

# Improved Techniques for Valuing Large-Scale Projects

BENJAMIN C. ESTY

**BENJAMIN C. ESTY** is an associate professor at Harvard Business School in Boston.

Sponsors typically use one of two methods to value equity investments in project finance deals. Either they value equity indirectly by discounting free cash flows (FCF) using the weighted average cost of capital (WACC) and subtracting debt value; or they value equity directly by discounting equity cash flows (ECF) using the cost of equity ( $K_E$ ). In either case, most use a *constant* discount rate for cash flows in all periods. Despite their prevalence, these approaches represent simple tools that were designed for simple applications. Most project finance investments, however, are not simple valuation problems. One feature that makes them complex is the fact that project leverage changes over time. For the typical project, the ratio of debt to total capitalization starts at 0%, rises to somewhere in the neighborhood of 60%–85%, and then falls back down to 0% in later years. Because the cost of equity is a function of leverage, both it and the WACC will change as leverage changes. Thus the use of a *single* discount rate for all years is inappropriate. A second, and related, problem with the standard approach is the measurement of leverage. Even though valuation theory dictates the use of market value leverage, most people measure leverage using book values. Failure to incorporate the effects of changing leverage or to measure leverage correctly can result in serious valuation errors.

In the first section of this article, I

describe these problems more fully and illustrate them using an equity cash flow valuation of a hypothetical project finance investment called PetroMexico. I then show how to address the problems using relatively simple techniques. The way to solve the problem of changing leverage is to use *multiple* discount rates rather than a single discount rate; the way to solve the problem of book value weights is to employ an alternative valuation technique called quasi-market valuation (QMV).

It is worth noting that both solutions — multiple discount rates and quasi-market valuation — are aimed at improving discount rate estimation and have nothing to do with improving cash flow projections. The failure to estimate cash flows accurately can also result in serious valuation errors. Although I do not address cash flow estimation in this paper for the simple reason that cash flow estimation tends to be project specific, I do show how Monte Carlo simulation can be used to analyze cash flow uncertainty. Continuing with my hypothetical example, I analyze PetroMexico's net present value (NPV), debt service coverage ratio (DSCR), and most likely year of default as functions of oil price volatility.

In the final section, I briefly discuss another new valuation tool called real options analysis, which can supplement discounted cash flow (DCF) analysis for valuing large-scale projects. Unlike DCF analysis,

real options analysis incorporates the benefits of managerial flexibility including the ability to defer action (not all investment decisions are "now or never" type decisions), to change strategy, and to abandon a bad project. Despite these benefits and the existence of well-developed theory showing that the real options approach is the right way to value large-scale investments, it can be difficult to implement accurately. As a result, it may take some time before it is generally accepted and commonly used in practice. Until that time, it is important to ensure that one uses the most accurate and efficient DCF tools possible, which is the point of this article.

## THE STANDARD APPROACH TO PROJECT VALUATION

To illustrate the problems associated with the standard valuation approaches and their solutions, I present a hypothetical project called PetroMexico, a \$2.02 billion oil-field development project in Mexico consisting of inland wells, a pipeline, and a coastal refinery. To build the facilities, the project requires capital expenditures of \$300 million, \$800 million, and \$600 million over the first three years; these expenditures will be depreciated using straight-line depreciation over fifteen years (see Exhibit 1). The sponsors, two large integrated oil companies, will fund 64% of the project's total cost with two rounds of non-recourse bank debt: \$700 million in the first year and \$600 million in the second year. The debt will carry a 10% coupon and will be fully amortized over fourteen years. The sponsors will fund the remaining 36% (\$724 million) with equity investments in years 0, 1, and 2. Total investment during the first three years of \$2.02 billion exceeds total capital expenditures of \$1.70 billion by \$324 million. This amount includes funds for start-up administrative expenses, interest expense during construction, net working capital, and a debt service reserve account (DSRA) equal to six months of principal and interest.

Beginning in year 3, the project will produce fifty million barrels of Maya crude per year at an operating cost of \$4.00 per barrel. One of the sponsors will buy all of PetroMexico's output at market prices assumed to be \$11.40 per barrel (in year 3). Both the operating costs and revenues are assumed to grow at 1.0% per year. For simplicity, I assume the project ends

in year 25 when all of the oil is depleted; the tax rate is constant at 35%; and the costs and revenues are denominated in U.S. dollars.

Because the FCF/WACC and the ECF/ $K_E$  approaches are theoretically equivalent, I could use either method to value the equity invested in PetroMexico. I prefer the ECF/ $K_E$  method for three reasons. First, it uses actual, estimated tax rates in the cash flows rather than incorporating a single, constant tax rate in the discount rate. Projects that generate net operating losses, receive tax credits, or are located in countries with non-linear tax regimes will not have a constant, marginal tax rates over time. Second, the FCF/WACC approach assumes the net present value of debt is zero. Under this assumption, it is correct to assume that firm or project value minus debt value equals equity value ( $V - D = E$ ). When this assumption does not hold, the FCF/WACC approach will result in errors. And finally, it is conceptually much more difficult to value multiple rounds of equity financing, particularly if different sponsors invest at different times, using the FCF/WACC approach. Because it is easier to implement the ECF approach correctly, I focus on it throughout the rest of the article.<sup>1</sup>

The first step in ECF valuation is to calculate the equity cash flows assuming all residual cash flows are distributed to the sponsors as dividends (see Exhibit 1).

$$\begin{aligned} \text{ECF} = & \text{Cash available for debt service (CADS)} \\ & - \text{Principal payments} \\ & - \text{Interest payments} \\ & - \text{Equity investments} \end{aligned}$$

where

$$\begin{aligned} \text{CADS} = & \text{Earnings before interest and taxes (EBIT)} \\ & + \text{Depreciation} \\ & - \text{Cash Taxes} \\ & - \text{Capital Expenditures} \\ & - \text{Increases in net working capital (NWC)} \\ & - \text{Funds for the debt service reserve account (DSRA)} \end{aligned}$$

The next step is to estimate the expected cost of equity ( $K_E$ ) using the capital asset pricing model (CAPM). According to the CAPM, the cost of equity, or the expected return on equity from the investors' perspective, is a function of the risk-free rate ( $R_f$ ), an equity or levered beta ( $\beta_E$ ), and a market risk premium ( $R_M - R_f$ ).

# EXHIBIT 1

Financial Structure and Cash Flow Statement for PetroMexico (US\$ in thousands, unless otherwise indicated)

Year	Forecasted Price of Crude Oil (per Barrel)	Total Revenue	Operating Expenses	Earnings Before Income Taxes (EBIT)	Depreciation	Capital Expenditures	Cash Taxes	Net Working Capital (NWC)	Increase in Net Working Capital (NWC)	Funding of the Debt Service Reserve Account
0		\$0	\$0	\$0	\$0	(\$300,000)	\$0	\$0	\$0	\$0
1		0	0	(20,000)	20,000	(800,000)	0	0	0	0
2		0	0	(73,333)	73,333	(600,000)	0	46,849	(46,849)	(77,500)
3	\$11.40	570,000	(200,000)	256,667	113,333	0	(44,333)	47,320	(471)	1,250
4	11.51	575,729	(202,000)	260,395	113,333	0	(46,513)	47,796	(476)	1,250
5	11.63	581,515	(204,020)	264,161	113,333	0	(48,707)	48,276	(480)	(11,250)
6	11.75	587,359	(206,060)	267,966	113,333	0	(50,913)	48,761	(485)	2,500
7	11.87	593,262	(208,121)	271,806	113,333	0	(54,008)	49,251	(490)	(10,000)
8	11.98	599,225	(210,202)	275,689	113,333	0	(57,116)	49,746	(495)	3,750
9	12.10	605,247	(212,304)	279,609	113,333	0	(61,113)	50,246	(500)	3,750
10	12.23	611,330	(214,427)	283,569	113,333	0	(65,124)	50,751	(505)	(8,750)
11	12.35	617,474	(216,571)	287,596	113,333	0	(69,149)	51,261	(510)	5,000
12	12.47	623,679	(218,737)	291,609	113,333	0	(74,063)	51,777	(515)	(7,500)
13	12.60	629,947	(220,924)	295,690	113,333	0	(78,991)	52,297	(520)	6,250
14	12.73	636,279	(223,134)	299,811	113,333	0	(84,809)	52,822	(526)	(6,250)
15	12.85	642,673	(225,365)	303,975	113,333	0	(0,641)	53,353	(531)	7,500
16	12.98	649,132	(227,619)	328,180	93,333	0	(104,363)	53,890	(536)	7,500
17	13.11	655,656	(229,895)	385,761	40,000	0	(129,766)	54,431	(542)	82,500
18	13.24	662,246	(232,194)	430,052	0	0	(150,518)	54,978	(547)	0
19	13.38	668,901	(234,516)	434,385	0	0	(152,035)	55,531	(553)	0
20	13.51	675,624	(236,861)	438,763	0	0	(153,567)	56,089	(558)	0
21	13.65	682,414	(239,229)	443,184	0	0	(155,115)	56,653	(564)	0
22	13.79	689,272	(241,622)	447,650	0	0	(156,678)	57,222	(569)	0
23	13.92	696,200	(244,038)	452,162	0	0	(158,257)	57,797	(575)	0
24	14.06	703,196	(246,478)	456,718	0	0	(159,851)	58,378	(581)	0
25	14.21	710,264	(248,943)	461,321	0	0	(161,462)	58,962	(584)	0

## EXHIBIT 1 (CONT'D)

Financial Structure and Cash Flow Statement for PetroMexico (US\$ in thousands, unless otherwise indicated)

Cash Available for Debt Service (CADS)	Debt Service			Total Debt Service	Debt Service Coverage Ratio (DSCR)*	Equity Cash Flow (ECF)
	Principal Outstanding	Principal Payments	Interest Payments			
(\$300,000)	\$0	\$0	\$0	\$0		(\$300,000)
(800,000)	700,000	0	70,000	(630,000)		(170,000)
(724,349)	1,300,000	0	130,000	(470,000)		(254,349)
326,446	1,300,000	25,000	130,000	155,000	2.11	171,446
327,990	1,275,000	25,000	127,500	152,500	2.15	175,490
317,058	1,250,000	25,000	125,000	150,000	2.11	167,058
332,401	1,225,000	50,000	122,500	172,500	1.93	159,901
320,643	1,175,000	50,000	117,500	167,500	1.91	153,143
335,161	1,125,000	75,000	112,500	187,500	1.79	147,661
322,523	975,000	75,000	97,500	172,500	1.87	150,023
336,243	900,000	100,000	90,000	190,000	1.77	146,243
322,864	800,000	100,000	80,000	180,000	1.79	142,864
335,761	700,000	125,000	70,000	195,000	1.72	140,761
321,560	575,000	125,000	57,500	182,500	1.76	139,060
333,636	450,000	150,000	45,000	195,000	1.71	138,636
324,114	300,000	150,000	30,000	180,000	1.80	144,114
377,953	150,000	150,000	15,000	165,000	2.29	212,953
278,987	0	0	0	0		278,987
281,798	0	0	0	0		281,798
284,638	0	0	0	0		284,638
287,506	0	0	0	0		287,506
290,403	0	0	0	0		290,403
293,330	0	0	0	0		293,330
296,286	0	0	0	0		296,286
299,275	0	0	0	0		299,275

\*DSCR = Cash Available for Debt Service (CADS) ÷ Total Debt Service.

$$K_E = R_f + \beta_E (R_M - R_f) \quad (1)$$

The equity beta is, in turn, a function of the project's asset or unlevered beta ( $\beta_A$ ) and its leverage ( $V/E$ ).<sup>2</sup>

$$\beta_E = \beta_A (V/E) \quad (2)$$

Here the leverage ratio is defined as firm value ( $V$ ) divided by equity value ( $E$ ), where firm value is the sum of debt and equity values ( $V = D + E$ ).

The project's asset beta can be estimated using

regression analysis and a set of comparable, publicly-traded firms. For the purposes of this analysis, I assume the asset beta for an integrated oil producer is 0.60. Given PetroMexico's funding mix of 64.2% debt ( $D/V$ ) and 35.8% equity ( $E/V$ ), its equity beta is:

$$\beta_E = \beta_A (V/E) = (0.60)(100.0\%/35.8\%) = 1.68 \quad (3)$$

Plugging this value into the CAPM (Equation 1) yields an expected cost of equity of 20.4%, assuming a long-term risk-free rate of 8.0% and a historical market-risk

premium of 7.4% over long-term treasuries.<sup>3</sup>

$$K_E = 8.0\% + (1.68)(7.4\%) = 20.4\% \quad (4)$$

Armed with a set of equity cash flow projections and the cost of equity, it is straightforward to calculate the project's net present value. Assuming year 0 cash flows occur immediately (i.e., the discount factor — the reciprocal of the discount rate — is 1.000), the NPV is negative \$60.3 million (see Exhibit 2). Because the NPV is negative, the sponsors should reject the project as failing to earn an appropriate risk-adjusted rate of return.<sup>4</sup>

## PROBLEMS WITH THE STANDARD APPROACH

The standard valuation approaches were developed at a time when financial analysis was both time consuming and costly in terms of computing resources, and when firms had stable, long-run capital structures. To reduce the computational complexity, practitioners used a single, *constant* discount rate. This approach is incorrect because the project's cost of equity is a function of its equity beta which, in turn, is a function of leverage (see Equations (1) and (2)). As seen in Exhibit 2, the project's leverage begins at 0%, rises to 64% in year 2, and then falls back down to 0% in year 17. When leverage changes, the discount rate must change. Exhibit 3A presents this phenomenon graphically. Calculating the project's cost of equity using the point of maximum leverage — typically the project's funding mix — causes the cost of equity to be overstated for most years because leverage is overstated for most years. Even simple adjustments, such as using the project's average debt ratio, are highly unlikely to yield the right answer (see Exhibit 3B).<sup>5</sup>

This miss-estimation of the cost of equity also distorts the calculation of the WACC.<sup>6</sup> So whether you use the FCF/WACC or the ECF approach, you will get incorrect answers with a single discount rate. Nevertheless, most valuation textbooks such as Ehrhardt [1994, p. 76], Finnerty [1996, chapter 7], and Copeland et al. [1996, pp. 249-250] recommend using the project's "target" capital structure to calculate the appropriate discount rate, a recommendation that will lead to errors when applied to project finance investments.

The way to solve this problem is to calculate a *different* discount rate for every year based on the lever-

age in existence at that time (see Exhibit 3C), an approach recommended by Damodaran [1994, p. 315] and Grinblatt and Titman [1998, pp. 323, 458]. For example, the correct discount rate for cash flows occurring in year two should be:

$$\text{Discount rate in year 2} = [1/(1 + K_{E,1})][1/(1 + K_{E,2})] \quad (5)$$

where  $K_{E,1}$  and  $K_{E,2}$  are the appropriate risk-adjusted costs of equity for years 1 and 2, respectively. Because the discount rate should incorporate *cumulative* risk through time, the following expression is not appropriate because it does not reflect the fact that year 1 risk may be different from year 2 risk (i.e., if leverage differs over time):

$$\text{Discount rate in year 2} = [1/(1 + K_{E,2})]^2 \quad (6)$$

Exhibit 2 shows the valuation using the multiple discount rate approach. The cost of equity ranges from 12.4% when the project is fully equity funded to 20.4% when the project is at maximum leverage. The final column shows the difference between the standard and multiple discount rate approaches: the cost of equity can differ by as much as 8% per year! For people familiar with the FCF/WACC approach, the fact that increasing leverage causes the cost of equity to rise, but causes the WACC to fall may appear counterintuitive. This phenomenon happens for two reasons. First, as leverage and the cost of equity increase, the weighted average falls because you are using less equity (the more expensive source of capital). Second, and more importantly, the standard WACC formula does not incorporate the effects of risky debt or the costs of financial distress, both of which would cause it to increase with leverage.

Interestingly, when the project is valued using the multiple discount rate approach, it has *positive* NPV of \$11.4 million, which implies the sponsors should *accept* the project. This example illustrates how the standard approach can result not only in valuation errors, but also in faulty managerial decision making.

What this alternative approach highlights is the critical links among leverage, equity risk, and equity returns. Because of this link, there is a need to calculate leverage ratios every year. According to finance theory, investors demand returns based on the market value of their investment, not on the historical book value. Except in very rare circumstances (when market

# EXHIBIT 2

Valuation Using Book Value Weights (U.S. dollar in thousands, unless otherwise indicated)

Year	Equity Cash Flows (ECF)	Total Equity	Total Debt	Debt to Value Ratio (D/V)	Standard Approach (Book Value Leverage)		Multiple Discount Rates Approach (Book Value Leverage)		Difference Between Costs of Equity (K <sub>E</sub> 's)
					Cost of Equity (K <sub>E</sub> )	Present Value (PV)	Cost of Equity (K <sub>E</sub> )	Present Value (PV)	
0	(300,000)	\$300,000	\$0	0.0%	20.4%	(\$300,000)	12.4%	(\$300,000)	8.0%
1	(170,000)	470,000	700,000	59.8%	20.4%	(141,186)	19.1%	(142,794)	1.4%
2	(254,349)	724,349	1,300,000	64.2%	20.4%	(175,435)	20.4%	(177,433)	0.0%
3	171,446	724,349	1,275,000	63.8%	20.4%	98,210	20.3%	99,455	0.2%
4	175,490	724,349	1,250,000	63.3%	20.4%	83,488	20.1%	84,762	0.3%
5	167,058	724,349	1,225,000	62.8%	20.4%	66,006	19.9%	67,270	0.5%
6	159,901	724,349	1,175,000	61.9%	20.4%	52,470	19.6%	53,817	0.8%
7	153,143	724,349	1,125,000	60.8%	20.4%	41,735	19.3%	43,191	1.1%
8	147,661	724,349	1,050,000	59.2%	20.4%	33,420	18.9%	35,032	1.5%
9	155,080	724,349	975,000	57.4%	20.4%	29,150	18.4%	31,070	2.0%
10	150,023	724,349	900,000	55.4%	20.4%	23,420	18.0%	25,482	2.5%
11	146,243	724,349	800,000	52.5%	20.4%	18,960	17.3%	21,168	3.1%
12	142,864	724,349	700,000	49.1%	20.4%	15,383	16.7%	17,715	3.7%
13	140,761	724,349	575,000	44.3%	20.4%	12,587	16.0%	15,052	4.4%
14	139,060	724,349	450,000	38.3%	20.4%	10,328	15.2%	12,908	5.2%
15	138,636	724,349	300,000	29.3%	20.4%	8,551	14.3%	11,261	6.1%
16	144,114	724,349	150,000	17.2%	20.4%	7,382	13.4%	10,326	7.0%
17	212,953	724,349	0	0.0%	20.4%	9,060	12.4%	13,570	8.0%
18	278,987	724,349	0	0.0%	20.4%	9,857	12.4%	15,811	8.0%
19	281,798	724,349	0	0.0%	20.4%	8,269	12.4%	14,204	8.0%
20	284,638	724,349	0	0.0%	20.4%	6,937	12.4%	12,760	8.0%
21	287,506	724,349	0	0.0%	20.4%	5,819	12.4%	11,462	8.0%
22	290,403	724,349	0	0.0%	20.4%	4,881	12.4%	10,297	8.0%
23	293,330	724,349	0	0.0%	20.4%	4,095	12.4%	9,250	8.0%
24	296,286	724,349	0	0.0%	20.4%	3,435	12.4%	8,309	8.0%
25	299,275	724,349	0	0.0%	0.4%	2,882	12.4%	7,465	8.0%
					NPV = (\$60,298)		NPV = \$11,410		



values and book values are exactly the same in all time periods), one cannot use the capital structure measured using book values. PetroMexico, like most approved investments, is expected to have a positive NPV, which implies that the market value of the project's equity will exceed its book value. When this is true, the overstated book value of leverage ( $D/V$ ) will cause the cost of equity to be overstated and the resulting NPV to be understated.

The solution to this problem, at least in theory, is to measure the capital structure using *market* values. Unfortunately, you do not know either the current or future market values of equity before you begin the project. In fact, the problem is inherently circular: you are discounting to get the present value of equity, but you need the present value of equity to calculate the right discount rate. A relatively new valuation technique called quasi-market valuation (QMV) can be used to solve this problem. Richard Ruback of the Harvard Business School developed this technique to value highly-leveraged transactions such as leveraged buyouts (LBOs); yet QMV works equally well, if not better, for project finance investments.

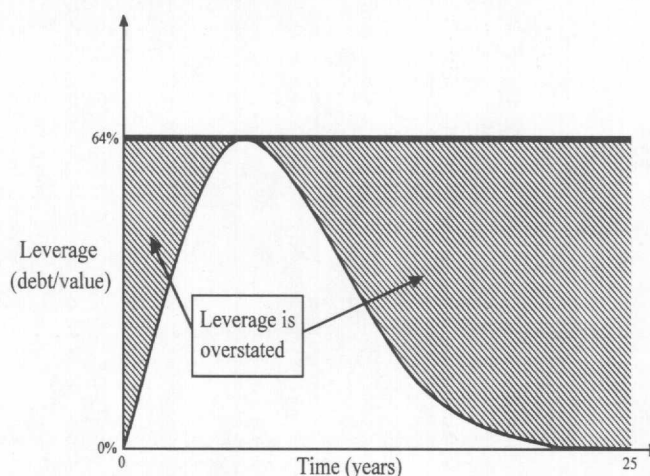
There are three key assumptions underlying quasi-market valuation. First, it assumes that the capital asset pricing model works; in other words, that equity earns its expected rate of return in each period. As a result, the market value of equity at the end of period  $t$  ( $E_t$ ) equals the market value at the beginning of the period ( $E_{t-1}$ ) plus the return on equity earned during the period ( $ROE_t = E_{t-1}K_{E, t-1}$ ), adjusted for equity infusions ( $INV_t$ ) and dividend payments made during the period ( $D_t$ ).

$$E_t = E_{t-1} + ROE + INV_t - D_t \quad (7)$$

The second assumption is that the book value of debt equals the market value of debt. This commonly-made assumption holds except for firms in financial distress. With this assumption, there is sufficient information to calculate a discount rate in all but the first year. Determining the capital structure at time zero requires the third assumption, an assumption regarding the efficiency of markets. The idea here is that market prices adjust to incorporate available information as soon as it is known. Accordingly, I assume that the project's NPV accrues to the initial equity holders on the day the project begins. Thus, the market value at the end of year 0 equals the initial equity investment plus the expected

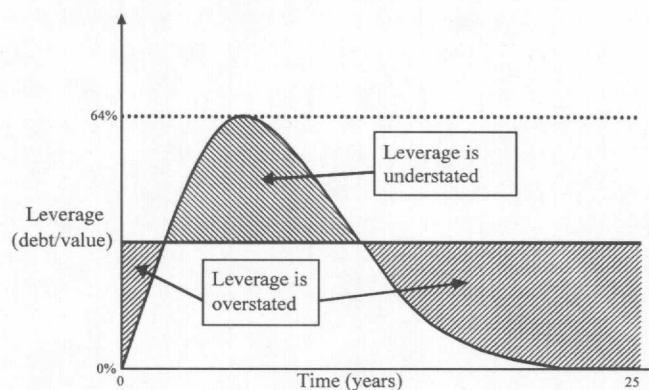
### EXHIBIT 3A

#### Cost of Equity Calculated Using Maximum Leverage



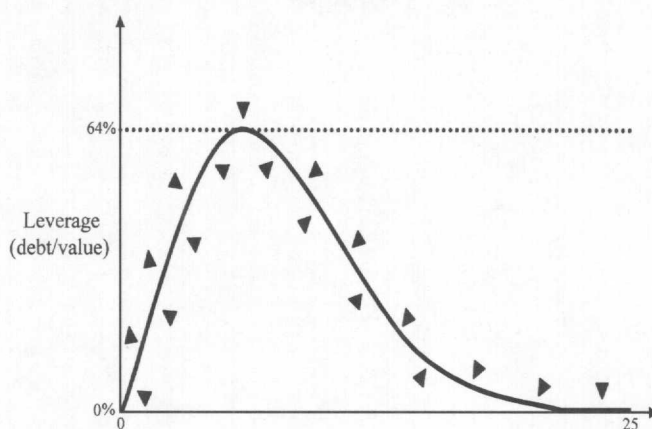
### EXHIBIT 3B

#### Cost of Equity Calculated Using Average Leverage



### EXHIBIT 3C

#### Cost of Equity Calculated Using Changing Leverage



NPV of the project which, of course, is not known at the time. Hence the circularity problem reappears. It can, however, be solved through iteration — for example, by using the “solver” function in Microsoft® Excel. The one thing that is known at time zero is that the project’s ending equity value in year 25 must be zero because the project ends.<sup>7</sup> Iteration will now solve the following problem: what initial net present value will create a capital structure and concomitant discount rates such that the final equity value will be equal to zero and the discounted present value of equity cash flows will equal the initial present value of equity?<sup>8</sup>

Exhibit 4 presents the quasi-market valuation of the PetroMexico project. Looking at year 1, the ending equity value of \$627.280 million can be calculated as follows:

Beginning Equity Value	= \$406,688
+ Equity Investment	= \$170,000
+ Return on Equity	= \$ 50,592
	= \$406,688 × 12.4%
- Dividends	= \$ 0
= Ending Equity Value	= \$627,280

where 12.4% is the year 0 expected return on equity (technically 12.44% = 8.0% + 0.60 × 7.4% from the CAPM). Under the QMV approach, discount rates are based on the cost of equity at the *beginning* of the year — what equity holders expect to earn over the year — which is equal to the cost of equity at the *end* of the prior year. In contrast, most people use the end-of-year capital structure to determine discount rates even though investors form expectations at the beginning of the year — the difference can be significant when leverage is changing.

The computer runs through these calculations for all years, discounts the equity cash flows to get an NPV, plugs the calculated NPV back in as the project’s NPV in year 0, and repeats until the valuation converges on a solution. Each time the project NPV changes, the capital structure changes, the discount rates change, and the resulting NPV changes until conversion. In this case, the valuation converged to an NPV of \$107 million, which is significantly above the NPV found using either the standard approach or the modified approach with multiple discount rates at *book* values. Based on the QMV approach, the managerial decision is to accept the project.

The final concern regarding the standard approach involves the use of discount rate adjustments

to reflect country-specific political risks such as expropriation.<sup>9</sup> Academics such as Lessard [1996], and investment bankers such as Abuaf and Chu [1994], Mariscal and Dutra [1996], and Godfrey and Espinosa [1997], recommend adding a country risk premium (CRP) to the cost of equity in Equation (1):

$$K_E^* = R_f + \beta_E(R_M - R_f) + \text{CRP} \quad (8)$$

When available, most people use the spread between dollar-denominated sovereign debt (Yankee, Euro, or Brady Bonds) and U.S. Treasury bonds as a proxy for country-specific credit risk. For PetroMexico, the country risk premium would be the spread between Mexican Brady bonds and ten-year U.S. Treasury bonds (see Exhibit 5).

Ignoring the problems of trading liquidity, changing spreads over time, and data availability (not all countries have dollar-denominated sovereign debt) as impediments to using this approach to measure country risk, there is the larger question of whether sovereign credit risk belongs in the discount rate in the first place. According to valuation theory for integrated capital markets, a project’s discount rate should incorporate only systematic or market risk, while the cash flows should reflect unsystematic or diversifiable risks. Technically the cash flows should be *expected* cash flows — i.e., the probabilistically weighted average of various future cash flow scenarios.<sup>10</sup> The key questions really are what risks are being considered and do they have a systematic component?

In the case of expropriation, I do not know of evidence that shows it is a systematic risk, which implies that any adjustment for expropriation should be done to the cash flows.<sup>11</sup> One can show, however, that adjusting the cash flows for a probability of default (i.e., non-payment for any reason including expropriation) is approximately equal to adding a country risk premium to the discount rate (see the Appendix). Assuming  $\lambda$  is the probability of default and CRP is the country risk premium, then the approaches illustrated in Equations (9) and (10) yield approximately equal present values when the country risk premium is small and equal to the probability of default (CRP =  $\lambda$ ).

Adjust the discount rate:

$$PV_0(ECF_t) = \sum_{t=1}^T \frac{ECF_t}{(1 + K_E + \text{CRP})^t} \quad (9)$$



# EXHIBIT 4

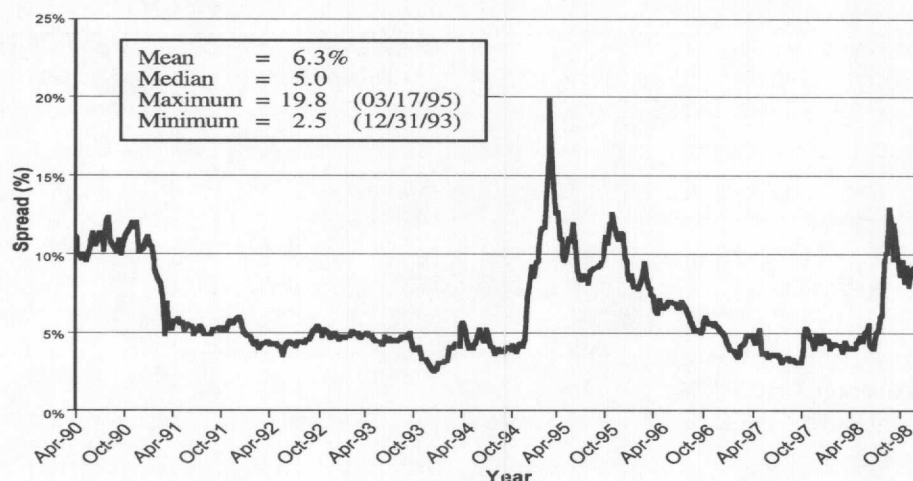
Quasi-market Valuation (U.S. dollars in thousands, unless otherwise indicated)

Year	Equity Cash Flows (ECF)	Beginning Equity Value (Quasi-market)	Project NPV	Equity Investment	Return on Equity (ROE)	Dividend Payments	Ending Equity Value (Quasi-market)	Total Debt (Book Value)	Debt to Value Ratio (D/V)	Equity Beta ( $\beta_E$ )	Expected Return on Equity ( $K_E$ )	Discount Factor	Present Value of ECF
0	(300,000)	\$0	\$106,688	\$300,000	\$0	\$0	\$406,688	\$0	0.0%	0.60	12.4%	1.0000	(\$300,000)
1	(170,000)	406,688		170,000	50,592	0	627,280	700,000	52.7%	1.27	17.4%	0.8894	(151,192)
2	(254,349)	627,280		254,349	109,114	0	990,743	1,300,000	56.8%	1.39	18.3%	0.7576	(192,691)
3	171,446	990,743			180,968	(171,446)	1,000,265	1,275,000	56.0%	1.36	18.1%	0.6406	109,824
4	175,490	1,000,265			181,043	(175,490)	1,005,819	1,250,000	55.4%	1.35	18.0%	0.5424	95,186
5	167,058	1,005,819			180,624	(167,058)	1,019,385	1,225,000	54.6%	1.32	17.8%	0.4598	76,818
6	159,901	1,019,385			181,201	(159,901)	1,040,685	1,175,000	53.0%	1.28	17.5%	0.3904	62,430
7	153,143	1,040,685			181,631	(153,143)	1,069,173	1,125,000	51.3%	1.23	17.1%	0.3324	50,907
8	147,661	1,069,173			182,955	(147,661)	1,104,467	1,050,000	48.7%	1.17	16.7%	0.2838	41,912
9	155,080	1,104,467			184,016	(155,080)	1,133,403	975,000	46.2%	1.12	16.3%	0.2433	37,732
10	150,023	1,133,403			184,285	(150,023)	1,167,665	900,000	43.5%	1.06	15.9%	0.2093	31,396
11	146,243	1,167,665			185,218	(146,243)	1,206,640	800,000	39.9%	1.00	15.4%	0.1806	26,415
12	142,864	1,206,640			185,626	(142,864)	1,249,402	700,000	35.9%	0.94	14.9%	0.1565	22,364
13	140,761	1,249,402			186,506	(140,761)	1,295,146	575,000	30.7%	0.87	14.4%	0.1362	19,173
14	139,060	1,295,146			186,646	(139,060)	1,342,732	450,000	25.1%	0.80	13.9%	0.1191	16,556
15	138,636	1,342,732			187,016	(138,636)	1,391,112	300,000	17.7%	0.73	13.4%	0.1045	14,487
16	144,114	1,391,112			186,374	(144,114)	1,433,372	150,000	9.5%	0.66	12.9%	0.0922	13,281
17	212,953	1,433,372			184,971	(212,953)	1,405,390	0	0.0%	0.60	12.4%	0.0816	17,381
18	278,987	1,405,390			174,831	(278,987)	1,301,234	0	0.0%	0.60	12.4%	0.0726	20,252
19	281,798	1,301,234			161,874	(281,798)	1,181,309	0	0.0%	0.60	12.4%	0.0646	18,193
20	284,638	1,181,309			146,955	(284,638)	1,043,627	0	0.0%	0.60	12.4%	0.0574	16,343
21	287,506	1,043,627			129,827	(287,506)	885,948	0	0.0%	0.60	12.4%	0.0511	14,681
22	290,403	885,948			110,212	(290,403)	705,756	0	0.0%	0.60	12.4%	0.0454	13,188
23	293,330	705,756			87,796	(293,330)	500,222	0	0.0%	0.60	12.4%	0.0404	11,848
24	296,286	500,222			62,228	(296,286)	266,164	0	0.0%	0.60	12.4%	0.0359	10,643
25	299,275	266,164			33,111	(299,275)	0	0	0.0%	0.60	12.4%	0.0319	9,561

Project NPV = \$106,688

## EXHIBIT 5

### The Spread of Mexican Brady Bonds Over 10-Year U.S. Treasury Bonds



Adjust the cash flows :

$$PV_0(ECF_t) = \sum_{t=1}^T \frac{ECF_t(1-\lambda)^t}{(1+K_E)^t} \quad (10)$$

Use of Equation (9) (adjusting the discount rate) instead of Equation (10) (adjusting the cash flows) is essentially an admission to the fact that the cash flows are not expected cash flows, which violates basic valuation theory. In much the same fashion venture capitalists add incremental risk premiums to counteract the effects of optimistic, "hockey stick" projections. The somewhat curious feature of this approach, however, is the belief that you can obtain *correct* values by using *incorrect* discount rates and *incorrect* cash flows.

Nevertheless, practitioners regularly use the adjustment shown in Equation (9) even though it is appropriate only when the following assumptions hold (see the appendix):

1. The probability of default or expropriation ( $\lambda$ ) is constant over time.
2. The project's capital structure is constant over time, which implies that the expected cost of equity ( $K_E$ ) is also constant over time.
3. The project's cash flows are constant and perpetual.

What is interesting about these assumptions is that none

seem true for large-scale projects. The probability of default is clearly not constant over time, capital structure is clearly not constant over time, and cash flows are not constant or perpetual.

There are probably four reasons why this approach has gained acceptance despite its theoretical flaws. First, like the use of a single discount rate discussed above, adjusting the discount rate greatly simplifies the analysis. It is far easier to adjust the discount rate by a constant term than to generate multiple cash flow scenarios, determine relevant probabilities for each scenario, and calculate an expected cash flow scenario. Second, the projections

used for valuation purposes are usually derived from and used for operating budgets. The problem with "expected" scenarios is that they do not resemble projects in any actual state of the world and it is time consuming to create multiple forecasts for multiple purposes. Third, Abuaf and Chu [1997] provide some evidence that local equity returns are related to local bond returns in emerging markets. And finally, the errors introduced by changing the discount rate are typically not large compared to other potential sources of error (cash flow estimation or beta estimation).<sup>12</sup> And so, while adjusting the discount rate to reflect country risk is probably not the worst mistake one can make in valuing large-scale investments, it is probably better to adjust the cash flows under the theory that it is worth eliminating errors where possible.

## MONTE CARLO SIMULATION

One of the most important realizations for people responsible for valuation analysis is that the process is filled with error. As a result, sensitivity analysis is an integral part of any valuation even though it is generally confined to static tests of particular inputs. A typical question for an oil-field project might be: "What happens to the project's NPV if oil prices are \$8.00 instead of \$11.40 per barrel?" Yet sensitivity analysis need not be confined to this kind of static analysis.

The alternative is to use Monte Carlo simulation

in which a computer generates a distribution of possible outcomes based on a large number of scenarios. Users can then analyze the full distribution of possible outcomes rather than a limited number of likely scenarios. For example, there might be two ways to structure a project with equivalent NPVs, but the NPV variance or probability of default is lower under one structure. Managers are likely to prefer this structure over the other one.

To illustrate the power of dynamic simulation analysis, I use a software package known as Crystal Ball<sup>®</sup> to analyze the effects of oil price volatility on PetroMexico's financial risk and present value.<sup>13</sup> In particular, I want to see how changing oil prices affect PetroMexico's NPV, debt service coverage ratio (DSCR = CADS/Total Debt Service), and most likely year of default.

The first step is to pick a distribution for the change in Maya prices — one can think of price changes as returns on holding a barrel of oil. For this example, I assume oil prices follow a random walk. In particular, I assume oil prices have a lognormal distribution, which means that the continuously compounded return is normally distributed.<sup>14</sup> Given this distribution, the next step is to generate random price changes over time: this year's price ( $P_t$ ) equals last year's price times one plus the random return:

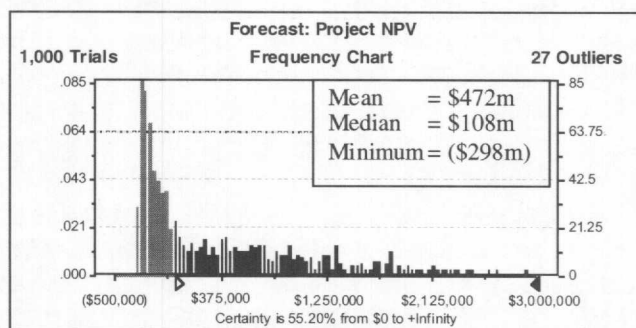
$$P_t = P_{t-1} (1 + \text{random return}) \quad (11)$$

Aggregating annual price changes over time results in a random price path. The simulation software uses these twenty-five-year price paths as inputs to the cash flow model in Exhibit 4 and calculates the various outputs of interest. Finally, the software repeats steps 2 and 3 1,000 times and generates a distribution of possible outcomes. For most applications, running 1000 scenarios takes only a few minutes, though more complicated scenarios such as quasi-market valuation can take longer.

Exhibits 6A-6C provides the results from a Crystal Ball<sup>®</sup> simulation assuming the volatility of oil price changes is 20% per year and the project has 80% leverage. Exhibit 6A shows the distribution of NPV's: the mean is \$472 million while the median is \$108 million — note the median value is quite close to the quasi-market NPV of \$107 million. The distribution clearly shows the option-like nature of leveraged equity: low oil prices generate losses which are truncated by an amount

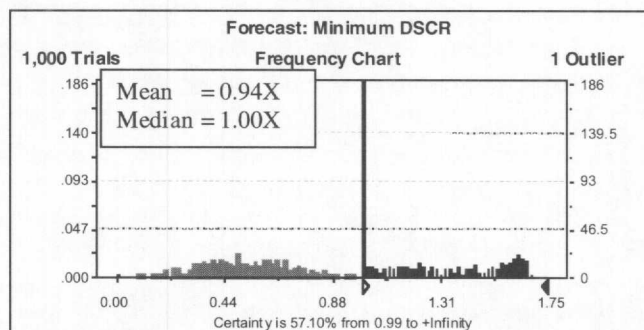
## EXHIBIT 6A

### Project NPV Distribution



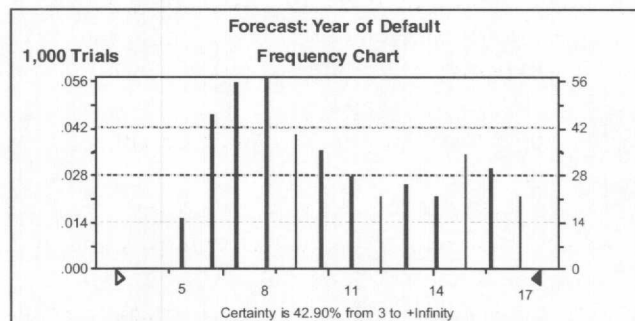
## EXHIBIT 6B

### Minimum Debt Service Coverage Ratio (DSCR) Distribution



## EXHIBIT 6C

### Year of Default Distribution



## EXHIBIT 7

### Monte Carlo Simulation Results

80%	Leverage (debt/value)	Mean NPV	=	\$192m	Mean NPV	=	\$472m
		Median NPV	=	\$110m	Median NPV	=	\$108m
		Prob (NPV>0)	=	61%	Prob (NPV>0)	=	55%
		Mean Minimum DSCR	=	1.13X	Mean Minimum DSCR	=	0.94X
		Prob (DSCR ( 1)	=	78%	Prob (DSCR ( 1)	=	57%
		Prob (Default)	=	22%	Prob (Default)	=	43%
50%	Leverage (debt/value)	Mean NPV	=	\$214m	Mean NPV	=	\$470m
		Median NPV	=	\$159m	Median NPV	=	\$236m
		Prob (NPV>0)	=	67%	Prob (NPV>0)	=	63%
		Mean Minimum DSCR	=	1.78X	Mean Minimum DSCR	=	1.37X
		Prob (DSCR ( 1)	=	94%	Prob (DSCR ( 1)	=	73%
		Prob (Default)	=	6%	Prob (Default)	=	27%
10%				20%			
Volatility of Oil Returns*							

\*Standard deviation of annual changes in Maya oil prices.

equal to the present value of the equity investment while high oil prices generate unlimited gains. Exhibit 6B shows the distribution of the minimum debt service coverage ratio (DSCR). Given the project's high leverage and oil-price volatility, it is not surprising the project defaults 57% of the time (the "certainty" of having a DSCR (1.00X is 57.10% from the exhibit). The large number of observations at 1.00X illustrates the importance of the debt service reserve account. If CADS is insufficient to cover total debt service, then the project relies on the debt service reserve account to cover the shortfall and the resulting coverage ratio is exactly 1.00X. The project defaults only when the CADS plus the debt service reserve account cannot cover the total debt service. Finally, Exhibit 6C shows the distribution of default years in those instances where the project defaults. Given that the project has its lowest coverage ratios of approximately 1.70 in years 11, 13, and 15 (see Exhibit 1), it is interesting to note that the project is most likely to default well before that time. In fact, the

project is most likely to default in years 7 and 8.

Having this kind of information allows project sponsors to restructure weak projects so as to minimize the probability of default and maximize expected equity returns. For example, Exhibit 7 shows the impact of structuring the project with 50% leverage versus 80% leverage, under two oil price volatility scenarios (10% and 20%). Neither the mean NPV nor the probability that the NPV is greater than 0 are very sensitive to the leverage choice (though they are very sensitive to the volatility scenario — see Exhibit 7). However, the minimum DSCR and the probability of default are both very sensitive to the amount of debt: in the 10% volatility scenario, the minimum DSCR falls from 1.78X to 1.13X and the probability of default increases from 6% up to 22% as you increase leverage from 50% to 80%. As the probability of default increases, value declines as seen by the fact that the median NPV falls. These same effects appear in the high volatility scenario although their impact is magnified.

## EXHIBIT 8

### Managerial Flexibility and Real Options Analysis

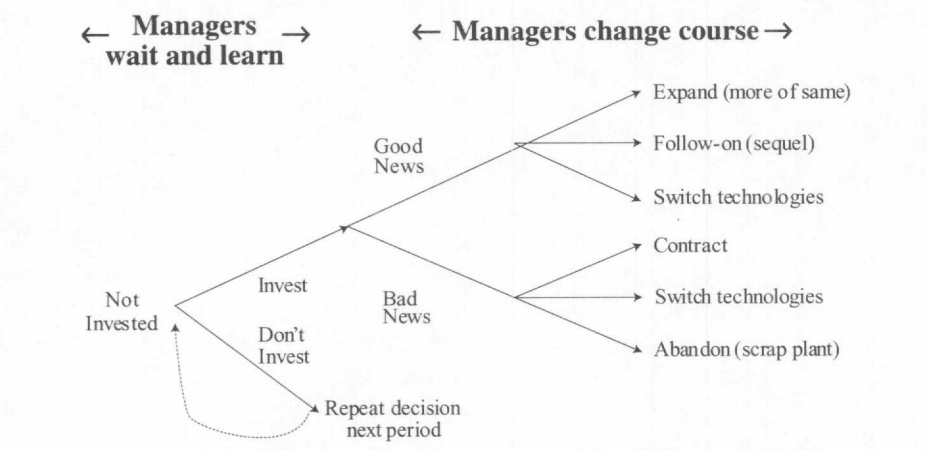


Exhibit 7 also shows how the project responds to different assumptions about oil price volatility. Intuitively, the project should be worth more, but default more often, as volatility increases. The reason NPV increases as oil price volatility increases is that the pay-offs to levered equity resemble a call option: equity is protected on the downside by limited liability but enjoys unlimited upside. As seen in Exhibit 7, the mean NPV more than doubles in the high volatility scenarios. On the other hand, the mean minimum DSCR falls and the probability of default increases. At 50% leverage, the minimum DSCR falls from 1.78X to 1.37X while the probability of default increases from 6% to 27%, again consistent with intuition.

Because Monte Carlo simulation shows the full distribution of outcomes, it is far more informative than traditional, static sensitivity analysis. While I have shown the power of this analysis for one model input, namely oil price volatility, the model can easily be adapted to illustrate the effects of uncertain exchange rates, output quantities, construction costs, or input prices. Nichols [1994] provides a good description of how one company, Merck, uses Monte Carlo analysis as part of its financial planning process.

### REAL OPTIONS ANALYSIS

The previous section illustrated the importance of understanding input uncertainty. What it did not do, however, is analyze how managers would respond to the

various scenarios, which is a fundamental flaw inherent in all discounted cash flow analysis. In fact, authors such as Dixit and Pindyck [1995], Trigeorgis [1996], Leslie and Michaels [1997], and Amram and Kulatilaka [1999] have criticized DCF valuation because it makes several unrealistic assumptions. First, it assumes that investments are now-or-never decisions rather than recognizing that managers have the option to wait. Often the key decision is not *whether* to invest, but *when*. Second, DCF analysis assumes that projects operate mecha-

nistically; in other words, DCF assumes that managers are passive. Yet in reality, managers add value by making mid-course corrections in response to new information — they have the option to change direction or strategy. Exhibit 8 illustrates the inherent flexibility managers have to defer investment and to change strategy in response to good or bad news having made an investment. Finally, DCF implicitly assumes that capital expenditures are reversible when, in fact, they are usually irreversible (i.e., sunk costs).

The authors mentioned above advocate a new tool, called real options analysis, for valuing large-scale investments. While the term “options” refers to the fact that managers have flexibility when making investment or operating decisions, the term “real” is meant to distinguish these options on tangible and intangible assets from options on financial instruments. The advantage of real options analysis is that it explicitly recognizes and incorporates the value of being able to defer investment, expand output, change technologies, or stop investing.

Real option analysis is built on the recognition that there is a correspondence between managing real assets and investing in financial assets. For example, deciding whether to invest in PetroMexico is analogous to deciding whether to exercise a call option on IBM stock. If the sponsors spend the capital expenditures necessary to develop the field, then they will have the right to receive an uncertain stream of future cash flows from a developed oil field.<sup>15</sup> They would only construct the field if the present value of expected future

cash flows exceeded the present value of development costs. Analogously, it is only rational to exercise a call option on IBM stock if the market price exceeds the exercise price. A related question, *when* to invest in PetroMexico, is akin to asking when it is rational to exercise a call option, in this case an American call option. The same tools that are used to value financial options, the Black-Scholes pricing formula, binomial trees, or numerical methods — can also be used to value real options. While a full explanation of this new and exciting valuation tool is beyond the scope of this article, it is important to know that such tools exist and are currently being used by companies like Merck, Enron, and New England Electric to analyze major strategic investments (see Nichols [1994] and Corman [1997]).

Despite its theoretical appeal, the disadvantage of real options analysis is that it is significantly more complex to implement which explains its limited use to date. As academics and practitioners alike learn how to adapt this tool to the complexities of the real world, however, it will become more common. Until that time, it is important to make the best use of the tools available which, for better or worse, is the DCF approach.

## CONCLUSION

Valuing complex investments requires complex tools. Unfortunately most of the tools in practice today were not designed to handle the complexities of today's investment decisions. In part, the problems lie in using the wrong tools — one should use real options analysis instead of DCF analysis — and in part the problems lie in using the existing tools incorrectly. The objective of this article has been to refine the project finance professional's valuation toolkit and to highlight some new tools. In particular, I discussed three ways to improve the discounting process in equity cash flow valuation (multiple discount rates, quasi-market valuation, and cash flow adjustments instead of ad hoc discount rate adjustments). I also discussed two new valuation tools, Monte Carlo simulation and real options analysis. Admittedly, these tools and techniques are more difficult to implement than the basic valuation tools people are used to, yet they provide more accurate valuations and, more importantly, better decision making. Given the low cost of personal computers and the relatively simplicity required to learn these tools, there is really

little justification for not adopting them.

## APPENDIX

### Evaluating Country Risk — The Country Risk Premium

The present value (PV) of a stream of equity cash flows (ECF) equals:

$$PV(ECF) = \sum_{t=1}^T \frac{ECF_t}{(1 + K_E)^t} \quad (A-1)$$

where the discount rate ( $K_E$ ) is the expected cost of equity as determined by the capital asset pricing model (CAPM). If one adds a country risk premium (CRP) to the cost of equity, then the adjusted cost of equity ( $K_E^*$ ) becomes:

$$K_E^* = K_E + CRP = R_f + \beta_E^*(R_m - R_f) + CRP \quad (A-2)$$

Substituting the adjusted cost of equity as a discount rate into Equation (A-1) yields:

$$PV(ECF) = \sum_{t=1}^T \frac{ECF_t}{(1 + K_E + CRP)^t} \quad (A-3)$$

Assuming the cash flows are constant over time ( $ECF = ECF_1 = ECF_2 \dots ECF_T$ ), then Equation (A-3) is a *stable* perpetuity. The present value when you adjust the discount rate is:

$$PV(\text{Adj. Discount Rate}) = \frac{ECF}{K_E + CRP} \quad (A-4)$$

Instead of adjusting the discount rate with country risk premium, one can adjust the cash flows to reflect a probability of default ( $\lambda$ ). Under the following assumptions, these two methods are approximately equal mathematically:

1. The probability of default ( $\lambda$ ) is constant over time, and  $0 \leq \lambda \leq 1$ ;
2. The capital structure and, therefore, expected cost of equity ( $K_E$ ) are constant over time, and
3. The cash flows are constant and perpetual, unless default occurs in which case they are zero.

With  $\lambda$  as the probability of default, the expected equity cash flow in year  $t$  is:

$$ECF_t = ECF[(1 - \lambda)^t + 0] \lambda = ECF(1 - \lambda)^t \quad (A-5)$$



which has a present value in year 0 of:

$$PV_0(ECF_t) = \frac{ECF_t(1-\lambda)^t}{(1+K_E)^t} \quad (A-6)$$

Assuming constant cash flows beginning in year 1 in the absence of default ( $ECF = ECF_1 = ECF_2 \dots ECF_T$ ), then the cash flows with default form a geometric series:

$$PV(\text{Adj. Cash Flows}) = \frac{ECF(1-\lambda)}{(1+K_E)} + \dots + \frac{ECF(1-\lambda)^t}{(1+K_E)^t} \dots \quad (A-7)$$

which can be rewritten as:

$$PV(\text{Adj. Cash Flows}) = (1-\lambda) \times \left( \frac{ECF}{(1+K_E)} + \dots + \frac{ECF(1-\lambda)^{t-1}}{(1+K_E)^t} + \dots \right) \quad (A-8)$$

Equation 8 is a *growing* perpetuity with a *negative* growth rate equal to  $\lambda$ . It can be written as:

$$PV(\text{Adj. Cash Flows}) = (1-\lambda) \times \left( \frac{ECF}{K_E - (-\lambda)} \right) = (1-\lambda) \times \left( \frac{ECF}{K_E + \lambda} \right) \quad (A-9)$$

When the default risk is equal to the country risk premium ( $\lambda = CRP$ ), then Equation (A-9) can be rewritten as:

$$PV(\text{Adj. Cash Flows}) = (1 - CRP) \left( \frac{ECF}{K_E + CRP} \right) \quad (A-10)$$

Substituting Equation 4 into Equation 10 yields:

$$PV(\text{Adj. Cash Flows}) = (1 - CRP)PV(\text{Adj. Discount Rate}) \quad (A-11)$$

Thus, the present value determined by adjusting the discount rate with a country risk premium (Equation (A-4)) is approximately equal to the present value determined by adjusting the cash flows for default risk (Equation (A-10)) when the country risk premia (or probability of default) is small.

## ENDNOTES

The author thanks Mathew Mateo Millett for invaluable assistance in writing this article and Rick Ruback for many helpful discussions on valuing highly-leveraged transactions including project finance transactions. The author also thanks the Division of Research at the Harvard Business School for supporting this research.

<sup>1</sup>Both methods are prone to similar implementation errors. For example, most people use the promised or contractual debt rate, not the expected rate; in the WACC. Similarly, they derive equity cash flows by subtracting promised, not expected, debt payments from free cash flows. Both errors cause equity value to be understated.

<sup>2</sup>Equation (2) is the Hamada formula for levering and unlevering betas assuming riskless debt (i.e., the debt has neither interest-rate nor credit risk which implies the beta of debt is zero,  $\beta_D = 0$ ). Theoretically, the formula is:  $\beta_A = \beta_D \times (D/V) + \beta_E \times (E/V)$ , which reduces to Equation (2) when the debt beta is zero. For simplicity I have assumed riskless debt, a simplification that is easily corrected.

<sup>3</sup>Ibbotson Associates [1998, Exhibit 2-1] reports that the difference in arithmetic means between market returns on large company stocks and long-term government bonds is 7.4% for the period 1926-1997.

<sup>4</sup>Note that I am calculating only the value of the project itself. To estimate the total value to the sponsors, I would need to include such things as the cost of providing debt guarantees, if any, during construction as well as the interest tax shields resulting from additional sponsor borrowing against project cash flows.

<sup>5</sup>Using the average debt ratio of 32%, the cost of equity is 14.6% and the NPV is \$213 million.

<sup>6</sup>The formula for the weighted average cost of capital (WACC) is:  $(D/V) \times (1 - \tau_c) \times K_D + (E/V)K_E$ , where  $(D/V)$  and  $(E/V)$  are the capital structure weights,  $\tau_c$  is the marginal corporate tax rate,  $K_D$  is the cost of debt, and  $K_E$  is the cost of equity.

<sup>7</sup>This assumption of zero ending equity value eliminates the need to estimate a final equity value which is why it is easier to apply QMV to project finance deals than to leveraged buyouts (LBOs).

<sup>8</sup>The QMV approach breaks down for projects with very negative NPVs because the negative NPV will exceed the initial equity contribution causing the project to have a negative equity value which is not possible with limited liability equity.

<sup>9</sup>Another common discount rate adjustment is the use of a "country beta" to adjust for emerging market volatility (see Lessard [1996] and Godfrey and Espinosa [1997]). Harvey [1995], and Bekaert and Harvey [1997] have shown that there is virtually no relation between

emerging market returns and betas measured with respect to a world market portfolio. Instead, emerging market returns are related to country volatility, which the authors hypothesize results from being segmented from world capital markets. Based on these results, certain practitioners advocate modifying the basic CAPM formula given in Equation 1 to reflect equity volatility:

$$K_E = R_f + (\beta_{EM} \times \beta_E \times R_p)$$

where  $\beta_{EM}$  = the emerging market beta =  $\rho_{EM/US} \times (\sigma_{EM}/\sigma_{US})$ ,  
 $\rho_{EM/US}$  = correlation between returns in the emerging market and the U.S. market,  
 $\sigma_{EM}/\sigma_{US}$  = volatilities of the emerging market and the U.S. market, respectively,  
 $\beta_E$  = the beta for a similar project in a developed market like the U.S., and  
 $R_p$  = the market risk premium

<sup>10</sup>For example, if a project is expected to have cash flow of \$100, but there is a 10% chance of expropriation, in which case the cash flow will be \$0, then the correct expected cash flow is \$90 = \$100 × 90% + \$0 × 10%.

<sup>11</sup>Diamonte et al. [1996] provide some evidence of a negative relation between political risk and stock returns, but do not show that political risk is a systematic risk.

<sup>12</sup>The errors are approximately equal to the size of the country risk premium (CRP), see the Appendix.

<sup>13</sup>Crystal Ball<sup>®</sup> (V. 3.0) is a graphically oriented forecasting and risk analysis software package which can be used with Microsoft Excel. It is produced by Decisioneering, Inc. (www.decisioneering.com).

<sup>14</sup>The continuously compounded return is defined as  $\ln(P_t/P_{t-1})$ .

<sup>15</sup>Brennan and Schwartz [1985] and Siegel, Smith, and Paddock [1987] have shown how option-pricing models can be used to value natural resource investments.

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