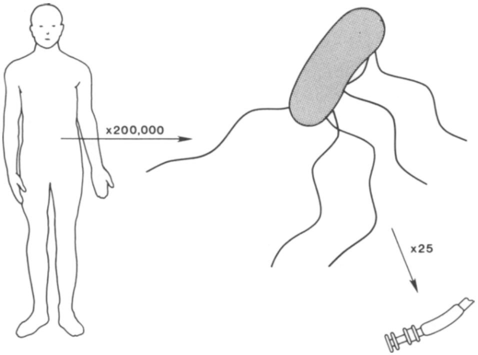
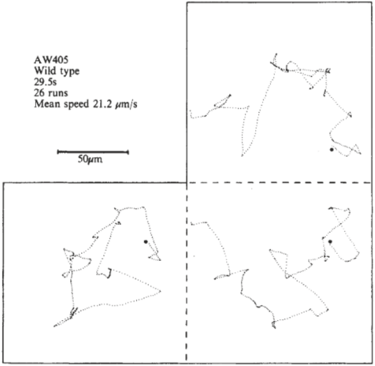
Physics/Neuro 141

Week 5

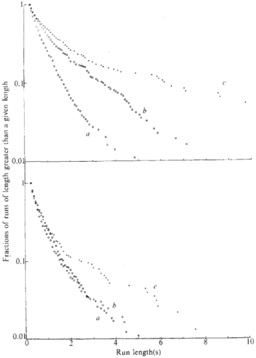
Berg and Brown, 1972



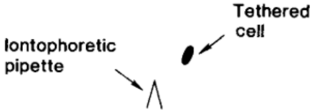
- Comparison of the size of man, *E. coli*, and part of *E. coli*'s flagellar motor.

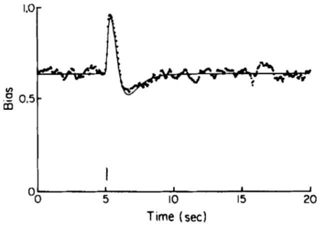


- Digital plots of the displacement of a wild type bacterium.
- Tracking began at the points indicated by the large dots.
- The plots are planar projections of three-dimensional paths.



- The data from the serine (top) and the aspartate (bottom) experiments plotted as the logarithm of the fractional number of runs of length greater than a given length.
- a, Runs in the control experiment; b, runs down the gradient; c, runs up the gradient

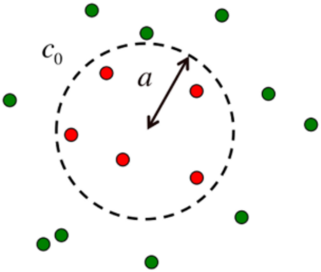




- A schematic illustration of experiments in which a tethered cell is simulated by ejection of a charged chemical from the tip of an iontophoretic pipette.
- The tethered cell is driven by one of its own flagellar motors.

- The response of tethered wild-type cells to a pulse of attractant (aspartate or a-methylaspartate) delivered iontophoretically.
- The dotted curve is the probability of CCW rotation (the bias). The stimuli were equivalent to a pulse that increases the receptor occupancy by 0.19 for a period of 0.02 sec.

Berg & Purcell 1977



- Simple measurement devices for concentration.
- The perfect monitor is permeable to ligand molecules and estimates the concentration c_0 by counting the molecules in its volume during time T

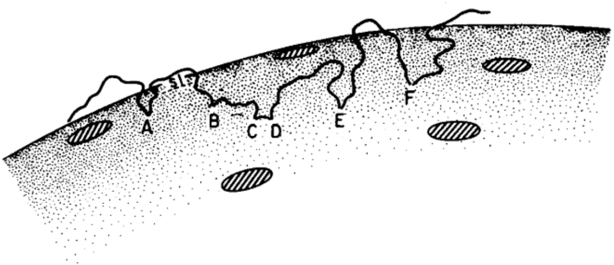
- Since the molecules diffuse independently, the number of molecules N will be Poisson distributed.
- Since for the Poisson distribution the variance equals the mean, i.e. $\delta N^2 = \bar{N}$:

$$\frac{\delta c^2}{c_0^2} = \frac{\delta N^2}{\bar{N}^2} = \frac{1}{\bar{N}} = \frac{1}{c_0 V}$$

- In time T, it can make $M \approx T/\tau_D$ independent measurements, where τ_D *approxa*$^2/D$ is the turnover time. This reduces uncertainty:

$$\frac{\delta c^2}{c_0^2} = \frac{1}{M\bar{N}} = \frac{1}{(T/\tau_D)c_0 V} \approx \frac{1}{Dac_0 T}$$

Patchy receptors



- The path of a diffusing molecule that has touched the surface of a cell of radius a at a sequence of points A, B, . . . F.
- The receptor patches, shown shaded, are of radius s.
- A and B constitute independent tries at hitting a patch, but C and D do not.
- Note between A and B the excursion of distance s perpendicular to the surface of the sphere.

- The probability P, that a molecule now located a distance s from the sphere of radius a will hit the surface of the sphere at least once before escaping to infinity is equivalent to the "capture probability" which we now rewrite as:

$$P_s = \frac{a}{a+s}$$

- The probability that a molecule now at $r = a + s$ will execute exactly n excursions to the surface, separated by reappearances at $r = a + s$ and followed by diffusion to infinity, is $P_s^n(1 - P_s)$. The average number of excursions is:

$$\bar{n} = \sum_{n=0}^{\infty} n P_s^n (1 - P_s) = \frac{P_s}{1 - P_s} = \frac{a}{s}$$

- The probability of not hitting a receptor patch in a single random encounter is $\beta = 1 - (Ns^2/4a^2)$.
- If the contacts we have just enumerated can be taken as independent tries, the probability that a molecule starting at $r = a + s$ survives all subsequent contacts until it escapes to infinity is:

$$P_{esc} = \sum_{n=0}^{\infty} \beta^n n P_s^n (1 - P_s) = \frac{1 - P_s}{1 - \beta P_s}$$
$$= \frac{4a}{4a + N_s}$$

- Since $1 - P_{esc}$ is the fraction of all arriving molecules that ultimately are captured, we have for the resulting current:

$$\frac{J}{J_{max}} = \frac{Ns}{4a + Ns}$$