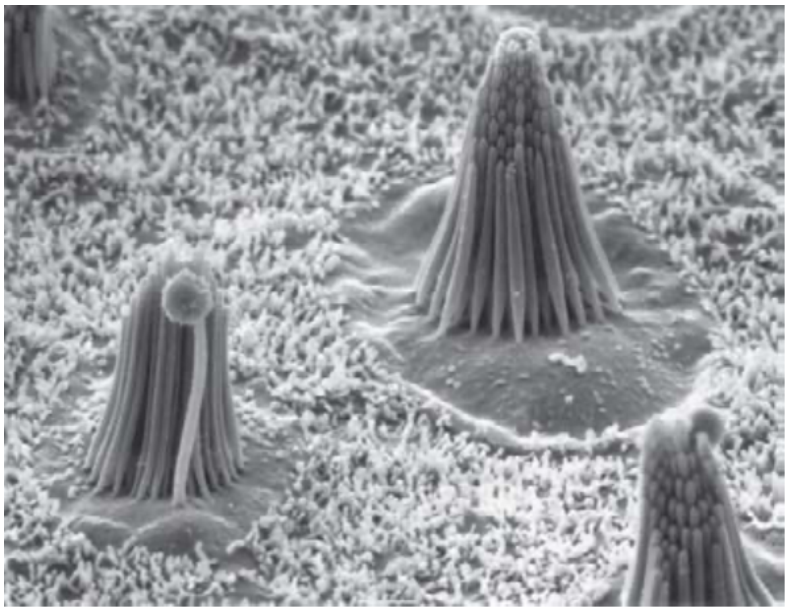
Hearing

Auditory illusions from nonlinear transduction

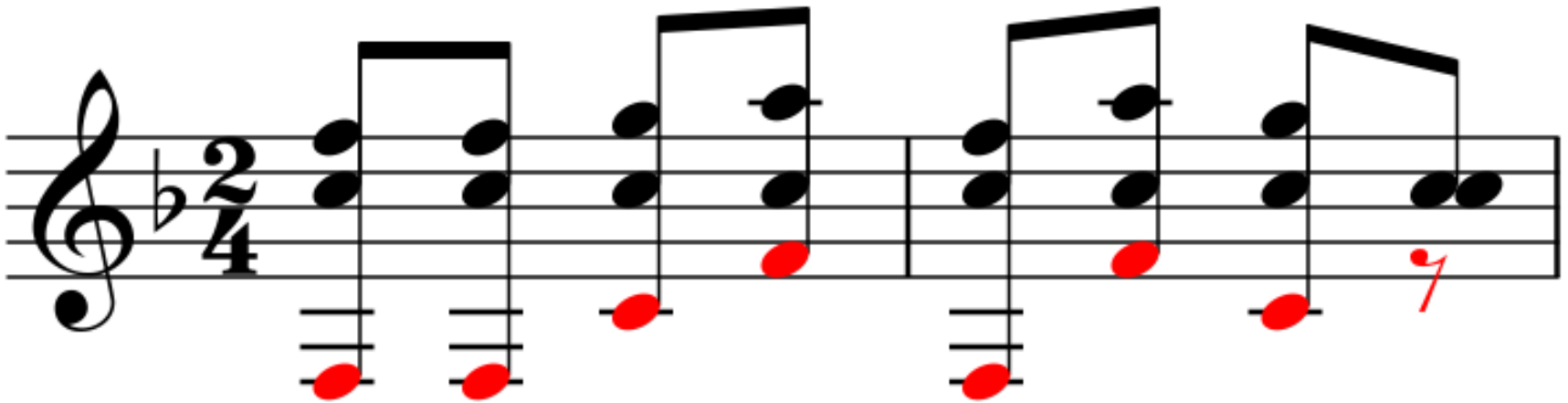
Aravi Samuel
David Zimmerman

Physics 141
Harvard University

Week 10

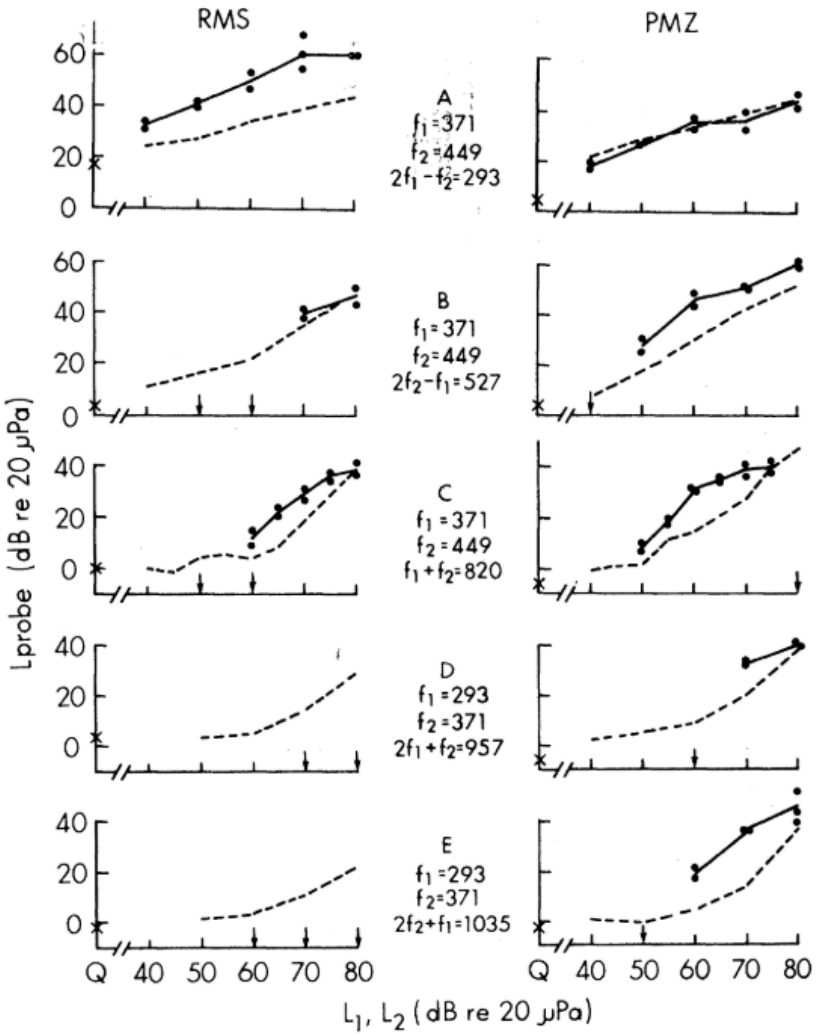


Combination tones



A combination tone is a psychoacoustic phenomenon of an additional tone or tones that are artificially perceived when two real tones are sounded at the same time. Their discovery is credited to the violinist Giuseppe Tartini.

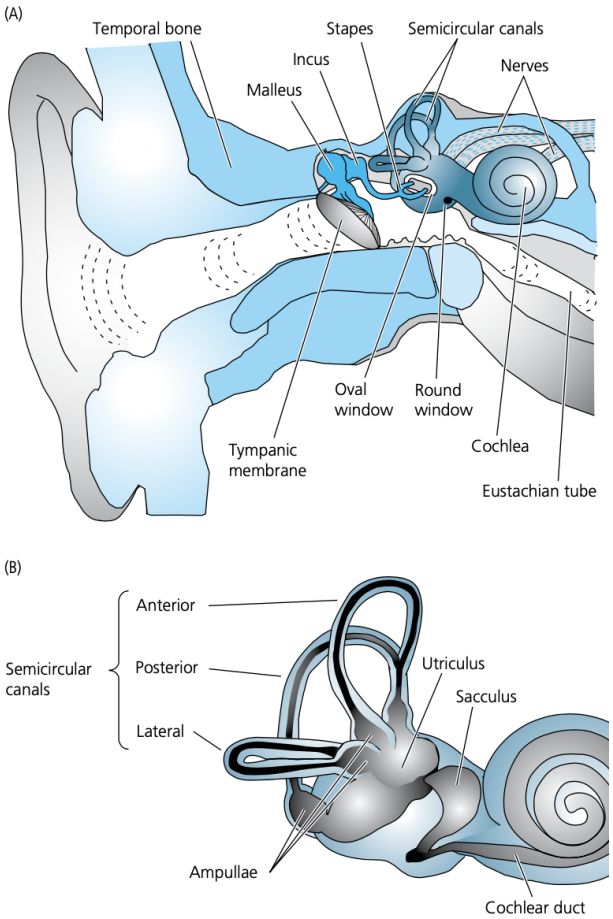
Combination tones

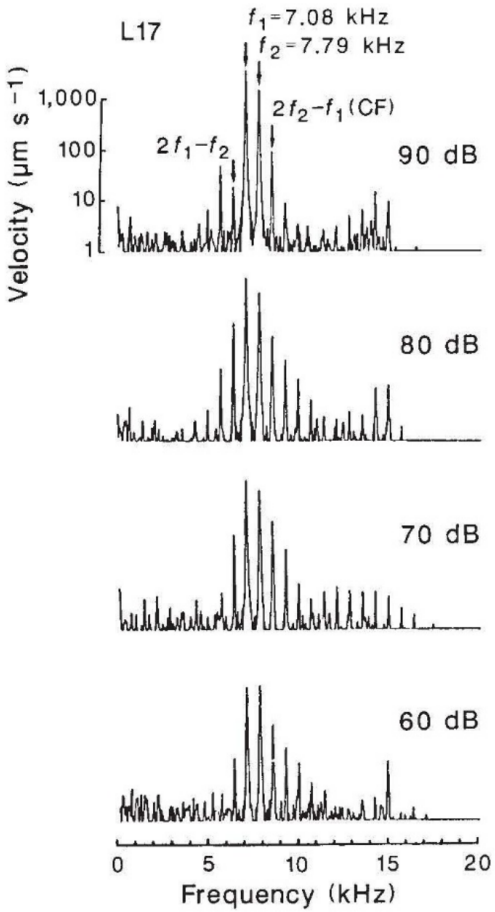


The existence of audible combinations at frequencies greater that the primary tones that generate them.

With primary tones at frequencies $\mathbf{f_1}$ and $\mathbf{f_2}$, combination tones at $\mathbf{f_1 + f_2}$, $\mathbf{2f_1 - f_2}$ and other frequencies can be demonstrated by using a contralateral probe tone to establish a binaural interaction with a given combination tone.

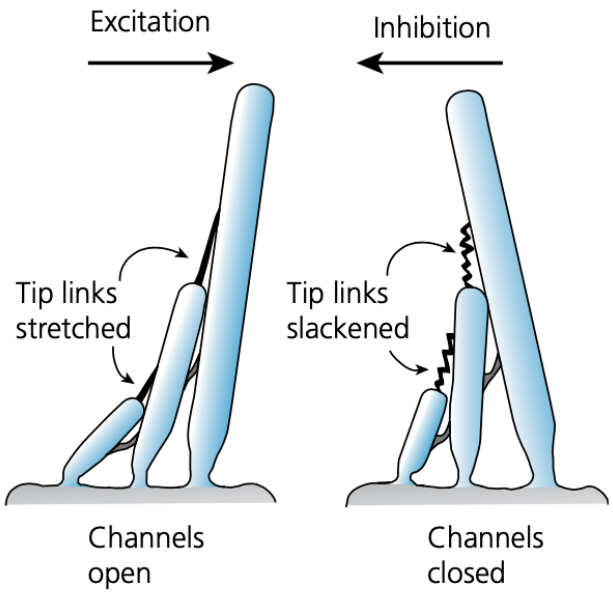
Two tone distortion in the basilar membrane of the cochlea





Frequency spectra of basilar membrane responses to two-tone stimuli measured using laser velocimetry at the basal turn of the cochlea

Deflection of the hair bundle acts to open mechanically sensitive ion channels

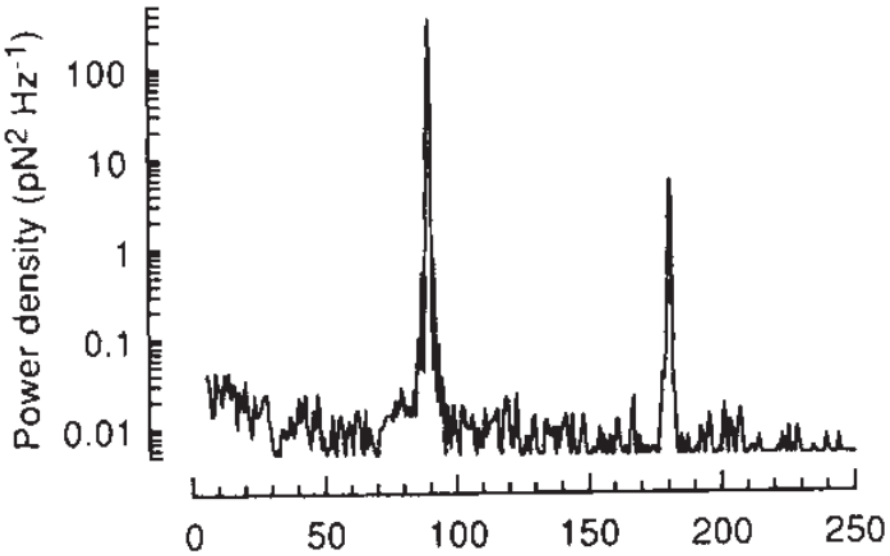


- A portion of the force F necessary to displace a hair bundle through a given distance X reflects the work done in pulling a channel's gates ajar.
- If a channel has three states, two closed and one open, then

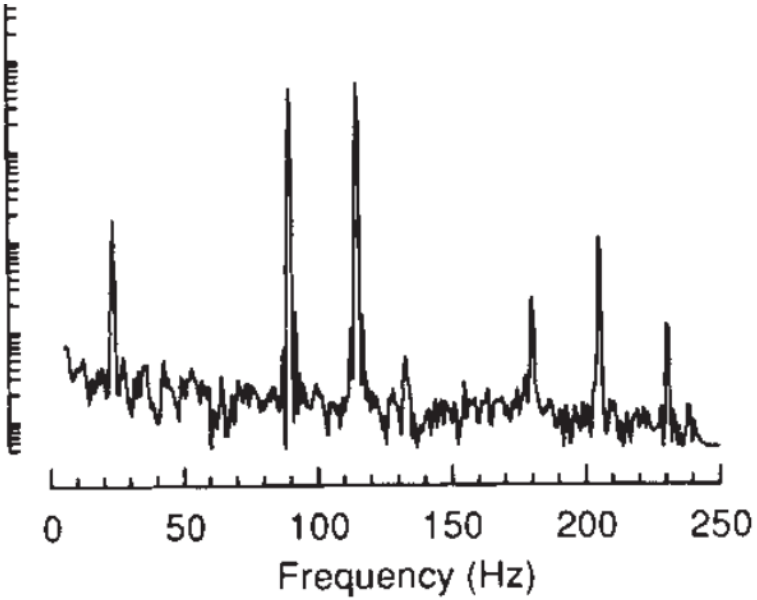
$$F = (\kappa_S + \kappa_G)X + C + [p_1(\Delta\kappa X + Nz_{12}) - p_3 Nz_{23}]$$

- Bundle deflection and force are linearly related when the bundle is extensively in the positive direction of negative direction.
- Over a middle range of bundle motions, compliance depends on displacement

Distortion products in the force produced by a hair bundle

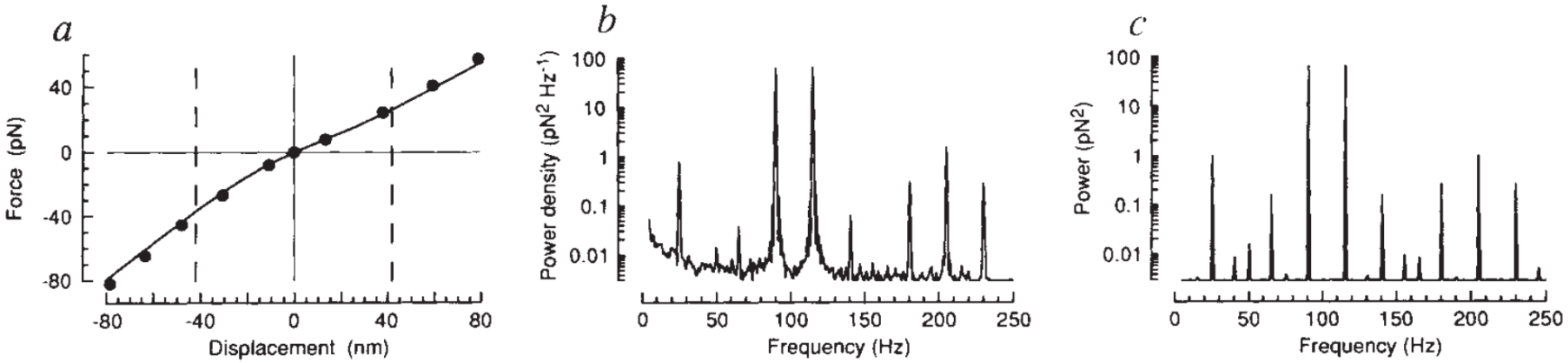


- When moved back and forth at 90 Hz, a hair bundle produced measurable force only at the stimulus frequency and second harmonic



- When a second frequency was added to the displacement command, the bundle produced forces at the difference and sum frequencies ($f_2 - f_1$ and $f_1 + f_2$) and at combination frequencies ($2f_1 - f_2$ and $2f_2 - f_1$).

Non-linearity creates distortion products



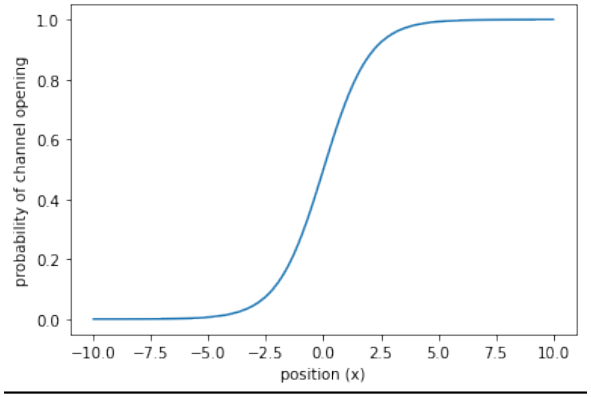
The relation between bundle displacement and force was determined by deflecting a bundle with stimulus pulses and measuring the flexion of the stimulus fiber.

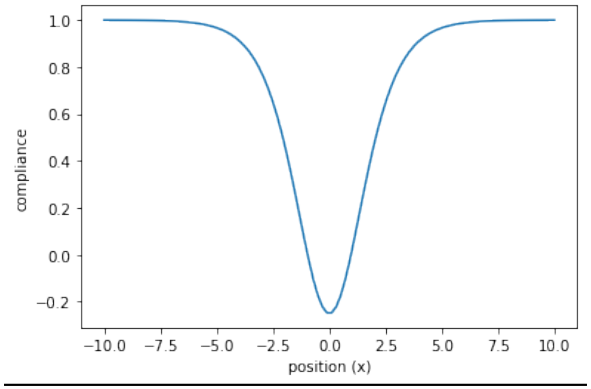
Stimulation of the hair bundle at 90Hz and 115 Hz elicited distortion products:

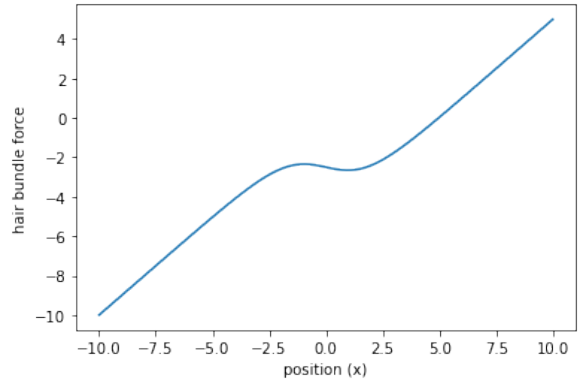
- $f_2 - f_1$ = 25 Hz
- $2f_2 - 2f_1$ = 50 Hz
- $2f_1 - f_2$ = 65 Hz
- $2f_2 - f_1$ = 140 Hz
- $2f_1$ = 180 Hz
- $f_1 + f_2$ = 205 Hz
- $2f_2$ = 230 Hz

Distortion products of similar frequency and magnitude arose in the power spectrum calculated from the bundle's measured stiffness and gating compliance. The primary frequencies and stimulus amplitudes were the same as those in (b).

My python experiment



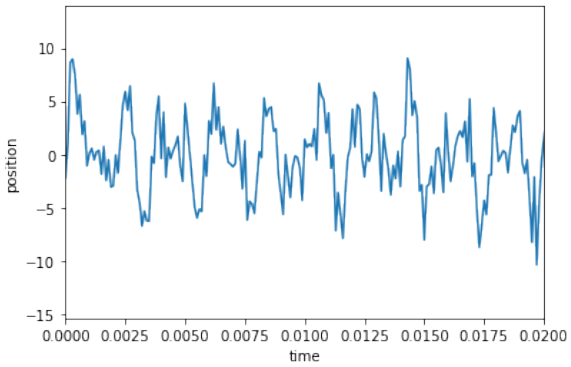


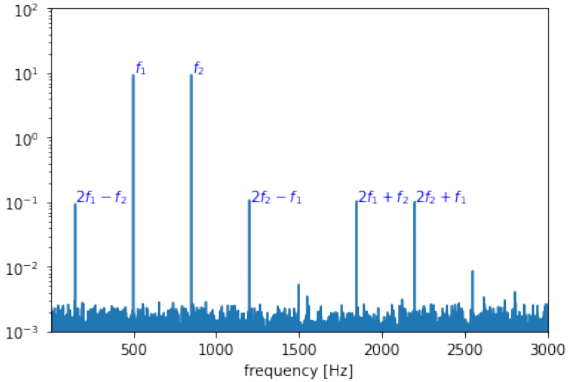


$$p = \frac{1}{1 + e^{-x}}$$

$$\frac{1}{k} = 1 - 5p(1-p)$$

$$F = 1 - 5p$$





Drive a hair bundle with sine waves at 500 Hz and 850 Hz
Power spectrum of the force contains distortion products!