# Search frictions and market power in negotiated price markets* 

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#### Abstract

This paper provides a framework for the empirical analysis of negotiated-price markets in which buying is single-source. These markets pose a challenge for empirical work since, although buyers potentially negotiate with many sellers, data-sets typically include only accepted offers. Moreover, negotiated-price markets feature search frictions, since consumers incur a cost to gather quotes, and long-term relationships between consumers and incumbent sellers, leading to the development of brand loyalty. Together, these characteristics imply that many consumers fail to consider more than one option, and that firms with extensive consumer bases have an incumbency advantage. We use data from the Canadian mortgage market and a model of search and negotiation to characterize the impact of search frictions on consumer welfare and to quantify the role of search costs and brand loyalty for market power. Our results suggest that search frictions reduce consumer surplus by almost $\$ 12$ per month per consumer, and that $28 \%$ of this reduction can be associated with discrimination, $22 \%$ with inefficient matching, and the remainder with the search cost. We also find that banks with large consumer bases have margins that are $70 \%$ higher than those with small consumer bases. The main source of this incumbency advantage is brand loyalty, however, the ability to price discriminate based on search frictions also accounts for almost a third of the advantage.


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## 1 Introduction

In a large number of markets, sellers post prices, but actual transaction prices are achieved via bilateral bargaining. This is the case for instance in the markets for new/used cars (Goldberg (1996), Scott-Morton et al. (2001), and Busse et al. (2006)), health insurance (Dafny (2010)), capital assets (Gavazza (2016)), financial products (Hall and Woodward (2012) and Allen et al. (2014a)), as well as for most business-to-business transactions (e.g. Joskow (1987), Town and Vistnes (2001) and Salz (2015)).

In this paper, we are interested in two key features characterizing many of these markets. First, since buyers incur a cost to gather price quotes, these markets are characterized by important search frictions. Second, the repeated relationship that develops between a buyer and a seller creates a loyalty advantage, which increases the value of transacting with the same seller. This can be because of switching costs associated with changing suppliers, cost advantages of the incumbent sellers, or because of complementarities from the sale of related products.

Search frictions and brand loyalty have implications for market power. Search costs open the door to price discrimination: the seller offering the first quote is in a quasi-monopoly position, and can make relatively high offers to consumers with poor outside options and/or high expected search costs. Brand loyalty reduces the bargaining leverage of consumers, because incumbent sellers provide higher value, which creates a form of lock-in. Together, these features imply that a firm with an extensive consumer base has an incumbency advantage over rival firms in the same market.

We study one particular negotiated-price setting, the Canadian mortgage market, for which we have access to an administrative data-set on a large number of individually negotiated mortgage contracts, which we use to estimate a model of search and price negotiation. In this market, national lenders post common interest rates, but in-branch loan officers have considerable freedom to negotiate directly with borrowers. Importantly, there is evidence of search frictions and brand loyalty in this setting. About $70 \%$ of consumers in this market combine day-to-day banking and mortgage services at their main financial institution and $80 \%$ get a rate quote from this lender. Moreover, despite the fact that approximately $60 \%$ of consumers search for additional quotes, only about $28 \%$ obtain a mortgage from a different lender than their main institutions.

In this setting, we consider two questions. First, what is the impact of search frictions on the welfare of consumers? Second, what are the sources and magnitude of market power? We focus on quantifying the incumbency advantage that stems from a large consumer base and decomposing it into two parts: (i) a first-mover advantage arising from price discrimination and search frictions, and (ii) a loyalty advantage originating from long-term relationships.

To address these questions, we estimate a structural model of demand and supply applied to negotiated-price markets. Our contribution is to develop a framework that accounts for the fact that in these settings buyers negotiate prices with potentially many differentiated sellers, but often sign exclusive contracts with just one. As a result, researchers typically only observe transaction
prices, and the identity of sellers, including whether or not buyers remain loyal to their current supplier. This poses a serious challenge for empirical work, since the outside option of buyers is unobserved. ${ }^{1}$ In our case-study, we do not observe rejected offers, or whether consumers search for more than one lender. This is not unique to mortgage lending; most data-sets used in previous empirical work on price negotiation in consumer goods and business-to-business markets share these same features. ${ }^{2}$

To overcome these challenges, we use a two-stage search model in which individuals are initially matched with their home bank for a quote, and then decide, based on their expected net gain from searching, whether or not to gather additional quotes. The home bank uses this initial quote to price discriminate by screening high search-cost consumers. If it is rejected, consumers pay a search cost, and local lenders compete via an English auction for the contract. Using an auction to characterize the competition stage represents a tractable approach to address the missing prices problem. In particular, it accounts for the fact that sellers, in most price negotiation markets, are able to counter rivals' offers by lowering prices; a process which mimics an English auction. This modeling strategy is related to search and bargaining models with asymmetric information developed by Wolinsky (1987), Chatterjee and Lee (1998), and Bester (1993), in which consumers negotiate with one firm, but can search across stores for better prices.

The tractability of the model also allows us to analyze the identification of the parameters in a transparent way. In particular, we discuss the conditions under which the distributions of searchand lending-costs are non-parametrically identified, using insights from the labor, discrete-choice and empirical auctions literatures. The identification argument is generalizable to other negotiatedprice settings in which researchers have access to data on transaction prices and switching decisions (but not necessarily search). Although the search- and lending-cost distributions could in theory be non-parametrically estimated, we instead estimate a parametric version of the model using maximum likelihood. This allows us to more easily incorporate observable differences between consumers and firms.

The results can be summarized as follows. We find that firms face relatively homogeneous lending costs for the same borrower. In contrast, we find that borrowers face significant search costs and brand loyalty advantage. On average, consumers in our sample face an upfront search cost of $\$ 1,150$. In addition, the incumbent bank has an average cost advantage of $\$ 17.10 /$ month (for a $\$ 100 \mathrm{~K}$ loan) generating a sizeable loyalty advantage.

[^1]We use the model estimates to characterize the impact of search frictions on consumer welfare and to measure market power. To quantify the welfare cost of search frictions, we perform a set of counter-factual experiments in which we eliminate the search costs of consumers. The surplus loss from search frictions originates from three sources: (i) misallocation of buyers and sellers, (ii) price discrimination, and (iii) the direct cost of gathering multiple quotes. Our results suggest that search frictions reduce average consumer surplus by almost $\$ 12$ per month, over a five year period. Approximately $28 \%$ of the loss in consumer surplus comes from the ability of incumbent banks to price discriminate with their initial quote. A further $22 \%$ is associated with the misallocation of contracts, and $50 \%$ with the direct cost of searching. We also find that the presence of a posted-rate limits the ability of firms to price discriminate, thereby reducing the welfare cost of search frictions. Competition also amplifies the adverse effects of search frictions on consumer welfare.

Our results also suggest that the market is fairly competitive. The average profit margin is estimated to be just over 20 basis points (bps), which corresponds to a Lerner index of $3.2 \%$. However, margins vary considerably depending on whether consumers search and/or switch. On average, firms charge a markup that is $90 \%$ higher for consumers who are not searching. Banks' profits from switching consumers are $\$ 14.99 /$ month ( 17.1 bps ), compared to $\$ 20.22 /$ month from loyal consumers ( 24.6 bps ).

The increased profits earned from loyal consumers correspond to the incumbency advantage, and are directly related to the size of the bank's consumer base. To measure the source and magnitude of the advantage we use the simulated model to evaluate the correlation between consumer base and its profitability. We find that banks with the largest consumer bases earn, on average, $62 \%$ of the profits generated in their markets, compared to only $2 \%$ for those with the smallest. This difference is driven by the fact that large consumer-base lenders control a large share of the mortgage market, and earn significantly more profit per contracts than smaller banks.

We measure the incumbency advantage as the increased market power of banks with large consumer bases relative to those with the smallest. Our estimates suggest that banks with large consumer bases have margins that are $70 \%$ higher than those with small consumer bases. To identify the importance of the two sources of the incumbency advantage we simulate a series of counterfactual experiments aimed at varying the first-mover advantage and the differentiation component independently. Our results suggest that about $50 \%$ of the incumbency advantage can be directly attributed to brand loyalty, $30 \%$ to search frictions and the remaining $20 \%$ to their interaction.

Our paper is related to three strands of literature. First are the recent empirical papers based on the complete-information multi-lateral negotiation game proposed by Horn and Wolinsky (1988) mentioned above (see for instance Grennan (2013) and Gowrisankaran et al. (2015)). This method for measuring the buyers' outside options is suitable for the case of bargaining between buyers and their network of suppliers, but is not applicable when buyers transact with a single seller. We are also related to the industrial organization search literature (see Sorensen (2001), Hortaçsu
and Syverson (2004), Hong and Shum (2006), Wildenbeest (2011), De Los Santos et al. (2012), Honka (2014), Alexandrov and Koulayev (2017), and Marshall (2016)). Although these papers take into account concentration and differentiation, they have mostly focused on cases where firms offer random posted prices to consumers irrespective of their characteristics (as opposed to targeted negotiated price offers). Lastly, our findings contribute to the literature on the advantages accruing to incumbent firms from demand inertia and brand loyalty. Bronnenberg and Dubé (2016) provide an extensive survey of this literature in I.O. and marketing. Our model allows us to quantify the relative importance of two sources of state dependence, and the market-power associated with the incumbency advantage.

The paper is organized as follows. Section 2 presents details on the Canadian mortgage market and introduces our data sets. Section 3 presents the model, and Section 4 discusses conditions for non-parametric identification of the primitives. Section 5 discusses the estimation strategy and Section 6 describes the empirical results. Section 7 analyzes the impact of search friction and brand loyalty on consumer welfare and market power. Finally, section 8 concludes.

## 2 Institutional details and data

### 2.1 Institutional details

The Canadian mortgage market is dominated by six national banks (Bank of Montreal, Bank of Nova Scotia, Banque Nationale, Canadian Imperial Bank of Commerce, Royal Bank Financial Group, and TD Bank Financial Group), a regional cooperative network (Desjardins in Québec), and a provincially owned deposit-taking institution (Alberta's ATB Financial). Collectively, they control $90 \%$ of banking industry assets. For convenience we label these institutions the "Big 8."

Canada features two types of mortgage contracts - conventional, which are uninsured since they have a low loan-to-value ratio, and high loan-to-value, which require insurance (for the lifetime of the mortgage). Most new home-buyers require mortgage insurance. The primary insurer is the Canada Mortgage and Housing Corporation (CMHC), a crown corporation with an explicit guarantee from the federal government. During our sample period a private firm, Genworth Financial, also provided mortgage insurance, and had a $90 \%$ government guarantee. CMHC's market share during our sample period averages around $80 \%$. Both insurers use the same insurance guidelines, and charge lenders an insurance premium, ranging from $1.75 \%$ to $3.75 \%$ of the value of the loan, which is passed on to borrowers. ${ }^{3}$

The large Canadian banks operate nationally and their head offices post prices that are common across the country on a weekly basis in both national and local newspapers, as well as online. Throughout our entire sample period the posted rate is nearly always common across lenders, and

[^2]represents a ceiling in the negotiation with borrowers. ${ }^{4}$
According to data collected by marketing firm Ipsos-Reid, the majority of Canadians have a main financial institution where they combine checking and mortgage accounts. Therefore, potential borrowers can accept to pay the rate posted by their home bank, or search for and negotiate over rates. Borrowers bargain directly with local branch managers or hire a broker to search on their behalf. ${ }^{5}$ Our model excludes broker transactions and focuses only on branch-level transactions.

### 2.2 Mortgage data

Our main data set is a $10 \%$ random sample of insured contracts from the CMHC, from January 1999 to October 2002. The data-set contains information the terms of the contract (transaction rate, loan size, and house price), as well as financial and demographic characteristics of the borrower. In the empirical analysis we focus in particular on the income of the borrower, the FICO risk score, the loan-to-value ratio, and the 5 -year bond-rate valid at the time of negotiation. In addition, we observe the closing date of the contract and the location of the purchased house up to the forward sortation area (FSA). ${ }^{6,7}$

The data set contains the lender information for twelve of the largest lenders during our sample period. For mortgage contracts where we do not have a lender name but only a lender type, these are coded as "Other credit union", and "Other trusts". The credit-union and trust categories are fragmented, and contain mostly regional financial institutions. ${ }^{8}$ We therefore combine both into a single "Other Lender" category. Overall, therefore, consumers face 12 lending options.

We restrict our sample to contracts with homogenous terms. In particular, from the original sample we select contracts that have the following characteristics: (i) 25 -year amortization period, (ii) 5 -year fixed-rate term, (iii) newly issued mortgages (i.e. excluding refinancing), (iii) contracts that were negotiated individually (i.e. without a broker), (iv) contracts without missing values for key attributes (e.g. credit score, broker, and residential status).

The final sample includes around 26,000 observations, or about one-third of the initial sample. Approximately $18 \%$ of the initial sample contained missing characteristics; either risk type or business originator (i.e. branch or broker). This is because CMHC started collecting these transaction

[^3]characteristics systematically only in the second half of 1999 . We also drop broker transactions, $(28 \%)$, as well as short-term, variable rate and refinanced contracts ( $40 \%$ ).

We use the data to construct three main outcome variables: (i) monthly payment, (ii) negotiated discounts, and (iii) loyalty. The monthly payment, denoted by $p_{i}$, is constructed using the transaction interest rate, loan size, and the amortization period ( 60 months) specified in borrower $i$ 's contract. To construct negotiated discounts, we must first identify the posted rate valid at the time of negotiation. Since our contract data include only the closing date, to pin down the appropriate posted rate we infer the negotiation week that maximizes the aggregate fraction of consumers paying the posted rate (or 33 days prior to closing). Lastly, the loyalty variable is a dummy variable equal to one if a consumer has prior experience dealing with the chosen lender. Since $75 \%$ consumers are new home buyers, this most likely identifies the bank with which the borrower possess a savings or checking account. Note that this variable is not available for one lender, and we therefore treat the loyalty outcome as partly missing when constructing the likelihood function

Finally, since the main dataset does not provide direct information on the number of quotes gathered by borrowers, we supplement it with survey evidence from the Altus Group (FIRM survey). The survey asks 841 people who purchased a house during our sample period about their shopping habits. We use the aggregate results of this survey to construct auxiliary moments characterizing the fraction of consumers who report searching for more than one lender, by demographic groups. We focus in particular on city size, regions, and income groups.

### 2.3 Market-structure data

The market structure is described by the consumer base of each bank, and the number of lenders available in consumers' choice sets. The consumer base of a lender is defined by its share of the market for day-to-day banking services. In the model, this is used to approximate the fraction of consumers in a given market that have prior experience with each potential lender. To construct this variable, we use micro-data from a representative survey conducted by Ipsos-Reid. ${ }^{9}$ Each year, Ipsos-Reid surveys nearly 12,000 households in all regions of the country. We group the data into by year, regions (10), and income categories (4). Within these sub-samples we estimate the probability of a consumer choosing one of the twelve largest lenders as their main financial institution, or home bank denoted by $h$. We use $\psi_{h}\left(x_{i}\right)$ to denote the probability that a consumer with characteristics $x_{i}$ has prior experience with bank $h$.

The choice set of consumers is defined by the location of the house being purchased. In particular, we assume that consumers have access to lenders that have a branch located within 10 KM of the centroid of their FSAs. ${ }^{10}$ This choice is justified by the data: over $90 \%$ of loans are originated

[^4]Table 1: Descriptive statistics on mortgage contracts and loyalty in the selected sample

| (a) Summary statistics |  |  |  |  |  | (b) Reduced-form regression |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (5) |  | (1) | (2) |
| VARIABLES | Mean | Std-dev. | P25 | P50 | P75 | VARIABLES | Rate | 1(Loyal) |
| Interest rate spread | 120 | 59.3 | 81 | 115 | 161 | 1(Loyal) | $0.097{ }^{a}$ |  |
| Positive discounts | 95.3 | 45.4 | 70 | 95 | 125 |  | (0.0079) |  |
| 1(Discount=0) | . 127 | . 333 | 0 | 0 | 0 | Previous owner | $0.025^{a}$ | $0.11{ }^{a}$ |
| Monthly payment | 925 | 385 | 619 | 858 | 1169 |  | (0.0084) | (0.0072) |
| Total loan (\$/100K) | 136 | 57.6 | 90.4 | 126 | 174 | Branch network | $0.023^{a}$ | $0.026^{a}$ |
| Income (\$/100K) | 68.4 | 27.9 | 48.5 | 64.1 | 82.1 |  | (0.0046) | (0.0045) |
| FICO score | 669 | 74 | 650 | 700 | 750 | \# Lenders (log) | $-0.13^{a}$ | $-0.076^{a}$ |
| LTV | 91 | 4.38 | 89.7 | 90 | 95 |  | (0.022) | (0.019) |
| 1(LTV=Max) | . 385 | . 487 | 0 | 0 | 1 |  |  |  |
| 1(Previous owner) | . 251 | . 433 | 0 | 0 | 1 | Observations | 20,619 | 20,619 |
| 1(Loyal) | . 651 | . 477 | 0 | 1 | 1 | R-squared | 0.612 | 0.095 |
| Number of Lenders | 8.65 | 1.44 | 8 | 9 | 10 | Marg. effect: income | 0.29 | 0.18 |
| Branch network | 1.6 | 1.02 | . 989 | 1.37 | 1.93 | Marg. effect: loan | 0.47 | -0.19 |

[^5]by a lender present within 10 KM of each FSA. In addition, the fact that rates are negotiated directly with loan officers limits the ability of consumers to perform the transaction online. Indeed, CMHC reports that less than $2 \%$ of mortgages are originated through the internet or phone.

The location of each financial institution's branches is available annually from MicromediaProQuest. We use this data set to match the new house location with branch locations, and construct each consumer's choice set. Formally, a lender is part of consumer $i$ 's choice set if it has a branch located within less than 10 KM of the house location. We use $\mathcal{N}_{i}$ to denote the set of rival lenders available to consumer $i$ (excluding the home bank), while $n_{i}$ is the number of banks in $\mathcal{N}_{i}$.

### 2.4 Market features

Before introducing the model, we provides descriptive evidence outlining the key features of the Canadian mortgage market that we want to capture. Table 1a describes the main financial and demographic characteristics of the borrowers in our sample. Table 1 b reports a subset of the coefficients of two reduced-form regressions describing the relationship between transaction characteristics and negotiation rates, as well as the probability of remaining loyal to the home bank.

The estimation sample corresponds to a fairly symmetric distribution of income and loansize. The average loan-size is about $\$ 136,000$ which is twice the average annual household income.

The loan-to-value (LTV) variable shows that many consumers are constrained by the minimum down-payment of $5 \%$ imposed by the government guidelines. Nearly $40 \%$ of households invest the minimum. Our focus is on the monthly payment made by a borrower, and so when we talk about quotes and rates, they will be based on a given monthly payment. The average monthly payment made by borrowers in our sample is $\$ 925$.

In what follows we present five key features that characterize shopping behavior and outcomes in the Canadian mortgage market and most negotiated-price markets:

Feature 1: Mortgage transaction rates are dispersed. There is little within-week dispersion in posted prices, especially among the big banks, where the coefficient of variation on posted rates is very close to zero. In contrast, the coefficient of variation on transaction rates is $50 \%$, and there is substantial residual dispersion as illustrated by the $R^{2}$ of 0.61 in Table 1 b . See Allen et al. (2014b) for more details.

Feature 2: Consumers who are loyal and located in concentrated markets tend to pay higher rates. The rate regression shows that clients who remain loyal to their home bank receive discounts that are about 9.1 bps smaller than do new clients. It also shows that discounts are increasing in the number of local lenders and decreasing in relative network size.

Feature 3: Consumers search more than they switch. The search and negotiation process typically begins with the consumer's main financial institution-about $80 \%$ of consumers get a quote from their main institution (see Allen et al. (2014a)). A little over $60 \%$ of consumers search, but only about $28 \%$ switch away from their main institution.

Feature 4: Consumers are more loyal in concentrated markets and to banks with larger branch networks. The loyalty regression shows that the likelihood of remaining loyal is decreasing in the number of lenders present in the market and increasing in relative network size.

Feature 5: Lenders with strong retail presence have larger market shares. On average consumers face 8.6 lenders within their neighborhood. Consumers tend to choose lenders with large branch networks; transacting with lenders that are nearly $60 \%$ larger than their competitors in terms of branches. Lenders with larger branch networks also tend to have a bigger share of the day-to-day banking market, generating a link between day-to-day market share and mortgagemarket share that provides large banks with an incumbency advantage.

## 3 Model

In this section we build a model that captures the five key features just mentioned. Consumers receive an initial quote from their main financial institution and then decide whether to accept or
reject this quote based on their heterogeneous search costs and their expected gain from gathering multiple quotes, which depends, among other things, on how competitive is their local market.

In addition to capturing these features, the model takes into account the fact that, during negotiation, loan officers can lower previously made offers in an effort to attract or retain potential clients. Furthermore, competition takes place locally between managers of competing banks, since consumers must contact loan officers directly to obtain discounts. We also suppose that branches that are part of the same network do not compete for the same borrowers, a feature of the Canadian mortgage market and of some, but not all, negotiated-price markets.

The next three subsections describe the model. First, we present preferences and cost functions, and the bargaining protocol. Then, we solve the model backwards, starting with the second stage of the game in which banks compete for consumers. Finally, we describe the consumer search decision, and the process generating the initial quote. All variables introduced in the model vary at the consumer level, $i$, based on observed or unobserved characteristics. To simplify notation we omit the borrower's index $i$, and will add it back in the next section for random variables and consumer characteristics.

### 3.1 Preferences and cost functions

Consumers solve a discrete-choice problem over which lender to use to finance their mortgage:

$$
\begin{equation*}
\max _{j \in \mathcal{J}} v_{j}-p_{j} \tag{1}
\end{equation*}
$$

where $\mathcal{J}$ is the set of lenders offering a quote, $p_{j}$ denotes the monthly payment offered by lender $j$, and $v_{j}$ denotes the maximum willingness-to-pay (or WTP) associated with bank $j$.

The choice set $\mathcal{J}$ is defined both by where consumers live, and by their search decision. In particular, consumers can obtain a quote from their home bank $(h)$ and from the $n$ lenders in $\mathcal{N}$. We assume that the cost of obtaining a quote from the home bank is zero, while the cost of getting additional quotes is $\kappa>0$. This search cost does not depend on the number of quotes, and is distributed in the population according to CDF $H(\cdot)$.

The WTP of consumers is a combination of differentiation and mortgage valuation:

$$
v_{j}= \begin{cases}\bar{v}+\lambda & \text { if } j=h \\ \bar{v} & \text { else }\end{cases}
$$

The valuation for a mortgage, $\bar{v}$, is common across all lenders. Throughout we assume that it is large enough not to affect the set of consumers present in our sample. The parameter $\lambda \geq 0$ measures consumers' willingness to pay for their home bank relative to other lenders.

We also assume that banks have a constant borrower-specific marginal cost of lending. This measures the direct lending costs for the bank (i.e. default and pre-payment risks), net of the
future benefits associated with selling complementary services to the borrower. ${ }^{11}$ Since we do not observe the performance of the contract along the risk and complementarity dimensions, we use a reduced-form function to approximate the net present value of the contract. In particular, the monthly cost for bank $j$ to lend to the consumer is:

$$
c_{j}= \begin{cases}c-\Delta & \text { If } j=h,  \tag{2}\\ c+\omega_{j} & \text { If } j \neq h,\end{cases}
$$

where $c$ is the common cost of lending to the consumer; $\omega_{j}$ the cost differential of lender $j$ relative to the home bank (or its match value); and $\Delta$ is the home bank's cost advantage. This advantage arises because of the multi-product nature of financial institutions and the fact that the home bank is potentially already selling profitable products to the consumer. ${ }^{12}$ It could come from real complementarities generated by bundling products (economies of scope), and/or from the fact that costs include not just the direct cost of mortgage lending, but also revenues/costs derived from the sale of additional products. ${ }^{13}$ In contrast to the home bank, competing lenders may need to offer discounts on these products to overcome the switching costs on these products, or may not earn any revenues at all from them if consumers do not switch.

As we will see below, the importance of brand loyalty in the market is driven by the sum of the cost and willingness-to-pay advantage of the home bank: $\gamma=\Delta+\lambda$. We refer to $\gamma$ as the home-bank loyalty advantage.

The idiosyncratic component, $\omega_{j}$, is distributed according to $G(\cdot)$, with $E\left(\omega_{j}\right)=0$. We use subscript ( $k$ ) to denote the $k^{\text {th }}$ lowest cost match value amongst the non home-bank lenders. The CDF of the $k^{t h}$ order statistic among the $n$ lenders is given by $G_{(k)}(w \mid n)=\operatorname{Pr}\left(\omega_{(k)}<w \mid n\right)$.

Finally, lenders' quotes are constrained by a common posted price $\bar{p} .{ }^{14}$ The posted price determines both the reservation price of consumers (i.e. $\bar{v}>\bar{p}$ ), and whether or not consumers qualify for a loan at a given lender (i.e. $\bar{p}>c_{j}$ ).

[^6]
### 3.2 Bargaining protocol, information and timing of the game

In an initial period outside the model, consumers choose the type of house they want to buy, the loan size, $L$, and the timing of the home purchase (including closing date). Our focus is on the negotiation process, which we model as a two-stage game. In the first-stage, the home bank makes an initial offer $p^{0}$. At this point, the borrower can accept the offer, or search for additional quotes by paying the search cost $\kappa$. If the initial quote is rejected, the borrower organizes an English auction among the home bank and the $n$ other banks present in their neighborhood. The lender choice maximizes the utility of consumers, as in equation (1).

Information about costs and preferences is revealed sequentially. At the initial stage, all parties observe the posted price $\bar{p}$, the number of rival banks $n$, the common component of the lending cost $c$, and the home-bank cost and WTP advantages $(\lambda, \Delta)$. These variables define the observed state vector: $s=(c, \lambda, \Delta, \bar{p}, n)$. This information is common to all players. The search cost is privately observed by consumers. The home bank knows only the distribution, which can vary across consumers based on observed demographic attributes. Finally, in the second stage of the game, each lender learns its idiosyncratic lending cost, $\omega_{j}$.

Before solving the game, two remarks are in order. First, consumers are price takers in the model, and so lenders have full bargaining power. This does not mean, however, that consumers have no bargaining leverage, since they have an informational advantage from knowing their search cost. This prevents the home bank from extracting the entire surplus of consumers, as in Allen et al. (2014a). ${ }^{15}$ Second, consumers are assumed to pay the cost of generating offers at the auction stage (rather than firms). Therefore banks that are not competitive relative to the home bank are, in theory, indifferent between submitting and not submitting a quote. In these cases we assume that banks always submit a truthful offer that is consistent with their realized match values.

Next, we describe the solution of the negotiation by backward induction, starting with the competition stage.

### 3.3 Competition stage

Conditional on rejecting $p^{0}$, the home bank competes with lenders in the borrower's choice set. We model competition as an English auction with heterogeneous firms, and a cost advantage for the home bank. ${ }^{16}$ Since the initial quote can be recalled, firms face a reservation price: $p^{0} \leq \bar{p}$.

We can distinguish between two cases leading to a transaction: (i) $\bar{p}<c-\Delta$, and (ii) $c<$

[^7]$p^{0}+\Delta \leq \bar{p}+\Delta$. In the first case the borrower does not qualify at the home bank. A borrower not qualifying at their home bank, must search and their reservation price is $\bar{p}$. This borrower may qualify at other banks because of differences in $\omega_{j}$. The lowest qualifying cost bank wins by offering a price equal to the lending cost of the second most efficient qualifying lender:
\[

$$
\begin{equation*}
p^{*}=\min \left\{c+\omega_{(2)}, \bar{p}\right\} . \tag{3}
\end{equation*}
$$

\]

This occurs if and only if, $0<\bar{p}-c-\omega_{(1)}$.
If the borrower qualifies at the home bank, the highest surplus bank wins, and offer a quote that provides the same utility as the second best option. The equilibrium pricing function is:

$$
p^{*}= \begin{cases}p^{0} & \text { If } \bar{v}+\lambda-p^{0} \geq \bar{v}-c-\omega_{(1)}  \tag{4}\\ c+\omega_{(1)}+\lambda & \text { If } \bar{v}+\lambda-p^{0}<\bar{v}-c-\omega_{(1)}<\bar{v}-c+\gamma \\ c-\gamma & \bar{v}-c-\omega_{(1)}>\bar{v}-c+\gamma>\bar{v}-c-\omega_{(2)} \\ c+\omega_{(2)} & \text { If } \bar{v}-c-\omega_{(2)}>\bar{v}-c+\gamma .\end{cases}
$$

This equation highlights the fact that, at the competition stage, lenders directly competing with the home bank will on average have to offer a discount equal to the loyalty advantage in order to attract new customers. ${ }^{17}$ In cases 1 and 2 the home bank provides the highest utility and so wins the auction. In case 1, the initial quote provides higher utility than does the next best lender's quote and so the consumer pays $p^{0}$. In case 2 , the reverse is true and so the consumer pays $c+\omega_{(1)}+\lambda$ and gets utility of $\bar{v}-c-\omega_{(1)}$. In cases 3 and 4 , the home bank is not the highest surplus lender and the consumer pays $c-\gamma$ or $c+\omega_{(2)}$ depending on whether the home bank is the second or third highest surplus lender.

### 3.4 Search decision and initial quote

The borrower chooses to search by weighing the value of accepting $p^{0}$, or paying a sunk cost $\kappa$ in order to lower their monthly payment. The utility gain from search is:

$$
\begin{aligned}
\bar{\kappa}\left(p^{0}, s\right) & =\underbrace{\bar{v}+\lambda\left[1-G_{(1)}(-\gamma)\right]-E\left[p^{*} \mid p^{0}, s\right]}_{2^{n d} \text { stage expected utility }}-\underbrace{\left[\bar{v}+\lambda-p^{0}\right]}_{1^{s t} \text { stage utility }} \\
& =p^{0}-E\left[p^{*} \mid p^{0}, s\right]-\lambda G_{(1)}(-\gamma),
\end{aligned}
$$

[^8]where $1-G_{(1)}(-\gamma)$ is the retention probability of the home bank in the competition stage. A consumer will reject $p^{0}$ if and only if the gain from search is larger than the search cost. Therefore, the search probability is:
\[

$$
\begin{equation*}
\operatorname{Pr}\left(\kappa<p^{0}-E\left[p^{*} \mid p^{0}, s\right]-\lambda G_{(1)}(-\gamma \mid n)\right) \equiv H\left(\bar{\kappa}\left(p^{0}, s\right)\right) \tag{5}
\end{equation*}
$$

\]

Lenders do not commit to a fixed interest rate, and are open to haggling with consumers based on their outside options. This allows the home bank to discriminate by offering the same consumer up to two quotes: (i) an initial quote $p^{0}$, and (ii) a competitive quote $p^{*}$ if the first is rejected.

The price discrimination problem is based on the expected value of shopping and the distribution of search costs. More specifically, anticipating the second-stage outcome, the home bank chooses $p^{0}$ to maximize its expected profit:

$$
\max _{p^{0} \leq \bar{p}} \quad\left(p^{0}-c+\Delta\right)\left[1-H\left(\bar{\kappa}\left(p^{0}, s\right)\right)\right]+H\left(\bar{\kappa}\left(p^{0}, s\right)\right) E\left(\pi_{h}^{*} \mid p^{0}, s\right)
$$

where $E\left(\pi_{h}^{*} \mid p^{0}, s\right)=\left(p^{0}-c+\Delta\right)\left(1-G_{(1)}\left(p^{0}-\lambda-c\right)\right)+\int_{-\gamma}^{p^{0}-c-\lambda}\left(\omega_{(1)}+\gamma\right) d G_{(1)}$, are the expected profits from the auction for the home bank. The first term represents the case where the initial quote provides higher utility than the next highest surplus lender, while the second is the reverse.

Importantly, the home bank will offer a quote only if it makes positive profit at the posted-price: $0<\bar{p}-c+\Delta$. In the interior solution, the optimal initial quote is implicitly defined by the following first-order condition:

$$
p^{0}-c+\Delta=\underbrace{\frac{1-H\left(\bar{\kappa}\left(p^{0}, s\right)\right)}{H^{\prime}\left(\bar{\kappa}\left(p^{0}, s\right)\right) \bar{\kappa}_{p^{0}}\left(p^{0}, s\right)}}_{\begin{array}{c}
\text { Search cost }  \tag{6}\\
\text { distribution }
\end{array}}+\underbrace{E\left(\pi_{h}^{*} \mid p^{0}, s\right)}_{\begin{array}{c}
\text { Cost and quality } \\
\text { Differentiation }
\end{array}}+\underbrace{\frac{H\left(\bar{\kappa}\left(p^{0}, s\right)\right)}{H^{\prime}\left(\bar{\kappa}\left(p^{0}, s\right)\right) \bar{\kappa}_{p^{0}}\left(p^{0}, s\right)} \frac{\partial E\left(\pi_{h}^{*} \mid p^{0}, s\right)}{\partial p^{0}}}_{\text {Reserve price effect }}
$$

where $\bar{\kappa}_{p^{0}}\left(p^{0}, s\right)=\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial p^{0}}$. Equation (6) implicitly defines the home bank's profit margins from price discrimination. It highlights three sources of profits: (i) positive average search costs, (ii) market power from differentiation in cost and quality (i.e. match value differences and home-bank cost advantage), and (iii) the reserve price effect. If firms are homogenous, the only source of profits will stem from the ability of the home bank to offer higher quotes to high search cost consumers.

Although the initial quote does not have a closed-form solution, in the following proposition (proven in Appendix B) we claim that, in the interior, it is additive in the common cost shock. This simplifies the problem, since we need to numerically solve the first-order condition for only one value of $c$ per consumer.

Proposition 1. The optimal initial quote, $p^{0}$, is additive in $c$ in the interior: $p^{0}=c+\mu(\Delta, \lambda, n)$.

From this proposition, we can characterize the initial quote as follows:

$$
p^{0}(s)= \begin{cases}\bar{p} & \text { If } c>\bar{p}-\mu(\Delta, \lambda, n) \\ c+\mu(\Delta, \lambda, n) & \text { Else }\end{cases}
$$

To summarize, the model predicts three equilibrium functions: (i) the initial quote $p^{0}(s)$, (ii) the search-cost threshold $\bar{\kappa}(s)$, and (iii) the competitive price $p^{*}(\omega, s)$. Although it is difficult to characterize these functions analytically, the separability of the initial quote in the interior leads to a series of useful predictions that are summarized in Corollary 1. We use these implications in the identification section below.

Corollary 1. The following predictions about the distribution of prices and search probability in the interior, when $p^{0}(s)<\bar{p}$, follow from Proposition 1:
(i) The equilibrium search probability is independent of $c$.
(ii) The equilibrium search probability is affected symmetrically by $\lambda$ and $\Delta$.
(iii) The distribution of $p^{*}$ for switchers is only a function of $\gamma=\lambda+\Delta$.
(iv) The average transaction price paid by loyal consumers is affected asymmetrically by $\lambda$ and $\Delta$, and the effect of $\lambda$ is stronger.

## 4 Identification

The model contains four primitives: (i) the distribution of the common lending cost conditional on observed attributes of borrower $i$ and region and period fixed effects $\left(x_{i}\right), F\left(c_{i} \mid x_{i}\right)$, (ii) the distribution of idiosyncratic cost differences, $G\left(\omega_{i j}\right)$, (iii) the search-cost distribution, $H\left(\kappa_{i}\right)$, and (iv) the loyalty-advantage parameters, $(\lambda, \Delta)$. From the model description, we maintain the assumptions that the lending cost function is additively separable in $c$ and $\omega$, and that $\kappa_{i}$ and $\omega_{i j}$ are IID. We also assume that loyalty parameters are common across consumers, and that ( $\kappa_{i}, \omega_{i j}$ ) are independent of observed borrower characteristics $x_{i}$ (this last assumption is partially relaxed in the empirical analysis).

In this section we discuss nonparametric identification of the search and cost distributions, as well as the identification of the loyalty parameters. In the next, we estimate a parametric version of the model, which allows us to more easily incorporate observable differences between consumers and firms.

The data correspond to a cross section of transaction prices, borrower characteristics, and lender choices (including whether or not the lender is the home bank). The characteristics of the borrower allow us to infer the posted price valid at the time of the transaction $\left(\bar{p}_{t(i)}\right)$, as well as the number of available options in the borrower's neighborhood $\left(n_{i}\right)$. From the data, we can therefore characterize the probability of switching lenders conditional on $\left(x_{i}, \bar{p}_{t(i)}, n_{i}\right)$, as well as the
distribution of transaction prices given $\left(x_{i}, \bar{p}_{t(i)}, n_{i}\right)$ separately for switching and loyal consumers. These three distributions correspond to the reduced form of the model.

We face two challenges when when discussing the mapping from the reduced form to the primitives of the model. First, since we only observe accepted offers, and we must infer the distributions of the two unobserved heterogeneity components ( $c_{i}$ and $\omega_{i}$ ) from a single price. Second, since we do not observe search and switch decisions separately, we need to distinguish between two sources of attachment to the home bank-search costs and the loyalty advantage-solely using the conditional probability of remaining loyal to the home bank.

To overcome these challenges, in addition to the assumptions listed above, we rely on two exclusion restrictions. We assume that the number of lenders and the posted price are independent of the $c_{i}$, conditional on the observed attributes of the borrower, $x_{i}$. Furthermore, we require that both variables exhibit enough variation across borrowers. We formally introduce these assumptions in Appendix C, and propose a sequential approach to show that they are sufficient to guarantee identification of the model. The argument can be summarized as follows.

1. Consider first the distribution of prices for switching borrowers facing very high posted prices: $\bar{p} \rightarrow \infty$. These transactions are generated from the auction, and reflect the cost of the second most efficient lender (including potentially the home bank). Furthermore, since the postedprice constraint is not binding, selection into the competition stage is independent of the realization of $c_{i}$ (from Corollary $1(i)$ ). This eliminates the selection bias that arises from looking separately at switching consumers. ${ }^{18}$

In this sub-sample, the distribution of transaction prices across markets with different $n$ 's can be used to separately identify $F\left(c_{i} \mid x_{i}\right), G\left(\omega_{i}\right)$ and the sum of the two loyalty parameters $(\gamma=\Delta+\lambda)$. To see this, note that when $n=2$, the transaction price is equal to $p_{i}^{*}=c_{i}+\gamma$, which can be used to identify $F\left(c_{i} \mid x_{i}\right)$ given $\gamma$. Next, consider markets with a small number of lenders $n>2$. In such markets, the presence of a positive loyalty advantage implies that prices paid by switchers mostly reflect the common cost component, which is independent of $n$. As the number of lenders increases, the probability that a rival lender, and not the home bank, is the next-best alternative, converges to one. For large $n$, the distribution of the idiosyncratic cost component is identified using standard English auction arguments. In between, the correlation between the number of rivals and the price paid by switchers depends on the magnitude of the loyalty advantage parameter. Therefore, $\gamma$ is identified from the strength of the correlation between the number of rivals and $p^{*}$, as the number of competitors becomes large.
2. Consider next data on the probability of remaining loyal to the home bank, conditional on $\left(x_{i}, n_{i}, \bar{p}_{t(i)}\right)$. In the model, this probability corresponds to the product of the search

[^9]probability, and the probability that the home bank retains the consumer at the auction stage (i.e. $\left.G_{(n)}(-\gamma)\right)$. The previous argument suggests that the gain from search and the retention probability, which are functions of $G(\omega)$ and $\gamma$, can be computed directly from the distribution of prices for unconstrained switching consumers. However, absent the constraint imposed by the posted price, the switching probability only takes discrete values in equilibrium; one for each $n \in\{2,3, \ldots, \bar{n}\}$. This is because $c_{i}$ does not affect the search probability. These moments would be sufficient to test the null hypothesis that search costs are zero, but not to identify the distribution $H\left(\kappa_{i}\right)$ nonparametrically. ${ }^{19}$

The presence of a binding posted-price constraint breaks this independence, and creates dispersion in the search-cost thresholds across consumers within the same market. In particular, for consumers receiving $p^{0}=\bar{p}$, the search probability is monotonically increasing in $\bar{p}$. Therefore, exogenous variation in $\bar{p}$ can be used to nonparametrically identify the distribution of search costs, by varying the search-cost threshold across consumers with similar $x_{i}$ and $n$.
3. Finally, the observed distribution of prices among loyal consumers can be used to separate the effect of loyalty on cost (i.e. $\Delta$ ) and willingness-to-pay (i.e. $\lambda$ ). This distribution is a mixture of initial quote offers and auction prices. We know from Corollary $1(i v)$ that $\lambda$ and $\Delta$ have different impacts on the average transaction price of loyal consumers. In contrast, $\lambda$ and $\Delta$ affect symmetrically the equilibrium search probability in the interior (Corollary $1(i i))$, and the distribution of prices for switchers (Corollary $1(i i i))$. Therefore, while both parameters influence in the same way the observed retention probability, they have different effects on the average price difference between loyal and switching consumers. This moment can thus be used to identify $\lambda$ separately from $\Delta$.

This identification argument relies on the existence of important variation in the number of lenders and the posted price across consumers. This is particularly relevant for the identification of the search cost distribution (step 2). In practice, we observe fairly limited time-series variation in the posted rate, and very few consumers with fewer than 4 lenders in their neighborhoods. Given these shortcomings of the data, we incorporate additional aggregate moments measuring the fraction of borrowers gathering more than one quote, conditional on (limited) demographic characteristics (from the FIRM survey). With this additional information, the separate identification of the search and loyalty parameters becomes even more transparent. We now have two measures of state dependence: the average switching probability $(\bar{S})$ and the average search probability $(\bar{H})$. Using these measures, one can use the predicted switching probability in equation (20) to estimate the

[^10]aggregate retention probability of the home bank at the auction stage:
$$
\bar{S}=\bar{H} \times G_{(1)}(-\gamma), \quad G_{(1)}(-\gamma)=\frac{\bar{S}}{\bar{H}}
$$

For instance, in our sample the average switching probability is approximately $30 \%$, while the aggregate search probability from the FIRM survey is $65 \%$. On average, the home bank therefore wins the auction with probability $46 \%$. Since, on average, the number of lenders per neighborhood is 8 , this implies that the loyalty advantage is positive and large relative to the dispersion of idiosyncratic cost differences.

## 5 Estimation method

In this section we describe the steps taken to estimate the model parameters. We begin by describing the functional form assumptions imposed on consumers' and lenders' unobserved attributes. We then derive the likelihood function induced by the model, and discuss the sources of identification.

### 5.1 Distributional assumptions and functional forms

The lending cost function differs slightly from the model presentation. In particular, we account for loan size differences across borrowers, and we allow observed bank characteristics to affect the distribution of cost differences across lenders (i.e. $\omega_{i j}$ and $\Delta_{i}$ ).

We model the monthly cost of lending $\$ L_{i}$ over a 25 year amortization period using a linear function of borrower and lender characteristics:

$$
\begin{equation*}
c_{i j}=L_{i} \times\left(c_{i}+\omega_{i j}\right), \tag{7}
\end{equation*}
$$

where the common cost component is normally distributed, $c_{i} \sim N\left(x_{i} \beta, \sigma_{c}^{2}\right)$, and the idiosyncratic cost differences are distributed according to a lender-specific type-1 extreme value distribution, $\omega_{i j} \sim \operatorname{T1EV}\left(\xi_{i j}-e \sigma_{\omega}, \sigma_{\omega}\right) .{ }^{20}$

The location parameter of the idiosyncratic cost difference distribution, $\xi_{i j}$, varies across lenders due to the presence of bank fixed-effects, and the size of the branch network in the neighborhood of the consumer (normalized by the average network size of rivals). The type-1 extreme-value distribution assumption leads to analytical expressions for the distribution functions of the firstand second-order statistics, and is often used to model asymmetric value distributions in auction settings (see for instance Brannan and Froeb (2000)).

The loan size is normalized so that the per-unit lending cost in equation (7) measures the monthly cost of a $\$ 100,000$ loan. The vector $x_{i}$ controls for observed financial characteristics of

[^11]the borrower (e.g. income, loan size, FICO score, LTV, etc), the bond-rate, as well as period and location fixed-effects. The location fixed-effects identify the region of the country where the house is located, defined using the first digit of the postal code (i.e. postal-code district). The period fixed-effects are defined at the quarter-year level.

The lending cost of the home bank is expressed slightly differently, because of the home-bank cost-advantage parameter:

$$
c_{i, h(i)}=L_{i} \times\left(c_{i}+\Delta_{i, h(i)}\right),
$$

where $h(i)$ is the home-bank index of borrower $i$, and $\Delta_{i, h(i)}=\xi_{i, h(i)}-\Delta\left(z_{i}^{2}\right)$ is consumer $i$ 's homebank deterministic cost differential. In the application, we allow the cost-advantage parameter to depend on the borrower's income and home-ownership status:

$$
\Delta\left(z_{i}^{2}\right)=L_{i} \times\left(\Delta_{0}+\Delta_{\text {inc }} \text { Income }_{i}+\Delta_{\text {owner }} \text { Previous } \text { Owner }_{i}\right) .
$$

The WTP component of the loyalty advantage is defined analogously as a linear function of income and home-ownership status:

$$
\lambda\left(z_{i}^{2}\right)=L_{i} \times\left(\lambda_{0}+\lambda_{\text {inc }} \text { Income }_{i}+\lambda_{\text {owner }} \text { Previous } \text { Owner }_{i}\right) .
$$

Finally, we assume that the search cost is exponentially distributed with a consumer-specific mean that depends on income and home-ownership status:

$$
H\left(\kappa \mid z_{i}^{1}\right)=1-\exp \left(-\frac{1}{\alpha\left(z_{i}^{1}\right)} \kappa\right), \quad \log \alpha\left(z_{i}^{1}\right)=\alpha_{0}+\alpha_{\text {inc }} \log \text { Income }_{i}+\alpha_{\text {owner }} \text { Previous } \text { Owner }_{i} .
$$

### 5.2 Likelihood function

We estimate the model by maximum likelihood. The endogenous outcomes of the model are: the chosen lender and monthly payment $\left\{b(i), p_{i}\right\}$, as well as whether consumers remain loyal to their home bank or switch. The observed prices are either generated from consumers accepting the initial quote (i.e. $p_{i}=p^{0}(s)$ ), or accepting the competitive offer (i.e. $p_{i}=p^{*}(\omega, s)$ ). Importantly, only the latter case is feasible if consumers switch financial institutions, while both cases have a positive likelihood for loyal consumers.

Moreover, the identity of the home bank is known for loyal consumers, while it unobserved for switching consumers. To construct the likelihood function, we first condition on the identity of the home bank for both types of transactions, and then integrate out $h$ using the empirical distribution of $h$ defined in Section 2 .

In order to derive the likelihood contribution of each individual, we first condition on the
choice-set $\mathcal{N}_{i},{ }^{21}$ the observed characteristics $x_{i}$, the identity of home bank $h$, the posted price valid at the time consumer $i$ negotiated the contract $\bar{p}_{t(i)}$, and the model parameter vector $\theta=$ $\left\{\beta, \xi, \sigma_{\omega}, \sigma_{c}, \alpha, \Delta, \lambda\right\}$. Let $\mathcal{I}_{i}=\left\{\mathcal{N}_{i}, x_{i}, \bar{p}_{t(i)}\right\}$ summarize the information known by the econometrician about consumer $i$.

In order to simplify notation, we use individual subscripts $i$ for the borrower characteristics and random variables, with the understanding that all functions and variables are consumer-specific and depend on $\mathcal{I}_{i}$ and the parameter vector $\theta$. For instance, $\Delta_{i, h}=\xi_{i, h}-\Delta\left(z_{i}^{2}\right)$ and $\lambda_{i}=\lambda\left(z_{i}^{2}\right)$ denote the home-bank cost and WTP advantages, and $\mu_{i} \equiv \mu\left(\mathcal{N}_{i}, \Delta_{i, h}, \lambda_{i}\right)$ is used to denote the initial quote markup (interior solution). In addition, we use $c_{i}$ to summarize the state variable in the initial stage of the game, instead of $s_{i}=\left\{c_{i}, \bar{p}_{t(i)}, \mathcal{N}_{i}, \Delta_{i, h}, \lambda_{i}\right\}$. For instance, $\bar{\kappa}\left(c_{i}\right) \equiv \bar{\kappa}\left(s_{i}\right)$ and $p^{0}\left(c_{i}\right) \equiv p^{0}\left(s_{i}\right)$ correspond to the equilibrium search-cost threshold and initial quote, respectively.

Next we summarize the likelihood contribution for loyal and switching consumers. Appendix D describes in greater details the derivation of the likelihood function.

Likelihood contribution for loyal consumers The main obstacle in evaluating the likelihood function is that we do not observe whether or not consumers search. The unconditional likelihood contribution of loyal consumers is therefore:

$$
\begin{align*}
& L\left(p_{i}, b(i)=h \mid \mathcal{I}_{i}, h, \theta\right) \\
& \quad=L\left(p_{i}=p^{0}\left(c_{i}\right), b(i)=h \mid \mathcal{I}_{i}, h, \theta\right)+L\left(p_{i}=p^{*}\left(\omega_{i}, c_{i}\right), b(i)=h \mid \mathcal{I}_{i}, h, \theta\right) \tag{8}
\end{align*}
$$

The first term is a function of the solution to the optimal initial quote: $p^{0}\left(c_{i}\right)=\min \left\{\bar{p}_{t(i)}, c_{i}+\right.$ $\left.\mu_{i}\right\}$. Since the markup is independent of $c_{i}$ in the interior, the distribution of $p_{i}$ takes the form of a truncated distribution:

$$
L\left(p_{i}=p^{0}\left(c_{i}\right), b(i)=h \mid \mathcal{I}_{i}, h, \theta\right)= \begin{cases}f\left(p_{i}-\mu_{i} \mid x_{i}\right)\left[1-H\left(\bar{\kappa}\left(p_{i}-\mu_{i}\right)\right)\right] & \text { If } p_{i}<\bar{p}_{t(i)}  \tag{9}\\ \int_{\bar{p}_{t(i)}-\mu_{i}}^{\bar{p}_{t(i)}+\Delta_{i, h}}\left[1-H\left(\bar{\kappa}\left(c_{i}\right)\right)\right] d F\left(c_{i} \mid x_{i}\right) & \text { If } p_{i}=\bar{p}_{t(i)}\end{cases}
$$

The second element measures the probability of observing a constrained initial quote. This event occurs if $c_{i}>\bar{p}_{t(i)}-\mu_{i}$, and the consumer qualifies for a loan at its home bank (i.e. $c_{i}<\bar{p}_{t(i)}-\Delta_{i, h}$ ).

In addition to the search cost and the common lending cost, the likelihood contribution from searching consumers reflects the realization of the lowest cost differential in $\mathcal{N}_{i}$ (i.e. $\left.\omega_{i,(1)}\right)$. In particular, the transaction price is given by: $p_{i}=p^{0}\left(c_{i}\right)$ if $\omega_{i,(1)}>p^{0}\left(c_{i}\right)-c_{i}-\lambda_{i}$, or by $p_{i}=$

[^12]$c_{i}+\omega_{i,(1)}+\lambda_{i}$ otherwise.
\[

$$
\begin{align*}
L\left(p_{i}=\right. & \left.p^{*}\left(\omega_{i}, c_{i}\right), b(i)=h \mid \mathcal{I}_{i}, h, \theta\right) \\
& = \begin{cases}\left(1-G_{(1)}\left(\mu_{i}-\lambda_{i} \mid \mathcal{N}_{i}\right)\right) H\left(\bar{\kappa}\left(p_{i}-\mu_{i}\right)\right) f\left(p_{i}-\mu_{i} \mid x_{i}\right) & \text { If } p_{i}<\bar{p}_{t(i)}, \\
+\int_{p_{i}+\mu_{i, h}}^{p_{i}+\Delta_{i, h}} g_{(1)}\left(p_{i}-c_{i}-\lambda_{i}\right) H\left(\bar{\kappa}\left(c_{i}\right)\right) d F\left(c_{i} \mid x_{i}\right) & \\
\int_{\bar{p}_{t(i)}+\Delta_{i}}^{\bar{p}_{i, h}}\left(1-G_{(1)}\left(\bar{p}_{t(i)}-c_{i}-\lambda_{i} \mid \mathcal{N}_{i}\right)\right) H\left(\bar{\kappa}\left(c_{i}\right)\right) d F\left(c_{i} \mid x_{i}\right) & \text { If } p_{i}=\bar{p}_{t(i)} .\end{cases} \tag{10}
\end{align*}
$$
\]

Likelihood contribution for switching consumers For switching consumers, the likelihood contribution depends on the relative position of the home bank in the surplus distribution of lenders belonging to $\mathcal{N}_{i}$. We use $g_{b}(\omega)$ to denote the density of the cost differential of the chosen lender, and $g_{-b}\left(\omega \mid \mathcal{N}_{i}\right)$ to denote the density of the most efficient lender in $\mathcal{N}_{i}$ other than $b .{ }^{22}$

If the observed price is unconstrained, the transaction price is equal to the minimum of $c_{i}-$ $\left(\Delta_{i, h}+\lambda_{i}\right)$ and $c_{i}+\omega_{i,-b}$. If the consumer does not qualify for a loan at their home bank, the transaction price is the minimum of the posted price, and the second-lowest cost. This occurs if $c_{i}>\bar{p}_{t(i)}+\Delta_{i, h}$. Therefore, the transaction price for switching consumers is equal to $\bar{p}$ if and only if the chosen lender is the only qualifying bank. This leads to the following likelihood contribution:

$$
\begin{align*}
& L\left(p_{i}, b(i) \neq h \mid \mathcal{I}_{i}, h, \theta\right) \\
& = \begin{cases}1\left(\bar{p}_{t(i)}>p_{i}+\lambda_{i}\right)\left[\begin{array}{c}
\left(1-G_{-b}\left(-\Delta_{i, h}-\lambda_{i} \mid \mathcal{N}_{i}\right)\right) G_{b}\left(-\Delta_{i, h}-\lambda_{i}\right) \\
\times H\left(\bar{\kappa}\left(p_{i}+\Delta_{i, h}+\lambda_{i}\right)\right) f\left(p_{i}+\Delta_{i, h}+\lambda_{i} \mid x_{i}\right)
\end{array}\right] & \text { If } p_{i}<\bar{p}_{t(i)}, \\
+\int_{p_{i}+\Delta_{i, h}+\lambda_{i}}^{\infty} G_{b}\left(p_{i}-c_{i}\right) H\left(\bar{\kappa}\left(c_{i}\right)\right) g_{-b}\left(p_{i}-c_{i} \mid \mathcal{N}_{i}\right) d F\left(c_{i} \mid x_{i}\right) & \\
\int_{\bar{p}_{t(i)}+\Delta_{i, h}}^{\infty} G_{b}\left(\bar{p}-c_{i}\right)\left(1-G_{-b}\left(\bar{p}_{t(i)}-c_{i} \mid \mathcal{N}_{i}\right)\right) d F\left(c_{i} \mid x_{i}\right) & \text { If } p_{i}=\bar{p}_{t(i)} .\end{cases} \tag{11}
\end{align*}
$$

Note that the first term is equal to zero if $\bar{p}_{t(i)}<p_{i}+\lambda_{i} .{ }^{23}$ This condition ensures that the home bank's lending cost is below $\bar{p}_{t(i)}$ at the observed transaction price.

Integration of the home bank identity and selection The unconditional likelihood contribution of each individual is evaluated by integrating out the identity of the home bank. Recall, that $h$ is missing for a sample of contracts, and is unobserved for switchers. In the former case we use the unconditional distribution of home banks, while in the latter case we condition on the fact

[^13]that the chosen lender is not the home bank. This leads to the following unconditional likelihood:
\[

L\left(p_{i}, b(i) \mid \mathcal{I}_{i}, \theta\right)= $$
\begin{cases}L\left(p_{i}, b(i) \mid \mathcal{I}_{i}, h=b(i), \theta\right), & \text { If } 1\left(\operatorname{Loyal}_{i}\right)=1  \tag{12}\\ \sum_{h \neq b(i)} \frac{\psi_{h}\left(x_{i}\right)}{\sum_{j \neq b} \psi_{j}\left(x_{i}\right)} L\left(p_{i}, b(i) \mid \mathcal{I}_{i}, h, \theta\right) & \text { If } 1\left(\operatorname{Loyal}_{i}\right)=0 \\ \sum_{h} \psi_{h}\left(x_{i}\right) L\left(p_{i}, b(i) \mid \mathcal{I}_{i}, h, \theta\right) & \text { If } 1\left(\operatorname{Loyal}_{i}\right)=\mathrm{M} / \mathrm{V}\end{cases}
$$
\]

where $\psi_{h}\left(x_{i}\right)$ is the unconditional probability distribution for the identity of the home bank.
In addition, the fact that we only observe accepted offers implies that the unconditional likelihood suffers from a sample selection problem. The probability that consumer $i$ is in our sample is given by the probability of qualifying for a loan from at least one bank in $i$ 's choice set. This is given by the probability that the minimum of $c_{i}-\Delta_{i, h}$ and $c_{i}+\omega_{i,(1)}$.

$$
\begin{equation*}
\operatorname{Pr}\left(\text { Qualify } \mid \mathcal{I}_{i}, \theta\right)=\sum_{h} \psi_{h}\left(x_{i}\right) \int F\left(\bar{p}_{t(i)}-\min \left\{\omega_{i,(1)},-\Delta_{i, h}\right\} \mid x_{i}\right) d G_{(1)}\left(\omega_{i,(1)} \mid \mathcal{N}_{i}\right) \tag{13}
\end{equation*}
$$

Using this probability, we can evaluate the conditional likelihood contribution of individual $i$ :

$$
\begin{equation*}
L^{c}\left(p_{i}, b(i) \mid \mathcal{I}_{i}, \theta\right)=L\left(p_{i}, b(i) \mid \mathcal{I}_{i}, \theta\right) / \operatorname{Pr}\left(\text { Qualify } \mid \mathcal{I}_{i}, \theta\right) \tag{14}
\end{equation*}
$$

Aggregate likelihood function To construct the likelihood function we need to aggregate the information contained in equation (14) across the $N$ observed contracts, while incorporating additional external aggregate information on search effort. We use the results of the annual FIRM survey (described in Section 2) to match the probability of gathering more than one quote along four dimensions: city-size, region, and income group.

Using the model and the observed new-home buyer characteristics we calculate the probability of rejecting the initial quote; integrating over the model shocks and the identity of the incumbent bank. Let $\bar{H}_{g}(\theta)$ denote this function for demographic group $g$. Similarly, let $\hat{H}_{g}$ denote the analog probability calculated from the survey. The difference between the two, $m_{g}(\theta)=\bar{H}_{g}(\theta)-\hat{H}_{g}$, is a mean-zero error under the null hypothesis that the model is correctly specified. We use $G=8$ aggregate moments.

Several econometric approaches have been proposed in the literature for combining data from multiple surveys. When individual data from independent surveys are available, a standard approach is to maximize a joint likelihood that is defined as the product of density functions calculated from separate data sets (e.g. Van den Berg and van der Klaauw (2001)). This approach is not feasible in our case, since we only observe aggregate moments, and do not have access to the micro-data from the search probability survey. Alternatively, we could use a constrained maximum likelihood estimator that maximizes the sum of individual likelihood contributions subject to the constraint that the aggregate moment conditions are satisfied exactly (see Ridder and Moffitt (2007)). The
disadvantage of this approach is that it ignores the fact that the aggregate moments are themselves measured with error. In our application the number of observations used to measure the aggregate moments is less than 500 , compared to close to 30,000 in the contract data. A third approach, which takes the relative sample sizes of the two data-sets into account, is the GMM estimator proposed by Imbens and Lancaster (1994). This approach combines moment restrictions obtained from the score of the log likelihood function, with the vector of aggregate errors obtained by matching moments from the survey.

Although this third option would be a natural choice, it can be difficult to implement in practice and does not perform well numerically for our specific problem. This is because in order to evaluate the GMM objective function we must rely on numerical derivatives to compute the score function. This is challenging since the likelihood function involves repeatedly solving a nested fixed-point and numerically approximating several integrals. With over 60 parameters this represents a non-trivial increase in computation time relative to evaluating the likelihood function once. Furthermore, the numerical score function is less smooth than the likelihood function, making optimization of the GMM problem numerically more prone to convergence problems. We experimented with different optimization routines without success, and decided to use an alternative estimating procedure instead.

We use a quasi-likelihood estimator that relies on a normal approximation to the density of the aggregate residuals. Let $\sigma_{g}^{2}$ denotes the predicted variance in the search probability across consumers in group $g$ (calculated from the model). From the central-limit theorem, $\sqrt{M}_{g} m_{g}(\theta) / \sigma_{g}$ is a sample average that is normally distributed when $M_{g}$ is large enough (i.e. the number of consumers surveyed in group $g$ ). In our case, the number of households surveyed by the Altus Group in each group ranges between 265 and 441.

Under this assumption, the combined quasi-likelihood is the product of the conditional likelihood function obtained from the contract data (product of equation 14 across $N$ ) and the normal densities associated with each of the aggregate moments. This leads to the following aggregate log likelihood function: ${ }^{24}$

$$
\begin{equation*}
\max _{\theta} \quad \sum_{i} \log L\left(p_{i}, b_{i} \mid \mathcal{I}_{i}, \theta\right)-m(\theta)^{T} \hat{W}_{2}^{-1} m(\theta), \tag{15}
\end{equation*}
$$

where $m(\theta)$ is a $K \times 1$ vector of errors from the auxiliary moments, and $\hat{W}_{2}$ is a diagonal matrix with the estimated asymptotic variance of the moments. ${ }^{25}$

[^14]Note that the constrained MLE problem takes a similar form:

$$
\begin{equation*}
\max _{\theta, \rho} \sum_{i} \log L\left(p_{i}, b_{i} \mid \mathcal{I}_{i}, \theta\right)-\rho m(\theta)^{T} \hat{W}_{2}^{-1} m(\theta) \tag{16}
\end{equation*}
$$

where $\rho \geq 0$ is a Lagrangian multiplier. Intuitively, as the number of observations in the auxiliary survey goes to infinity (holding fixed $N$ ), $\hat{W}_{2}^{-1}$ goes to infinity (in equation 15), and our quasilikelihood estimator forces the aggregate moments to be satisfied with equality almost surely (just like with constrained MLE).

By setting $\rho=1$, the weight that the quasi-likelihood puts on the auxiliary moments depends on the sample size. ${ }^{26}$ In that sense, our approach is similar to the GMM estimator proposed by Imbens and Lancaster (1994). However, the two estimators cannot be nested in any sense. The moment conditions in Imbens and Lancaster (1994) are not the same as the score of the quasilikelihood defined in equation 15 . When using a block-diagonal weighting matrix for each set of moment conditions, the GMM estimator minimizes the sum of the square of the scores minus a penalty function to account for the sum of square of the moment residual, while our estimator maximizes the sum the log-likelihood function minus the same quadratic penalty function. We have conducted a series of Monte Carlo simulations to analyze the small sample performance of both estimators, and found that our quasi-likelihood estimator performs equally well or better than GMM. These results are available in Appendix E. The appendix also provides additional details as to the differences between GMM and our quasi-likelihood approach.

Computation In order to evaluate the aggregate likelihood function, we must first solve the optimal initial offer defined implicitly by equation (6). This non-linear equation needs to be solved separately for every consumer/home-bank combination. We perform this operation numerically using a Newton algorithm that uses the first and second derivatives of firms' expected profits. We use starting values defined as the expected initial quote from the complete information problem, for which we have an analytical expression. This procedure is robust and converges in a small number of steps. Notice that since the interior solution is additive in $c$, this equation needs to be solved only once for each evaluation of the likelihood contribution of each household, $L\left(l_{i}, b(i) \mid \mathcal{I}_{i}, h, \theta\right)$. In addition, the integrals are evaluated numerically using a quadrature approximation.

## 6 Estimation results

### 6.1 Parameter estimates

Table 2 summarizes the maximum-likelihood estimates from three specifications, each one varying the source of the loyalty advantage. In Specification (1), the loyalty advantage takes the form of a

[^15]WTP term, $\lambda$, for the home bank. In Specification (2), the home bank has a cost advantage, $\Delta$, over competing lenders. Specification (3) nests both models.

Each specification implies that the home bank is more likely to "win" against rival banks at the competition stage, but have different implications for the price differences between loyal and switching borrowers. Holding fixed the magnitude of the idiosyncratic cost differences between lenders $\left(\sigma_{\omega}\right)$, the WTP model implies a larger average price difference between loyal and switching borrowers, relative to the cost advantage model. This difference is relatively small in the data: loyal borrowers pay about 10 bps more than switching borrowers, or about $10 \%$ of the standard-deviation of residual rates. In Specification (1), the model reconciles these two features with small estimates of $\sigma_{\omega}$ and $\lambda_{0}$. In contrast, the cost-advantage model leads to larger estimates of the differentiation parameters, $\Delta$ and $\sigma_{\omega}$. Also, the cost-advantage model fits the data significantly better.

We formally assess the performance of the two modeling choices by estimating Specification (3). The last row reports the results of two likelihood-ratio tests testing the null-hypothesis that $\lambda_{i}=0$ and $\Delta_{i}=0$. We can easily reject the null hypothesis that the cost advantage parameters are zero; the test statistics is more than 40 times larger than the $1 \%$ critical value (i.e. 660.7 vs 16.3 ). In contrast, the null hypothesis of zero home-bank WTP parameters is much more modestly rejected (i.e. 45.7 vs 16.3 ).

A closer look at the estimates of $\lambda$ in Specification (3) reveals that the intercept and owner parameters are not significantly different from zero statistically or economically, while the estimated cost advantage parameters are large and precisely estimated. The reverse is true for the interaction of income and loyalty. This suggests that the relationship between loyalty and income is better explained by the WTP model. Still, the effect of income on the loyalty advantage is economically small and imprecise in all three specifications. Since the data do not support the WTP model, we use to the cost-advantage model as our baseline specification.

Table 11 in the Appendix, evaluates the robustness of the results to the weight assigned to the auxiliary search moments. Specifically, we re-estimated the model with weights of 0 and 100 on the auxiliary search moments. A weight of 100 is analogous to increasing the sample size of the search survey to be roughly on par with the number of observations in the mortgage contract data. Doing so tends to increase the magnitude and heterogeneity of the loyalty-advantage parameters (i.e $\lambda$ and $\Delta$ ), and changes the sign of the income coefficient in the search cost function. This allows the model to better match the observed heterogeneity in the search probability across market-size and income groups (see goodness of fit discussion below).

By setting a zero weight, the parameters are identified solely using the mortgage contract data. The results from Specifications (4) and (5) are similar to the results presented in Table 2, which is not surprising given the fact that the sample size in the contract data is much larger than in the search survey. The most noticeable differences between the two estimates are that the average search cost is lower with a weight of zero (by about $15 \%-20 \%$ ), and that the dispersion of costs
across lenders is larger (e.g. $\sigma_{\omega}=0.12$ instead of $\sigma_{\omega}=0.1$ ). Both features imply a larger predicted search probability in Specifications (4) and (5), relative to (2) and (3) (approximately 3 percentage points). The fact that these differences are fairly minor confirms that the model's key parameters can be identified without using direct information on search behavior.

Next, we discuss the economic magnitude of the parameter estimates, focusing on the lending cost function and the search cost distribution. To better understand the magnitude of the estimates, recall that consumers choose a lender by minimizing their monthly payment net of the search cost. The monthly cost of supplying a $\$ 100,000$ loan is a linear function of borrowers' observed and unobserved characteristics, and the parameters are expressed in $\$ 100$ per month. For instance, in Table 2 the variance parameter of the common shock, $\sigma_{c}=0.358$, implies that the common lending-cost standard-deviation for a $\$ 100,000$ loan with fixed attributes is equal to $\$ 35.80 /$ month .

Lending cost function The first two parameters, $\sigma_{c}$ and $\sigma_{\omega}$, measure the relative importance of consumer unobserved heterogeneity with respect to the cost of lending. The standard-deviation of the common component is $64 \%$ larger than the standard-deviation of idiosyncratic shock (i.e. 0.358 versus 0.128 ), suggesting that most of the residual price dispersion is due to consumer-level unobserved heterogeneity rather than to idiosyncratic differences across lenders. ${ }^{27}$

The estimate of $\sigma_{\omega}$ has key implications for our understanding of the importance of market power in this market. Abstracting from systematic differences across banks, the average cost difference between the first- and second-lowest cost lender, $c_{(1)}$ and $c_{(2)}$, is equal to $\$ 20$ in duopoly markets, $\$ 17$ with three lenders, and approaches $\$ 14$ when $N$ is equal 11.

In the model, market-power also arises because of systematic cost differences across banks: (i) bank fixed-effects, (ii) network size, and (iii) home-bank cost advantage. The estimates of the fixed-effects reveal relatively small differences across banks. Three of the eleven coefficients are not statistically different from zero (relative to the reference bank), and the range of fixed-effects is equal to $\$ 15 /$ month in our baseline specification, or about the same scale as the standard-deviation of the idiosyncratic components.

We incorporate network size in the model by allowing the lending cost to depend on the relative branch network size of lenders in the same neighborhood. The estimates reveal that a lender with 3 times more branches than the average would experience a cost advantage of about $\$ 12 /$ month (compared to a single-branch institution). This is consistent with our interpretation of the lending cost function, as capturing elements of profits from complementary banking services that are increasing in branch-network size.

Turning to the estimate of $\Delta_{i}$, we find that the presence of the loyalty advantage corresponds to an average cost advantage of $\$ 17.10 /$ month (for a loan size of $\$ 100,000$ ). This cost advantage is substantial, given the fact that $\sigma_{\omega}$ is relatively small. At the estimated parameters, the probability that the home bank has a cost lower than the most efficient lender in $\mathcal{N}_{i}$ is equal to $G_{(1)}\left(\omega_{h}\right)=51 \%$;

[^16]Table 2: Maximum likelihood estimation results

|  | Specification 1 |  | Specification 2 Baseline |  | Specification 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Estimate | (S.E.) | Estimate | (S.E.) | Estimate | (S.E.) |
| Heterogeneity and preferences |  |  |  |  |  |  |
| Common shock ( $\sigma_{c}$ ) | 0.356 | (0.003) | 0.358 | (0.003) | 0.358 | (0.003) |
| Idiosyncratic shock ( $\sigma_{\omega}$ ) | 0.047 | (0.002) | 0.102 | (0.002) | 0.094 | (0.002) |
| Avg. search cost (log) |  |  |  |  |  |  |
| $\alpha_{0}$ | -1.539 | (0.042) | -1.506 | (0.026) | -1.592 | (0.034) |
| $\alpha_{\text {inc }}$ | 0.458 | (0.052) | 0.401 | (0.038) | 0.356 | (0.045) |
| $\alpha_{\text {owner }}$ | 0.184 | (0.054) | 0.086 | (0.059) | 0.143 | (0.059) |
| Home-bank WTP |  |  |  |  |  |  |
| $\lambda_{0}$ | 0.064 | (0.003) |  |  | 0.010 | (0.007) |
| $\lambda_{\text {owner }}$ | 0.032 | (0.002) |  |  | -0.016 | (0.008) |
| $\lambda_{\text {inc }}$ | 0.002 | (0.003) |  |  | 0.023 | (0.01) |
| Home-bank cost advantage |  |  |  |  |  |  |
| $\Delta_{0}$ |  |  | 0.146 | (0.005) | 0.126 | (0.008) |
| $\Delta_{\text {owner }}$ |  |  | 0.066 | (0.004) | 0.075 | (0.008) |
| $\Delta_{\text {inc }}$ |  |  | 0.012 | (0.006) | -0.010 | (0.01) |
| Cost function |  |  |  |  |  |  |
| Intercept | 5.332 | (0.229) | 5.495 | (0.229) | 5.479 | (0.23) |
| Bond rate | 0.307 | (0.026) | 0.306 | (0.026) | 0.306 | (0.026) |
| Range posted-rate | -0.147 | (0.017) | -0.145 | (0.017) | -0.145 | (0.017) |
| Total loan | -0.220 | (0.073) | -0.208 | (0.073) | -0.208 | (0.073) |
| Income | -0.228 | (0.026) | -0.214 | (0.026) | -0.228 | (0.027) |
| Loan/Income | -0.100 | (0.01) | -0.102 | (0.01) | -0.102 | (0.01) |
| Previous owner | -0.003 | (0.007) | 0.047 | (0.007) | 0.051 | (0.008) |
| House price | 0.222 | (0.066) | 0.211 | (0.066) | 0.211 | (0.066) |
| FICO | -0.662 | (0.038) | -0.656 | (0.038) | -0.660 | (0.038) |
| LTV | 1.111 | (0.157) | 1.092 | (0.158) | 1.093 | (0.157) |
| $1(L T V=95 \%)$ | 0.029 | (0.008) | 0.029 | (0.008) | 0.029 | (0.008) |
| Rel. network size | -0.019 | (0.001) | -0.039 | (0.002) | -0.036 | (0.002) |
| Range of Bank FE | [ -0.041 | 0.038 ] | [ -0.088 | $0.063]$ | [-0.08, | . 059 ] |
| Quarter-year FE | Y |  | Y |  | Y |  |
| Region FE | Y |  | Y |  | Y |  |
| Sample size | 26, |  | 26, |  | 26, |  |
| LLF/N | -2.01 |  | -2.010 |  | -2.01 |  |
| Search moments weight | 1 |  | 1 |  | 1 |  |
| Likelihood ratio test ( $\chi^{2}(3)$ ) | 660. |  | 45. |  |  |  |

[^17]substantially more than the uniform probability of choosing a lender at random in the average choice set (i.e. $1 / 8=12 \%$ ).

Table 3: Summary statistics on the home-bank cost advantage, search and interest costs.

| (a) Home-bank cost advantage |  |  |  |  |  | (b) Search and interest cost |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| VARIABLES | Mean | SD | P25 | P50 | P75 | VAR | BLES | Mean | SD | P25 | P50 | P75 |
| NHB |  |  |  |  |  | NS | Total SC | 2.3 | 1.3 | 1.47 | 2 | 2.82 |
| Inc. $<60 \mathrm{~K}$ | 15.1 | . 111 | 15.1 | 15.1 | 15.2 |  | Interest cost | 44.2 | 18.3 | 30.1 | 40.9 | 55.3 |
| Inc. $\geq 60 \mathrm{~K}$ | 15.6 | . 273 | 15.4 | 15.5 | 15.7 | S | Total SC | . 549 | . 443 | . 203 | . 461 | . 809 |
| PO |  |  |  |  |  |  | Interest cost | 45.6 | 19 | 30.6 | 42.6 | 57.6 |
| Inc. $<60 \mathrm{~K}$ | 21.7 | . 107 | 21.6 | 21.7 | 21.8 | Total | Total SC | 1.15 | 1.19 | . 323 | . 784 | 1.58 |
| Inc. $\geq 60 \mathrm{~K}$ | 22.3 | . 336 | 22 | 22.2 | 22.4 |  | Interest cost | 45.1 | 18.8 | 30.4 | 42 | 56.9 |

Acronyms: $\mathrm{NHB}=$ New home buyers; $\mathrm{PO}=$ Previous owners; $\mathrm{NS}=$ Non searchers; $\mathrm{S}=$ Searchers; Total $\mathrm{SC}=$ Total search cost. Units Table 3a: $\$ /$ Month. The cost advantage is measured for a $\$ 100,000$ loan. Units Table 3b: $\$ / 1,000$. The search and interest costs correspond to the total over the term of the mortgage contract ( 60 months).

As mentioned, this cost advantage arises from the presence of switching costs, and/or complementarities between mortgage lending and other financial services, since the home bank enjoys a cost advantage relative to rival lenders due to its profits from other services. To capture these gains, rival lenders must offer (costly) discounts on other products to get consumers to switch institutions.

Table 3a summarizes the distribution of $\Delta_{i}$ across borrowers. Recall that the loyalty advantage is a deterministic function of income and prior-ownership status. We find that the home-bank cost advantage is particularly important for previous owners, suggesting that underlying switching costs are more important for older borrowers with longer prior experience. In comparison, the effect of income on the loyalty advantage is positive, but much smaller (less than $\$ 0.5 /$ month).

Search cost distribution Table 2 reports the parameters of the average search-costs. Recall that we use an exponential distribution, and model the mean as a log-linear function of income and prior-ownership status. We find that search costs are increasing in income and ownership experience. New home-buyers are estimated to have lower search costs on average (8.6\%), and a $1 \%$ increase in income leads to $0.4 \%$ increase in the average search cost of consumers. This is consistent with an interpretation of search costs as being proportional to the time cost of collecting multiple quotes.

Since search costs are not paid on a monthly basis, Table 3 b summarizes the simulated distribution of search costs expressed over the 5 -year term of the mortgage contract. ${ }^{28}$ The bottom panel reports the unconditional distribution, and the top two panels illustrate the selection effect of consumers' search decisions. On average, we estimate that the cost of searching for multiple offers is equal to $\$ 1,150$ (with a median of $\$ 784$ ). The difference between searchers and non-searchers is substantial. We estimate that the search cost of "searchers" is $\$ 549$, while "non-searchers" decided to accept the initial offer in order to avoid paying on average $\$ 2,300$ in search costs.

[^18]To put these numbers in perspective, we also report in Table 3 b the total interest cost over 5 years, as well as the total loan size. While the search cost estimates are nominally very important, they represent on average only $2.5 \%$ of the overall cost of the contracts (i.e. $2.5 \%=1.15 / 45.1$ ).

An important feature of the model, is that consumers financing larger loans are more likely to search. This is because the gains from search are increasing in loan size, while the search cost is fixed. As a result, in Table 3b we find that searchers incur $3 \%$ larger total interest costs. This is because they finance loans that are on average $\$ 11,000$ larger than those of non-searchers. This is despite paying on average 20 basis-points lower rates.

Are these number realistic? Hall and Woodward (2012) calculate that a U.S. home buyer could save an average of $\$ 983$ on origination fees by requesting quotes from two brokers rather than one. Our estimate of the search cost distribution is consistent with this measure. Our results are also comparable to those in Allen et al. (2014a), where, using a simpler complete-information analogue to the bargaining model employed here, results suggest that for the Canadian mortgage market search costs represent about $4 \%$ of the overall cost.

How do our results compare to existing estimates of search costs in the literature? Perhaps the closest point of comparison comes from Honka's (2014) analysis of the insurance market. She estimates the cost of searching for policies to be $\$ 35$ per online search and a little over $\$ 100$ per offline search. These numbers represent roughly $6 \%$ and $20 \%$ of annual insurance premia respectively, and are therefore somewhat larger than the $2.5 \%$ reported above.

We can also compare our findings to those of Salz (2015), Hortaçsu and Syverson (2004) and Hong and Shum (2006). Salz (2015) studies the New York City trade-waste market in which businesses contract with waste cartels for waste disposal and finds that search costs represent between $30 \%$ and $50 \%$ of total expenses. Hortaçsu and Syverson (2004) estimate a median search cost of 5 bps , yielding a ratio of $8 \%$. The average search cost across the four books considered by Hong and Shum (2006) is $\$ 1.58$ (for non sequential search), yielding a ratio of $33 \%$.

Although somewhat lower, our search-cost estimates are comparable with those found in the literature. This is despite the fact that, because of the negotiation process, it is more complicated to obtain information about mortgage prices than about most products studied until now.

### 6.2 Goodness of fit

We next provide a number of tests for the goodness of fit of the baseline model. To do so, we simulate 100,000 contracts from the model. We follow these steps:

1. Sample individual shocks from the estimated distributions: $\left(c_{i}, \omega_{i 1}, \ldots, \omega_{i n}, \kappa_{i}\right)$,
2. Sample borrower characteristics from the empirical distribution: $\left(L_{i}, \bar{p}_{t(i)}, x_{i}, h(i)\right)$,
3. Solve the model and compute the endogenous outcomes: $\left(p_{i}^{0}, p_{i}^{*}, 1\left(\kappa_{i}<\bar{\kappa}_{i}\left(p^{0}\right)\right), b_{i}\right)$,
4. Drop consumers who failed to qualify for a loan at any bank-about $5.5 \%$ of consumers.

Table 4: Summary statistics for simulated and observed data

|  |  | Spread <br> (bps) | Discounts (bps) | 1(Discount=0) | $\begin{aligned} & \text { Payment } \\ & \text { (\$/Month) } \end{aligned}$ | 1(Loyal) | Network size (relative) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \ddot{0} \\ & \stackrel{0}{0} \\ & 0.0 \\ & 0 \end{aligned}$ | Mean | 119.5 | 95.3 | 0.127 | 924.6 | 0.651 | 1.599 |
|  | SD | 59.3 | 45.4 | 0.333 | 385.0 | 0.477 | 1.015 |
|  | P25 | 81.0 | 70.0 |  | 619.3 |  | 0.989 |
|  | P50 | 115.0 | 95.0 |  | 857.9 |  | 1.370 |
|  | P75 | 161.0 | 125.0 |  | 1169.0 |  | 1.931 |
|  | Mean | 119.4 | 92.2 | 0.092 | 962.8 | 0.670 | 1.678 |
|  | SD | 62.0 | 53.4 | 0.289 | 397.3 | 0.470 | 1.136 |
|  | P25 | 78.1 | 51.0 |  | 647.4 |  | 0.969 |
|  | P50 | 123.0 | 86.7 |  | 896.2 |  | 1.400 |
|  | P75 | 165.1 | 126.7 |  | 1218.6 |  | 2.087 |

[^19]Table 4 presents summary statistics for the key endogenous outcomes of the model. The top panel summarizes the observed sample, while the bottom summarizes the simulated data. Overall, the baseline model is able to match well the unconditional distribution of interest-rate spread (transaction rate minus bond-rate) and monthly payments. The predicted and observed discount distributions are also very similar, but the model tends to under-predict the median discount (86.7 vs 95 bps ), as well as the fraction of borrowers paying the posted rate ( $9.2 \%$ vs $12.7 \%$ ).

The last two columns of Table 4 highlight how well the model matches aggregate lender-choice decisions. The model slightly over-predicts the fraction of loyal consumers ( $67 \% \mathrm{vs} 65 \%$ ), as well as the fact that borrowers tend to choose lenders with larger than average branch network sizes (1.678 vs 1.599).

Next we measure the ability of the model to fit the aggregate search moments measured from the national survey of new home buyers. On average, the baseline specification predicts that $65.7 \%$ of consumers reject the initial offer and search, compared to $62.5 \%$ in the survey. This difference is significantly different from zero only at a $10 \%$ significance level. We can also contrast the survey results with the predicted search probabilities from the two alternative specifications in Table 11, which vary the weight placed on the search moments (i.e. $\rho=0$ vs $\rho=100$ in equation (15)). When the search moments are not used in the estimation, the model tends to predict a larger search probability ( $69.4 \%$ ). In contrast, by assigning a weight of $\rho=100$ to the search moments, the model is able to reproduce almost perfectly the survey predictions (63.5\%).

The main takeaway of this simulation exercise is that the model estimated from the contract data alone tends to predict slightly more search than what the aggregate survey suggests. To understand the source of this discrepancy, it is useful to look at the ability of the model to explain the rate difference between loyal and switching consumers. Like in the data, the model predicts that
loyal consumers obtain lower discounts than do switching consumers, but the model predicts an even greater difference ( 16 bps vs 9 bps ). This is because, in the model, "switching" consumers must have rejected an initial offer and must pay a competitive price. This timing restriction is probably too restrictive. In practice, the timing of moves is likely heterogenous across consumers, in ways that we cannot identify in our data. Honka, Hortacsu, and Vitorino (2016), for instance, consider a richer search/matching model that exploits data on search and consideration set formation.

In Appendix F we provide more information on the fit of the model (including the two results discussed above). We show that the model reproduces very well the lenders' aggregate market shares. We also provide further details on the model's ability to match search probabilities across regions, city sizes and demographic groups. The model reproduces the general patterns of the survey across regions and city sizes, but tends to under-estimate the amount of heterogeneity across demographics groups. In the appendix we also evaluate the ability of the model to reproduce the reduced-form relationships observed in the data between rates, loyalty, and transaction characteristics. In general, the model does a good job of predicting the relationship between discounts and financial attributes.

## 7 Search frictions and market power

In this section, we use the model to quantify the effect of search frictions and market power on consumer surplus and firms' profits. In the model, market power and search frictions are tightly linked, since lenders are able to use the initial quote to screen high search-cost consumers. We start by quantifying the welfare impact of search frictions by computing the equilibrium allocation of contracts absent search costs. We then quantify the importance of market power in the industry, by focusing on the incumbency advantage.

### 7.1 Quantifying the effect of search frictions on welfare

The presence of search costs lowers the welfare of consumers for three reasons. First, it imposes a direct burden on consumers searching for multiple quotes. Second, it can prevent non-searching consumers from matching with the most efficient lender in their choice set, creating a misallocation of buyers and sellers. Lastly, it opens the door to price discrimination, by allowing the initial lender to make relatively high offers to consumers with poor outside options and/or high expected search costs. These factors can be identified by decomposing the change in consumer surplus caused by

Table 5: Decomposing the effect of search frictions on welfare


Each entry corresponds to an average over 100,000 simulated contracts. Statistics in lines 2-5 are calculated using the samples of consumers facing non-zero changes. Cumulative changes are the sum of all changes divided by the total number of qualifying consumers. The welfare decomposition in columns (1)-(3) corresponds to: $\Delta \mathrm{CS}_{i}=$ $\Delta V_{i}-\Delta m_{i}-\Delta \kappa_{i} S_{i}$. The last row reports the contribution of each component, in $\%$, to the cumulative change. Column (5) summarizes the effect of search frictions on the total interest payment over 5 years: Total interest cost $\left(\kappa_{i}>0\right)$ - Total interest cost ( $\kappa_{i}=0$ ). The last column reports the further reduction in consumer surplus arising from the presence of market power in the second-stage of the game: $C S\left(\kappa_{i}=0, m_{i}>0\right)-C S\left(\kappa_{i}=0, m_{i}=0\right)$.
the presence of search frictions:

$$
\begin{align*}
\Delta \mathrm{CS}_{i} & =\underbrace{\bar{v}-p_{i}-1\left(\kappa_{i}<\bar{\kappa}\left(p_{i}^{0}\right)\right) \kappa_{i}}_{\mathrm{CS}_{i}}-\underbrace{\left(\bar{v}-\tilde{p}_{i}\right)}_{\widetilde{\widetilde{\mathrm{CS}}_{i}}} \\
& =\left[\tilde{c}_{i, b}-c_{i, b}\right]-\left(m_{i}-\tilde{m}_{i}\right)-1\left(\kappa_{i}<\bar{\kappa}\left(p_{i}^{0}\right)\right) \kappa_{i} \\
& =\Delta V_{i}-\Delta m_{i}-1\left(\kappa_{i}<\bar{\kappa}\left(p_{i}^{0}\right)\right) \kappa_{i}, \tag{17}
\end{align*}
$$

where the $\sim$ superscript indicates the equilibrium outcomes without search cost, $\bar{v}$ is the WTP for mortgages (policy invariant), $V_{i j}=\bar{v}-c_{i j}$ is the transaction surplus (excluding the search cost), $m_{i j}=p_{i}-c_{i j}$ is the profit margin, and $1\left(\kappa_{i}<\bar{\kappa}\left(p_{i}^{0}\right)\right)$ is an indicator variable equal to one if the consumer rejects the initial offer. As before, we assume that the WTP for mortgages is large enough that the same group of consumers would enter the housing market with or without search frictions.

We label the three components misallocation, discrimination, and search cost, respectively. The sum of the first and third components measures the change in total welfare caused by search frictions. The discrimination component is related to the surplus split between firms and consumers.

We simulate the counter-factual experiments as before. The only difference between the baseline and the zero search-cost environments is that, absent search frictions, consumers do not obtain an initial quote. As a result, the posted rate becomes the reservation price in the competition stage. Table 5 presents the main simulation results. Columns 1 through 3 show the change in the misallocation, discrimination and search cost components respectively, while column 4 presents the
total change in consumer surplus. To illustrate the heterogeneity across consumers, the first line reports the fraction of simulated consumers experiencing zero changes, and the next four describe the conditional distribution of non-zero changes. To calculate the cumulative changes, we average the changes across all qualifying consumers. The percentage shares of each component are expressed relative to the cumulative changes.

We estimate that the cumulative reduction in consumer surplus associated with search frictions is equal to $\$ 12.80$ per month, or $2 \%$ of the total interest cost of mortgages in our data-set. The largest component ( $50 \%$ ) is attributed to the sunk cost of searching, followed by the increase in margins associated with price discrimination (28\%), and misallocation (21\%). Over $98 \%$ of consumers are affected. The sum of the misallocation and discrimination components corresponds to the effect of search frictions on monthly payments alone: $\$ 6.37 /$ month on average per borrower. This leads to an increase in interest payments of $\$ 503$ over 5 years (column (5)), or $\$ 1,569$ for consumers who are directly impacted by the price change.

The sum of the misallocation and search-cost components corresponds to the total welfare cost of search frictions: $\$ 9.15 /$ month per borrower. For these two components, the fraction of zero changes measures the percentage of buyers and sellers that are matched efficiently and the fraction of non-searchers in the presence of search frictions, respectively. Search frictions cause $17 \%$ of transactions to be misallocated, despite the fact that more than $32 \%$ of consumers do not search. Note that the difference between these two fractions would be close to zero if the loyalty advantage were null. Since banks' fixed-effects are not highly dispersed, this is mostly because consumers visit the highest expected surplus seller first, which reduces the fraction of inefficient matches.

Focusing directly on the change in profit margins, Column (2) shows that the relatively small contribution of the discrimination component is explained by the fact that some consumers pay higher markups in the frictionless market. The median change in profit margins is equal to $\$ 12.43$ per month; significantly more than the median increase in search costs (i.e. 12.43 vs 7.86 ). However, the 10 th percentile consumer benefits from a $\$ 7.59$ reduction in profit margins, which brings the cumulative effect down to $\$ 3.64$.

To understand this heterogeneity, recall that the initial quote is used both as a screening tool, and as a price ceiling in the competition stage. The home bank is in a monopoly position in the first stage, and can set individual prices based on consumers' expected outside options. This is analogous to first-degree price discrimination, and strictly increases the expected profit of the home bank. This adverse effect is weighed against the fact that the initial offer can be recalled, and so protects consumers against excessive market power in the competition stage. In the zero search-cost environment, the price ceiling is on average higher (i.e. it is the posted-rate), which explains why some consumers experience an increase in profit margins after eliminating search frictions.

To put these numbers in perspective, column (6) summarizes the distribution of consumersurplus changes arising from eliminating market power entirely, relative to the zero search-cost

Figure 1: Distribution of profit margins

environment. We calculate the difference in surplus between the zero search-cost environment, and one with no search frictions and zero profits margins. This is equivalent to shifting the bargaining power entirely to consumers in the competition stage, and therefore maximizing the surplus of consumers. Relative to the baseline environment, eliminating market power and search frictions would increase consumer surplus by $\$ 27.92 /$ month on average (i.e. $12.80+15.12$ ). Therefore, eliminating search frictions would allow consumers to reach $46 \%$ of their maximum surplus.

These results can be compared to those of Gavazza (2016), who performs a similar decomposition of the effect of search frictions on welfare in decentralized asset markets. Using data from the business aircraft market, he finds that, relative to his estimated model, when search costs are set to zero welfare falls slightly (by roughly $\$ 1$ million per quarter). This small decrease is the result of a reduction in direct search costs (of about $\$ 6$ million), a reduction of misallocation ( $\$ 3$ million) and an offsetting increase in dealer costs ( $\$ 11$ million).

### 7.2 Quantifying the importance of market power

Overall, we find that the market is competitive. Figure 1a plots the distribution profit margins. The average profit margins is 22.1 bps , which corresponds to a Lerner index of $3.2 \%$. This is consistent with our earlier findings that mortgage contracts are fairly homogenous across lenders, and search-costs represent a small share of consumers' overall mortgage spending. It is also fairly consistent with the findings in Allen et al. (2014a), which suggest margins of around 35 bps before the merger and 40 bps afterwards.

This implies that a large fraction of the observed spread between negotiated rates and the 5year bond-rate corresponds to transaction costs. In particular, we estimate that each contract costs roughly 100 bps to originate, beyond the financing cost, which is proxied by the bond rate. This cost
stems from a variety of sources: the compensation of loan officers (bonuses and commissions), the advantage associated with pre-payment risks, transaction costs associated with the securitization of contracts, as well as upstream profit margins from financing.

The distribution of profit margins is also very dispersed. The coefficient of dispersion of profit margins is equal to $72 \%$, and the range exceeds 100 bps. Figure 1b shows that part of this dispersion is caused by heterogeneous search efforts. On average, firms charge a markup that is $90 \%$ larger on consumers who are not searching (i.e. 32.1/16.9). The margin distribution for searchers also exhibits an important mass between 0 and 20 bps , and the median margin among searchers is only 13 bps (compared to 32 bps in the non-searcher sample).

The dispersion in profit margins also reflects the fact that market-power arises from a variety of sources: (i) price discrimination, (ii) loyalty advantage, (iii) observed cost differences (i.e. bank fixed effects and network size), and (iv) idiosyncratic cost differences (i.e. $\omega_{i j}$ ).

The last two components ensure positive profits margins in the competition stage. On average, the difference between the lowest and second-lowest cost among rival lenders is equal to $\$ 15.70 /$ month. This is the profit margin that would be realized if the home bank were not present and there were no posted-rate (i.e. ceiling), and therefore can be thought of as an upper bound on the market power of rival banks. In practice, rival lenders earn slightly less: banks' average profits from switching consumers are $\$ 14.99 /$ month (or 17.1 bps ), compared to $\$ 20.22 /$ month for loyal consumers (or 24.6 bps ).

The profit gain from loyalty corresponds to an incumbency advantage: Banks with a large consumer base have more market power because of a first-mover advantage and loyalty advantage (or differentiation). We find that the loyalty advantage is substantial: the average home-bank cost advantage is $33 \%$ larger than the standard-deviation of idiosyncratic cost differences. As a result the home bank is able to retain, on average, $51 \%$ of searching consumers. The first-mover advantage arises because the home bank is in a quasi-monopoly position in the first-stage of the game, and can price discriminate between consumers based on heterogeneity in their expected reservation prices. The ability to make the first quote allows the home bank to charge a higher markup and retain a larger fraction of consumers who, absent search costs, would choose another lender.

To measure the source and magnitude of the incumbency advantage, we use the simulated model to evaluate the correlation between the size of a lender's consumer base and its profitability. In the model, the consumer base of a given bank is defined as the share of consumers with whom it has an existing day-to-day banking relationship, and this base determines the fraction of consumers in a given market who start their search with the bank (i.e. $\psi_{i j}$ ). Recall that this matching probability is defined at the level of a neighborhood, income group (low, medium and high), and year. We use this definition to construct markets that each have a homogenous consumer base distribution, and we construct measures of profits and concentration at this level of aggregation. Doing so yields 8,075 unique markets.

To construct a measure of consumer base that is comparable across markets, we compute, for each market $i$, the ratio of the matching probability of lender $j$ over the average matching probability among banks in the market:

$$
\text { Matching probability ratio }=\frac{\psi_{i j}}{\bar{\psi}_{i}} .
$$

Table 6a summarizes the distribution of contracts and profits across different types of lenders. The table ranks banks from the smallest consumer base (i.e. between 0 and $25 \%$ of the average size in the same market), to the largest (i.e. between 4 and 7 times the average size). As we saw earlier, most consumers choose a mortgage lender with a large branch presence. This is reflected in the distribution of contracts shown in column (1): $46 \%$ of contracts are issued by banks with a consumer base between 1 and 2 times larger than the average bank in their market.

Columns (2) and (3) report the weighted average share of profits and contracts generated by each bank type. To get this number, for each market, we calculate the average share of profits and contracts generated by lenders with consumer bases belonging to one of the 6 categories. We then aggregate these shares across markets, using the total number of contracts originated in each market as weight.

If there were no relationship between banks' consumer bases and mortgages, contracts and profits would be uniformly distributed across categories (i.e. would be about $11 \%$ on average). The resulting distributions are significantly more concentrated. Banks in the top category ( 4 to 7 ) earn, on average, $62 \%$ of the profits generated in their respective markets, compared to only $2 \%$, on average, for the smallest banks. Note that the average profit share increases very quickly with the size of the consumer base.

In addition, the distribution of profits is more concentrated than the distribution of contracts. On average the top lenders originate $54 \%$ of contracts, but earn $62 \%$ of the profits. This pattern reflects the fact that banks with a large consumer base charge, on average, higher markups. Column (5) shows that the average profit margin for banks in the top category is equal to 30.7 bps , compared to only 16.6 bps for banks in the bottom category. This discrepancy is largely explained by the difference in markups between searchers and non-searchers. Banks in the smallest consumer-base category earn on average $90 \%$ of their profits from consumers reaching the second stage of the game, compared to $40 \%$ for banks in the largest category. This confirms the importance of the first-mover advantage as a source of market power for large consumer base lenders.

Identifying the relative importance of the first mover advantage and differentiation is not an easy task however, since the two interact to generate a correlation between profitability and size of consumer base. For instance, the profit gain from being able to make the first offer depends on the amount of differentiation, since lower-cost banks have more leverage in the initial negotiation stage. Similarly, the presence of a cost advantage reduces the incentive for consumers to search, and increases the fraction of profits generated from price discrimination.

Table 6: Incumbent advantage and market power
(a) Distribution of bank profitability and consumer base in the baseline environment

| Consumer <br> base | Matching <br> probability | Sample <br> frequency <br> ratio | Within <br> Profits <br> market shares | Second stage <br> Contracts <br> (2) | Margins <br> profits (\%) | (bps) <br> (bps) <br> (5) |
| ---: | :--- | :---: | :---: | :---: | :---: | :---: |
| Small | 0 to $1 / 4$ | 0.05 | 0.02 | 0.02 | 0.90 | 16.6 |
|  | $1 / 4$ to $1 / 2$ | 0.04 | 0.04 | 0.05 | 0.62 | 19.1 |
|  | $1 / 2$ to 1 | 0.17 | 0.07 | 0.08 | 0.51 | 18.9 |
|  | 1 to 2 | 0.46 | 0.16 | 0.16 | 0.50 | 20.4 |
|  | 2 to 4 | 0.25 | 0.34 | 0.30 | 0.51 | 24.0 |
| Large | 4 to 7 | 0.04 | 0.62 | 0.54 | 0.40 | 30.7 |

(b) Distribution of bank profitability in the baseline and counterfactual environments

| Statistics Variables | Baseline | $\begin{gathered} \text { CF-1 } \\ \Delta_{i}=0 \end{gathered}$ | $\begin{gathered} \text { CF-2 } \\ \psi_{i}=1 / N \end{gathered}$ | $\begin{gathered} \text { CF-3 } \\ \psi_{i}=1 / N \& \Delta_{i}=0 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| Ratio: Large base/Small base |  |  |  |  |
| Margins (bps) | 1.851 | 1.369 | 1.485 | 1.145 |
| Profit shares | 35.717 | 11.652 | 17.159 | 6.699 |
| Contract shares | 24.582 | 9.769 | 13.204 | 6.243 |
| Full sample averages |  |  |  |  |
| Search probability | 0.656 | 0.774 | 0.838 | 0.822 |
| $2^{\text {nd }}$ stage profits (\%) | 0.531 | 0.727 | 0.809 | 0.784 |
| Margins (bps) | 22.07 | 18.56 | 21.34 | 18.60 |
| Match prob. ratio | 1.709 | 1.546 | 1.605 | 1.439 |

(c) Decomposition of the incumbency advantage

| Large/Small Ratio | $\begin{gathered} \text { Incumbency adv. } \\ \text { Base - CF-3 } \end{gathered}$ | Loyalty premium CF-2-CF-3 | Price discrimination CF-1-CF-3 | Interaction |
| :---: | :---: | :---: | :---: | :---: |
| Margins | 0.707 | 0.340 | 0.224 | 0.142 |
|  |  | (0.48) | (0.32) | (0.2) |
| Profit share | 29.018 | 10.460 | 4.954 | 13.605 |
|  |  | (0.36) | (0.17) | (0.47) |
| Contract share | 18.339 | $\begin{aligned} & 6.961 \\ & (0.38) \end{aligned}$ | $\begin{array}{r} 3.526 \\ (0.19) \end{array}$ | $7.852$ (0.43) |

Each entry in Table 6a is the weighted average outcome of lenders belonging to each category (rows). The weights are proportional to the number of contracts originated in each market (i.e. neighborhood/year/income). Variable definitions: Matching probability ratio = Consumer base of bank j / Average consumer base; Sample frequency $=$ Market share of lenders in each category; Second-stage profit (\%) = Average share of profits originating from the searching consumers; Within market share $=$ Average share of profits or contracts generated by lenders in each category; Margins $=r_{i}-c_{i}$ in percentage basis points; Ratio: Large base/Small base $=$ Ratio of the average outcomes of lenders in the large group, over those in the small group. Counter-factual environments: (1) Zero home-bank cost advantage, (2) Uniform matching probability, (3) combination of (1) and (2).

To measure each of the components that generate the incumbency advantage, we simulate a series of counter-factual experiments aimed at varying the first-mover advantage and the differentiation component independently. In particular, to eliminate the differentiation component, CF-1 simulates a model in which the cost-advantage of the home bank is set to zero, which is analogous to
separating the provision of mortgages from other banking services. We eliminate the ability of firms to screen high search-cost consumers by imposing a uniform matching probability and breaking the link between the ability to make the first offer and the size of the consumer base (CF-2). Finally, CF-3 combines the previous two environments by assuming a uniform matching probability and zero loyalty advantage. ${ }^{29}$

Results are displayed in Table 6b. The top panel summarizes concentration in the industry, as well as the dispersion in profit margins between large and small banks. The bottom panel describes some of the key equilibrium outcomes in the baseline and counter-factual environments. The ratio of the profit margin of large and small banks is a measure of the incumbency advantage: how much more market power do banks with large consumer bases have relative to banks with small consumer bases. In the baseline environment, we estimate that large banks' profit margins are $85.1 \%$ larger. Eliminating the first-mover and the loyalty advantage shrinks the margin difference to $14.5 \%$ (CF-3), and so this is a measure of the market power of large banks that stem solely from brand and branch network differences. ${ }^{30}$ The difference, or $0.707=1.851-1.145$, is explained by the incumbency advantage.

The first column of Table 6c summarizes the incumbency advantage in terms of profit margins, profit shares, and market shares (or contract). Columns (2) to (4) use the uniform matching probability (CF-2) and the zero loyal advantage (CF-1) counter-factual environments to decompose the incumbency advantage into three terms:

$$
\underbrace{\text { Incumbency advantange }}_{0.707}=\underbrace{\text { Loyalty advantage }}_{0.34}+\underbrace{\text { Price discrimination }}_{0.22}+\underbrace{\text { Interaction }}_{0.14} .
$$

Therefore, relative to CF-3, almost $50 \%$ of the market-power of large banks is caused by the home-bank cost advantage, just over $30 \%$ by the first-mover advantage, and the remaining $20 \%$ is explained by the interaction of both elements.

The interaction term originates from the joint equilibrium effect of differentiation and the order of moves on the search probability. As the middle panel indicates, the combined effect of the home-bank cost and first-mover advantage is to lower the search probability from $82.2 \%$ to $65.6 \%$, which increases the profit margin ratio by an extra 14 percentage points through a change in the composition of loyal borrowers. Independently, the two elements have little or no effect on the search probability relative to the CF-3 environment.

The concentration of profits and contracts is similarly impacted. Eliminating both the loyalty advantage and the first-mover advantage substantially reduces the concentration of profits: large

[^20]banks' share of profits is 35.72 times larger than that of small banks in the baseline, compared to only 6.70 times in CF-3. As with margins, the loyalty advantage alone explains a bigger share of the drop in concentration (36\%) than the first-mover advantage ( $17 \%$ ). However, unlike with margins, a larger portion of the profit share ratio is explained by the interaction of differentiation and discrimination: $47 \%$ of the profit share difference between large and small banks is explained by the interaction term. This is because the increase in the search probability from letting the most efficient lender make the first offer has a very large effect on banks' retention probability, and therefore on their overall profitability.

## 8 Conclusion

The paper makes three main contributions. The first is to provide an empirical framework for studying markets in which prices are negotiated. The second is to show that search frictions are important and generate significant welfare losses for consumers that can be decomposed into misallocation, price discrimination, and direct search cost components. We also show that the welfare loss is mitigated by switching costs (loyalty advantage) and posted prices, but amplified by competition. Finally, the paper also demonstrates the importance of having a large consumer base for market power, and decomposes the effect into a first-mover advantage and brand loyalty. We find that brand loyalty is the most important source of market power, but that search frictions play an important role through the first-mover advantage.

A few caveats should be mentioned. First, the assumption that monthly payment has no effect on the loan size could imply (depending on the elasticity of loan demand) that the distortions arising from search and market power are larger than the ones we calculate. Second, although the overall fit of our model is good, it predicts that loyal consumers pay more than they are observed to in the data. This difference is directly related to our modeling assumptions: the timing and order of search are the same for all consumers, and all consumers have a single home bank. These are simplifying assumptions that closely link search and switching in the model.

The model also over-estimates the impact of competition on rates, likely because market structure is assumed to be independent of consumers' unobserved attributes, up to regional fixed-effects. Otherwise, our estimates of firms' cost differences would suffer from an attenuation bias, and our results would correspond to a lower bound on profit margins in this market. A related interpretation of the small reduced-form effect of competition on rates and discounts, is that consumers face heterogenous consideration sets, conditional on being located in the same postal-code area. This would create measurement error in the choice-set of consumers, which is computationally prohibitive to incorporate, since lenders are ex-ante heterogeneous. Moreover, we do not have data on the set, or identity of lenders considered by borrowers.

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## A Data description

Our data-set consists of a $10 \%$ random sample of insured contracts from CMHC. It covers the period from 1992 to 2004. We restrict our analysis to the 1999-2002 period for two reasons. First, between 1992 and 1999, the market transited from one with a larger fraction of posted-price transactions and loans originated by trust companies, to a decentralized market dominated by large multi-product lenders. Our model is a better description of the latter period. Second, between November 2002 and September 2003, TD-Canada Trust experimented with a new pricing scheme based on a "no-haggle" principle. Understanding the consequences of this experiment is beyond the scope of this paper, and would violate our confidentiality agreement.

We also have access to data from Genworth Financial, but do not use these contracts in the paper, since we are missing some key information for these contracts. We obtained the full set of contracts originated by the 12 largest lenders and further sampled from these contracts to match Genworth's annual market share.

Both insurers use the same guidelines for insuring mortgages. First, borrowers with less than $25 \%$ equity must purchase insurance. ${ }^{31}$ Second, borrowers with monthly gross debt service (GDS) payments that are more than $32 \%$ of gross income or a total debt service (TDS) ratio of more than $40 \%$ will almost certainly be rejected. Crucial to the guidelines is that the TDS and GDS calculations are based on the posted rate and not the discounted price. Otherwise, given that mortgages are insured, lenders might provide larger discounts to borrowers above a TDS of 40 in order to lower their TDS below the cut-off. The mortgage insurers charge the lenders an insurance premium, ranging from 1.75 to $3.75 \%$ of the value of the loan lenders pass this premium onto borrowers. Insurance qualifications (and premiums) are common across lenders and based on the posted rate. Borrowers qualifying at one bank, therefore, know that they can qualify at other institutions, given that the lender is protected in case of default.

Table 7: Definition of Household / Mortgage Characteristics

| Name | Description |
| :--- | :--- |
| FI | Type of lender |
| Source | Identifies how lender generated the loan (branch, online, broker, etc) |
| Income | Total amount of the borrower(s) salary, wages, and income from other sources |
| TDS | Total debt service ratio |
| GDS | Gross debt service |
| Duration | Length of the relationship between the borrower and FI |
| R-status | Borrowers residential status upon insurance application |
| FSA | Forward sortation area of the mortgaged property |
| Market value | Selling price or estimated market price if refinancing |
| Applicant type | Quartile of the borrowers risk of default |
| Dwelling type | 10 options that define the physical structure |
| Close | Closing date of purchase or date of refinance |
| Loan amount | Dollar amount of the loan excluding the loan insurance premium |
| Premium | Loan insurance premium |
| Purpose | Purpose of the loan (purchase, port, refinance, etc.) |
| LTV | Loan amount divided by lending value |
| Price | Interest rate of the mortgage |
| Term | Represents the term over which the interest rate applies to the loan |
| Amortization | Represents the period the loan will be paid off |
| Interest type | Fixed or adjustable rate |
| $C R E D I T$ | Summarized application credit score (minimum borrower credit score). |

[^21][^22]
## B Model predictions

We rewrite equation 4 as:

$$
p^{*}= \begin{cases}p^{0} & \text { If } \omega_{(1)}>p^{0}-c-\lambda \\ c+\lambda+\omega_{(1)} & \text { If }-\gamma<\omega_{(1)}<p^{0}-c-\lambda \\ c-\gamma & \omega_{(2)}>-\gamma>\omega_{(1)} \\ c+\omega_{(2)} & \text { If } \omega_{(2)}<-\gamma,\end{cases}
$$

and describe the model by the following functions:

- The expected second stage price:

$$
\begin{aligned}
E\left(p^{*} \mid p^{0}, s\right)= & p^{0}\left(1-G_{(1)}\left(p^{0}-c-\lambda\right)\right)+\int_{-\gamma}^{p^{0}-c-\lambda}\left(c+\lambda+\omega_{(1)}\right) d G_{(1)} \\
& +(c-\gamma)\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right]+\int_{-\infty}^{-\gamma}\left(c+\omega_{(2)}\right) d G_{(2)}
\end{aligned}
$$

- The expected second stage profit of the home bank:

$$
E\left(\pi_{h}^{*} \mid p^{0}, s\right)=\left(p^{0}-c+\Delta\right)\left(1-G_{(1)}\left(p^{0}-\lambda-c\right)\right)+\int_{-\gamma}^{p^{0}-c-\lambda}\left(\omega_{(1)}+\gamma\right) d G_{(1)}
$$

- The search-cost threshold is:

$$
\bar{\kappa}\left(p^{0}, s\right)=p^{0}-E\left(p^{*} \mid p^{0}, s\right)-\lambda G_{(1)}(-\gamma)
$$

- The first-order condition (FOC):

$$
\begin{aligned}
f\left(p^{0}, s\right)= & 1-H\left(\bar{\kappa}\left(p^{0}, s\right)\right)-\left(p^{0}-c+\Delta\right) H^{\prime}\left(\bar{\kappa}\left(p^{0}, s\right)\right) \frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial p^{0}} \\
& +H^{\prime}\left(\bar{\kappa}\left(p^{0}, s\right)\right) \frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial p^{0}} E\left(\pi_{h}^{*} \mid p^{0}, s\right)+H\left(\bar{\kappa}\left(p^{0}, s\right)\right) \frac{\partial E\left(\pi_{h}^{*} \mid p^{0}, s\right)}{\partial p^{0}}=0
\end{aligned}
$$

## B. 1 Comparative statics

Using these functions, we can derive the following comparative statics. The derivative of $E\left(p^{*} \mid p^{0}, s\right)$ wrt $p^{0}$ is given by:

$$
\begin{aligned}
\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial p^{0}} & =1-G_{(1)}\left(p^{0}-c-\lambda\right)-p^{0} g_{(1)}\left(p^{0}-c-\lambda\right)+g_{(1)}\left(p^{0}-c-\gamma\right)\left[c+\lambda+p^{0}-c-\lambda\right] \\
& =1-G_{(1)}\left(p^{0}-c-\lambda\right)
\end{aligned}
$$

The derivative of $E\left(p^{*} \mid p^{0}, s\right)$ wrt $c$ is given by:

$$
\begin{aligned}
\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial c}= & p^{0} g_{(1)}\left(p^{0}-c-\lambda\right)-\left(c+\lambda+p^{0}-c-\lambda\right) g_{(1)}\left(p^{0}-c-\lambda\right)+\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right] \\
& +\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right]+G_{(2)}(-\gamma) \\
= & G_{(1)}\left(p^{0}-c-\lambda\right)
\end{aligned}
$$

The derivative of $E\left(p^{*} \mid p^{0}, s\right)$ wrt $\lambda$ is given by:

$$
\begin{aligned}
\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial \lambda}= & p^{0} g_{(1)}\left(p^{0}-c-\lambda\right)-\left(c+\lambda+p^{0}-c-\lambda\right) g_{(1)}\left(p^{0}-c-\lambda\right) \\
& (c+\lambda-\gamma) g_{(1)}(-\gamma)+\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right] \\
& -\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right]-(c-\gamma)\left[g_{(1)}(-\gamma)-g_{(2)}(-\gamma)\right]-(c-\gamma) g_{(2)}(-\gamma) \\
= & \lambda g_{(1)}(-\gamma)+\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right]-\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right]
\end{aligned}
$$

The derivative of $E\left(p^{*} \mid p^{0}, s\right)$ wrt $\Delta$ is given by:

$$
\begin{aligned}
\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial \Delta}= & (c+\lambda-\gamma) g_{(1)}(-\gamma)-\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right] \\
& -(c-\gamma)\left[g_{(1)}(-\gamma)-g_{(2)}(-\gamma)\right]-(c-\gamma) g_{(2)}(-\gamma) \\
= & \lambda g_{(1)}(-\gamma)-\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right]
\end{aligned}
$$

The difference is given by:

$$
\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial \lambda}-\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial \Delta}=G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)
$$

The derivative of the expected profits wrt to $p^{0}$ is:

$$
\begin{aligned}
\frac{\partial E\left(\pi^{*} \mid p^{0}, s\right)}{\partial p^{0}} & =1-G_{(1)}\left(p^{0}-\lambda-c\right)-\left(p^{0}-c+\Delta\right) g_{(1)}\left(p^{0}-c-\lambda\right)+\left(p^{0}-c-\lambda+\Delta+\lambda\right) g_{(1)}\left(p^{0}-c-\lambda\right) \\
& =1-G_{(1)}\left(p^{0}-\lambda-c\right)
\end{aligned}
$$

The derivative of the expected profits wrt to $c$ is:

$$
\begin{aligned}
\frac{\partial E\left(\pi^{*} \mid p^{0}, s\right)}{\partial c} & =-\left(1-G_{(1)}\left(p^{0}-\lambda-c\right)\right)+\left(p^{0}-c+\Delta\right) g_{(1)}\left(p^{0}-c-\lambda\right)-\left(p^{0}-c-\lambda+\Delta+\lambda\right) g_{(1)}\left(p^{0}-c-\lambda\right) \\
& =G_{(1)}\left(p^{0}-\lambda-c\right)-1
\end{aligned}
$$

The derivative of the expected profits wrt to $\lambda$ is:

$$
\begin{aligned}
\frac{\partial E\left(\pi^{*} \mid p^{0}, s\right)}{\partial \lambda}= & \left(p^{0}-c+\Delta\right) g_{(1)}\left(p^{0}-\lambda-c\right)-\left(p^{0}-c-\lambda+\Delta+\lambda\right) g_{(1)}\left(p^{0}-c-\lambda\right) \\
& +(-\gamma+\gamma) g_{(1)}(-\gamma)+\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right] \\
= & {\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right] }
\end{aligned}
$$

The derivative of the expected profits wrt to $\Delta$ is:

$$
\begin{aligned}
\frac{\partial E\left(\pi^{*} \mid p^{0}, s\right)}{\partial \Delta} & =1-G_{(1)}\left(p^{0}-\lambda-c\right)+(-\gamma+\gamma) g_{(1)}(-\gamma)+\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right] \\
& =1-G_{(1)}(-\gamma)
\end{aligned}
$$

The difference is negative:

$$
\frac{\partial E\left(\pi^{*} \mid p^{0}, s\right)}{\partial \lambda}-\frac{\partial E\left(\pi^{*} \mid p^{0}, s\right)}{\partial \Delta}=G_{(1)}\left(p^{0}-c-\lambda\right)-1<0
$$

and,

$$
\begin{aligned}
\frac{\partial E\left(\pi^{*} \mid p^{0}, s\right)}{\partial p^{0}} & =\frac{\partial E\left(\pi^{*} \mid p^{0}, s\right)}{\partial \Delta}-\frac{\partial E\left(\pi^{*} \mid p^{0}, s\right)}{\partial \lambda} \\
& =-\frac{\partial E\left(\pi^{*} \mid p^{0}, s\right)}{\partial \lambda}+1-G_{(1)}(-\gamma)
\end{aligned}
$$

The derivative of $\bar{\kappa}\left(p^{0}, s\right)$ wrt $p^{0}$ is:

$$
\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial p^{0}}=1-\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial p^{0}}=G_{(1)}\left(p^{0}-c-\lambda\right)>0
$$

The derivative of $\bar{\kappa}\left(p^{0}, s\right)$ wrt $c$ is:

$$
\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial c}=-\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial c}=-G_{(1)}\left(p^{0}-c-\lambda\right)<0
$$

The derivative of $\bar{\kappa}\left(p^{0}, s\right)$ wrt $\lambda$ is:

$$
\begin{aligned}
\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial \lambda}= & -\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial \lambda}-G_{(1)}(-\gamma)+\lambda g_{(1)}(-\gamma) \\
= & -\lambda g_{(1)}(-\gamma)-\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right]+\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right] \\
& -G_{(1)}(-\gamma)+\lambda g_{(1)}(-\gamma) \\
= & -\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right]-G_{(2)}(-\gamma)<0
\end{aligned}
$$

The derivative of $\bar{\kappa}\left(p^{0}, s\right)$ wrt $\Delta$ is:

$$
\begin{aligned}
\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial \Delta} & =-\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial \Delta}+\lambda g_{(1)}(-\gamma) \\
& =-\lambda g_{(1)}(-\gamma)+\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right]+\lambda g_{(1)}(-\gamma) \\
& =\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right]
\end{aligned}
$$

The difference is:

$$
\begin{aligned}
\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial \lambda}-\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial \Delta} & =-\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right]-G_{(2)}(-\gamma)-\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right] \\
& =-G_{(1)}\left(p^{0}-c-\lambda\right)
\end{aligned}
$$

## B. 2 Proofs of Proposition 1 and Corollary 1

Using these results, we can prove Proposition 1 and Corollary 1. In order to do so we need to derive expressions for the second-order condition (SOC) and the partials of the FOC wrt $c, \lambda$ and $\Delta$.

The FOC can be re-arranged as follows:

$$
\begin{aligned}
f\left(p^{0}, s\right) & =1-H\left(\bar{\kappa}\left(p^{0}, s\right)\left[1-\frac{\partial E\left(\pi_{h}^{*} \mid p^{0}, s\right)}{\partial p^{0}}\right]-H^{\prime}\left(\bar{\kappa}\left(p^{0}, s\right)\right) \frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial p^{0}}\left[\pi^{0}-E\left(\pi_{h}^{*} \mid p^{0}, s\right)\right]\right. \\
& =1-H\left(\bar{\kappa}\left(p^{0}, s\right)\right) G_{(1)}\left(p^{0}-c-\lambda\right)-H^{\prime}\left(\bar{\kappa}\left(p^{0}, s\right)\right) G_{(1)}\left(p^{0}-c-\lambda\right)\left[\pi^{0}-E\left(\pi_{h}^{*} \mid p^{0}, s\right)\right] \\
& =1-G_{(1)}\left(p^{0}-c-\lambda\right)\left(H(\bar{\kappa})+H^{\prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]\right)
\end{aligned}
$$

where $\bar{\kappa} \equiv \bar{\kappa}\left(p^{0}, s\right), \pi^{0} \equiv p^{0}-c+\Delta$, and $E\left(\pi_{h}^{*}\right) \equiv E\left(\pi_{h}^{*} \mid p^{0}, s\right)$.

The SOC is given by:

$$
\begin{aligned}
f_{p^{0}}\left(p^{0}, s\right)= & -g_{(1)}\left(p^{0}-c-\lambda\right)\left(H(\bar{\kappa})+H^{\prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]\right) \\
& -G_{(1)}\left(p^{0}-c-\lambda\right)\left(H^{\prime}(\bar{\kappa}) \frac{\partial \bar{\kappa}}{\partial p^{0}}+H^{\prime \prime}(\bar{\kappa}) \frac{\partial \bar{\kappa}}{\partial p^{0}}\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]+H^{\prime}(\bar{\kappa})\left[1-\frac{\partial E\left(\pi_{h}^{*}\right)}{\partial p^{0}}\right]\right) \\
= & -g_{(1)}\left(p^{0}-c-\lambda\right)\left(H(\bar{\kappa})+H^{\prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]\right)-G_{(1)}\left(p^{0}-c-\lambda\right)^{2}\binom{2 H^{\prime}(\bar{\kappa})}{+H^{\prime \prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]}
\end{aligned}
$$

The derivative of the FOC wrt $c$ is:

$$
\begin{aligned}
f_{c}\left(p^{0}, s\right)= & g_{(1)}\left(p^{0}-c-\lambda\right)\left(H(\bar{\kappa})+H^{\prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]\right) \\
& -G_{(1)}\left(p^{0}-c-\lambda\right)\left(H^{\prime}(\bar{\kappa}) \frac{\partial \bar{\kappa}}{\partial c}+H^{\prime \prime}(\bar{\kappa}) \frac{\partial \bar{\kappa}}{\partial c}\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]+H^{\prime}(\bar{\kappa})\left[-1-\frac{\partial E\left(\pi_{h}^{*}\right)}{\partial c}\right]\right) \\
= & g_{(1)}\left(p^{0}-c-\lambda\right)\left(H(\bar{\kappa})+H^{\prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]\right)+G_{(1)}\left(p^{0}-c-\lambda\right)^{2}\binom{2 H^{\prime}(\bar{\kappa})}{+H^{\prime \prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]}
\end{aligned}
$$

The derivative of the FOC wrt $\lambda$ is:

$$
\begin{aligned}
f_{\lambda}\left(p^{0}, s\right)= & g_{(1)}\left(p^{0}-c-\lambda\right)\left(H(\bar{\kappa})+H^{\prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]\right) \\
& -G_{(1)}\left(p^{0}-c-\lambda\right)\left(H^{\prime}(\bar{\kappa}) \frac{\partial \bar{\kappa}}{\partial \lambda}+H^{\prime \prime}(\bar{\kappa}) \frac{\partial \bar{\kappa}}{\partial \lambda}\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]+H^{\prime}(\bar{\kappa})\left[-\frac{\partial E\left(\pi^{*}\right)}{\partial \lambda}\right]\right)
\end{aligned}
$$

The derivative of the FOC wrt $\Delta$ is:

$$
f_{\Delta}\left(p^{0}, s\right)=-G_{(1)}\left(p^{0}-c-\lambda\right)\left(H^{\prime}(\bar{\kappa}) \frac{\partial \bar{\kappa}}{\partial \Delta}+H^{\prime \prime}(\bar{\kappa}) \frac{\partial \bar{\kappa}}{\partial \Delta}\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]+H^{\prime}(\bar{\kappa})\left[1-\frac{\partial E\left(\pi^{*}\right)}{\partial \Delta}\right]\right)
$$

## B.2.1 Proof of Proposition 1

We first prove Proposition 1.
Proof of Proposition 1. From above, we have that

$$
f_{c}\left(p^{0}, s\right)=-f_{p^{0}}\left(p^{0}\right)
$$

Therefore,

$$
\frac{d p^{0}}{d c}=-\frac{f_{c}\left(p^{0}\right)}{f_{p^{0}}\left(p^{0}\right)}=1
$$

This proves that the initial quote is additive in $c$ as claimed.

## B.2.2 Proof of Proposition Corollary 1

Next, we prove Corollary $1(i)$
Proof of Corollary 1(i). The proof consists of three steps:
Step 1: If $p^{0}$ is additive in $c$, then, from equation (18), since $p^{0}$ and the value of the home bank and the second highest utility lender are additive in $c$, we have that $E\left(p^{*} \mid \bar{p}, s\right)$ is also additive in $c$.

Step 2: Since only the difference between $p^{0}$ and $E\left(p^{*} \mid \bar{p}, s\right)$ matters for determining the threshold of consumers, the search probability $H$ is independent of $c$. Specifically, if $p^{0}$ is in the interior, then the search threshold is implicitly defined by equation 6 and depends only on $n$ : $\bar{\kappa}(n)$. If, on the other hand, $p^{0}=\bar{p}$, then $\bar{\kappa}\left(p^{0}, c, n\right)=E\left(p^{*} \mid \bar{p}, c, n\right)-\bar{p}$.

This completes the proof of Corollary $1(i)$.
Proof of Corollary 1(ii). To prove Corollary $1(i i)$ we need to show that the marginal effect of $\lambda$ and $\Delta$ on $\bar{\kappa}\left(p^{0}, s\right)$ are equal in equilibrium. The total derivatives are given by:

$$
\begin{aligned}
\frac{d \bar{\kappa}\left(p^{0}, s\right)}{d \lambda} & =\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial \lambda}+\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial p^{0}} \frac{d p^{0}}{d \lambda} \\
& =-\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right]-G_{(2)}(-\gamma)+\frac{d p 0}{d \lambda}\left(1-\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial p^{0}}\right) \\
\frac{d \bar{\kappa}\left(p^{0}, s\right)}{d \Delta} & =\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial \Delta}+\frac{\partial \bar{\kappa}\left(p^{0}, s\right)}{\partial p^{0}} \frac{d p^{0}}{d \Delta} \\
& =\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right]+\frac{d p 0}{d \Delta}\left(1-\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial p^{0}}\right)
\end{aligned}
$$

The difference between the two is:

$$
\begin{aligned}
\frac{d \bar{\kappa}\left(p^{0}, s\right)}{d \lambda}-\frac{d \bar{\kappa}\left(p^{0}, s\right)}{d \Delta} & =-G_{(1)}\left(p^{0}-c-\lambda\right)+\left(1-\frac{\partial E\left(p^{*} \mid p^{0}, s\right)}{\partial p^{0}}\right)\left[\frac{d p 0}{d \lambda}-\frac{d p 0}{d \Delta}\right] \\
& =-G_{(1)}\left(p^{0}-c-\lambda\right)+G_{(1)}\left(p^{0}-c-\lambda\right)\left[\frac{f_{\Delta}\left(p^{0}, s\right)-f_{\lambda}\left(p^{0}, s\right)}{f_{p^{0}}\left(p^{0}\right)}\right]
\end{aligned}
$$

Therefore the proof of Corollary 3.2 requires that:

$$
\frac{f_{\Delta}\left(p^{0}, s\right)-f_{\lambda}\left(p^{0}, s\right)}{f_{p^{0}}\left(p^{0}\right)}=1
$$

The difference between $\frac{f_{\Delta}\left(p^{0}, s\right)}{f_{p^{0}}\left(p^{0}\right)}$ and $\frac{f_{\lambda}\left(p^{0}, s\right)}{f_{p^{0}}\left(p^{0}\right)}$ is:

$$
\begin{aligned}
f_{\Delta}\left(p^{0}, s\right)-f_{\lambda}\left(p^{0}, s\right)= & -g_{(1)}\left(p^{0}-c-\lambda\right)\left(H(\bar{\kappa})+H^{\prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]\right) \\
& -G_{(1)}\left(p^{0}-c-\lambda\right)\binom{H^{\prime}(\bar{\kappa})\left[\frac{\partial \bar{\kappa}}{\partial \Delta}-\frac{\partial \bar{\kappa}}{\partial \lambda}\right]+H^{\prime \prime}(\bar{\kappa})\left[\frac{\partial \bar{\kappa}}{\partial \Delta}-\frac{\partial \bar{\kappa}}{\partial \lambda}\right]\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]}{+H^{\prime}(\bar{\kappa})\left[1-\frac{\partial E\left(\pi^{*}\right)}{\partial \Delta}+\frac{\partial E\left(\pi^{*}\right)}{\partial \lambda}\right]} \\
= & -g_{(1)}\left(p^{0}-c-\lambda\right)\left(H(\bar{\kappa})+H^{\prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]\right) \\
& -G_{(1)}\left(p^{0}-c-\lambda\right)^{2}\left(2 H^{\prime}(\bar{\kappa})+H^{\prime \prime}(\bar{\kappa})\left[\pi^{0}-E\left(\pi_{h}^{*}\right)\right]\right) \\
= & f_{p^{0}\left(p^{0}, s\right)}
\end{aligned}
$$

Therefore,

$$
\frac{f_{\Delta}\left(p^{0}, s\right)-f_{\lambda}\left(p^{0}, s\right)}{f_{p^{0}}\left(p^{0}, s\right)}=1 .
$$

Proof of Corollary 1(iii). To prove that the distribution of prices for switchers is a function only of the sum of the two loyalty terms (i.e. $\gamma$ ), we consider the expected second stage price paid by switchers conditional on $c$.

$$
E\left(p_{S}^{*} \mid p^{0}, s, S\right)=\frac{1}{G_{(1)}(-\tilde{\Delta)}}\left[(c-\gamma)\left[G_{(1)}(-\gamma)-G_{(2)}(-\gamma)\right]+\int_{-\infty}^{-\gamma}\left(c+\omega_{(2)}\right) d G_{(2)}\right]
$$

Since $\lambda$ and $\Delta$ do not enter this equation separately, the distribution of prices for switchers is a function only of their sum $\gamma$.

Proof of Corollary 1(iv). The average transaction price paid by loyal consumers is given by:

$$
\begin{aligned}
E\left(p \mid p^{0}, s, L\right) & =E\left(p^{0}(1-H)+H E\left(p^{*} \mid p^{0}, s, L\right)\right) \\
& =E\left(p^{0}-H\left(p^{0}-E\left(p^{*} \mid p^{0}, s, L\right)\right)\right.
\end{aligned}
$$

We want to show that $\frac{d E\left(p \mid p^{0}, s, L\right)}{d \lambda} \neq \frac{d E\left(p \mid p^{0}, s, L\right)}{d \Delta}$.

$$
\begin{aligned}
\frac{d E\left(p \mid p^{0}, s, L\right)}{d \lambda}-\frac{d E\left(p \mid p^{0}, s, L\right)}{d \Delta} & =E\left[\frac{d p^{0}}{d \lambda}-\frac{d p^{0}}{d \Delta}-H\left[\frac{d p^{0}}{d \lambda}-\frac{d p^{0}}{d \Delta}-\left(\frac{d E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{d \lambda}-\frac{d E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{d \Delta}\right)\right]\right] \\
& =E\left(1-H\left(1-E\left(p^{*} \mid p^{0}, s, L\right)\right)\right.
\end{aligned}
$$

Therefore, to show that the average transaction price paid by loyal consumers is affected asymmetrically we need to compare $\frac{d E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{d \lambda}$ and $\frac{d E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{d \Delta}$, where $E\left(p_{L}^{*} \mid p^{0}, s, L\right)$ is the expected
second stage price for loyals and is given by:

$$
E\left(p_{L}^{*} \mid p^{0}, s, L\right)=\frac{1}{1-G_{(1)}(-\tilde{\Delta})}\left[p^{0}\left(1-G_{(1)}\left(p^{0}-c-\lambda\right)\right)+\int_{-\gamma}^{p^{0}-c-\lambda}\left(c+\lambda+\omega_{(1)}\right) d G_{(1)}\right] .
$$

We have that

$$
\begin{aligned}
& \frac{d E\left(p_{L}^{*} \mid p^{0}, s\right)}{d \lambda}=\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial \lambda}+\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial p^{0}} \frac{d p^{0}}{d \lambda} \\
& \frac{d E\left(p_{L}^{*} \mid p^{0}, s\right)}{d \Delta}=\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial \Delta}+\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial p^{0}} \frac{d p^{0}}{d \Delta} .
\end{aligned}
$$

The derivative of $E\left(p_{L}^{*} \mid p^{0}, s, L\right)$ wrt $\lambda$ is given by:

$$
\begin{aligned}
\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial \lambda}= & \frac{-g_{(1)}(-\gamma)}{\left[1-G_{(1)}(-\gamma)\right]^{2}}\left[p^{0}\left(1-G_{(1)}\left(p^{0}-c-\lambda\right)\right)+\int_{-\gamma}^{p^{0}-c-\lambda}\left(c+\lambda+\omega_{(1)}\right) d G_{(1)}\right] \\
& +\frac{1}{1-G_{(1)}(-\gamma)}\left[(c+\lambda-\gamma) g_{(1)}(-\gamma)+\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right]\right]
\end{aligned}
$$

The derivative of $E\left(p_{L}^{*} \mid p^{0}, s, L\right)$ wrt $\Delta$ is given by:

$$
\begin{aligned}
\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial \Delta}= & \frac{-g_{(1)}(-\gamma)}{\left[1-G_{(1)}(-\gamma)\right]^{2}}\left[p^{0}\left(1-G_{(1)}\left(p^{0}-c-\lambda\right)\right)+\int_{-\gamma}^{p^{0}-c-\lambda}\left(c+\lambda+\omega_{(1)}\right) d G_{(1)}\right] \\
& +\frac{1}{1-G_{(1)}(-\gamma)}\left[(c+\lambda-\gamma) g_{(1)}(-\gamma)\right]
\end{aligned}
$$

The difference is given by:

$$
\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial \lambda}-\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial \Delta}=\frac{1}{1-G_{(1)}(-\gamma)}\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right]
$$

We also have that $\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial p^{0}}=\frac{1}{1-G_{(1)}(-\Delta)}\left[1-G_{(1)}\left(p^{0}-c-\lambda\right)\right]$, such that

$$
\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial p^{0}} \frac{d p^{0}}{d \lambda}-\frac{\partial E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{\partial p^{0}} \frac{d p^{0}}{d \Delta}=\frac{1}{1-G_{(1)}(-\tilde{\Delta})}\left[1-G_{(1)}\left(p^{0}-c-\lambda\right)\right]
$$

since $\frac{d p^{0}}{d \lambda}-\frac{d p^{0}}{d \Delta}=1$.

Combining these differences we have that

$$
\begin{aligned}
\frac{d E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{d \lambda}-\frac{d E\left(p_{L}^{*} \mid p^{0}, s, L\right)}{d \Delta}= & \frac{1}{1-G_{(1)}(-\gamma)}\left[G_{(1)}\left(p^{0}-c-\lambda\right)-G_{(1)}(-\gamma)\right] \\
& +\frac{1}{1-G_{(1)}(-\tilde{\Delta})}\left[1-G_{(1)}\left(p^{0}-c-\lambda\right)\right] \\
= & \frac{1-G_{(1)}(-\tilde{\Delta})}{1-G_{(1)}(-\tilde{\Delta)}}=1
\end{aligned}
$$

Therefore

$$
\frac{d E\left(p \mid p^{0}, s, L\right)}{d \lambda}-\frac{d E\left(p \mid p^{0}, s, L\right)}{d \Delta}=1
$$

## C Identification

There are four model primitives: (i) the distribution of the common lending cost ( $c_{i}$ ), (ii) the distribution of idiosyncratic cost differences $\left(\omega_{i j}\right)$, (iii) the search-cost distribution ( $\kappa_{i}$ ), and (iv) the loyalty advantage parameters $\left(\lambda_{i}, \Delta_{i}\right)$. In addition, outcomes depend on the following observable state variables: (i) borrower financial characteristics $x_{i}$, (ii) the posted price $\bar{p}$, and (iii) the number of lenders $n$.

Crucial to our arguments presented below will be the existence of sufficient variation in $\bar{p}$ and $n$, conditional on $x_{i}$. In particular, our data set is a panel of locations and periods, over which we observe a (large) cross section of borrower characteristics $x_{i}$. For each consumer, the negotiation period $t(i)$ determines the posted price and other time-varying cost shifters, while the location determines the number of lenders available. Although we are not restricting the correlation between observable characteristics and $\left(\bar{p}_{t(i)}, n_{i}\right)$, it is essential that the supports of borrower characteristics be comparable across periods and markets.

Assumption 1 describes the relationship between the primitives of the model and observable characteristics of each transaction.

Assumption 1. The primitive distributions of the model satisfy the following assumptions:
(i) The distribution of the common lending cost is conditionally independent of $(n, \bar{p}): \operatorname{Pr}\left(c_{i}<\right.$ $\left.c \mid x_{i}, n, \bar{p}\right)=F\left(c \mid x_{i}\right)$.
(ii) The idiosyncratic cost differences are IID across consumers: $\operatorname{Pr}\left(\omega_{i j}<\omega \mid x_{i}, n, \bar{p}\right)=G_{j}(\omega)$.
(iii) The search cost distribution is a function of borrower characteristics $z_{i}^{1} \in x_{i}$ and independent of $(n, \bar{p}): \operatorname{Pr}\left(\kappa_{i}<\kappa \mid x_{i}, n, \bar{p}\right)=H\left(\kappa \mid z_{i}^{1}\right)$.
(iv) The loyalty advantage parameters are deterministic functions of borrower characteristics $z_{i}^{2} \in$ $x_{i}: \lambda_{i}=\lambda\left(z_{i}^{2}\right)$ and $\Delta_{i}=\Delta\left(z_{i}^{2}\right)$.

These assumptions clarify that the model is identified by two key exclusion restrictions, as well as one important assumption regarding the distribution of the common cost and WTP advantages. On the first point, we assume that the lending and search costs are independent of the posted price and the structure of local markets. Similarly, Assumption 1(iv) restricts the loyalty advantage to
be a deterministic function of observable attributes. This is crucial since we do not separately observe search and loyalty. This restriction could in theory be relaxed with richer data on search and firm choices.

In addition, we use the following support assumptions. We do not impose these assumptions in the empirical application, since we use parametric distributions to estimate the model.

Assumption 2. The data generating process is such that:
(i) The common lending cost is distributed over a known support: $c_{i} \in[\underline{c}, \bar{c}]$.
(ii) The posted price, $\bar{p}$, has a full support between a lower bound $\overline{\bar{p}}$ and $\infty$.
(iii) The number of lenders has full support between 2 and $\bar{n} \geq 4$.

The rest of the section is organized as follows. First, we formally define the relationship between observed and predicted outcomes. Then, we discuss the identification of the model primitives in two steps: (i) identification of the lending cost distributions $\left\{F\left(c \mid x_{i}\right), G(\omega)\right\}$ and loyalty advantage $\gamma$, and (ii) identification of the search cost distribution $(H(\kappa))$ and home-bank cost/WTP advantages $\{\lambda, \Delta\}$. In order to make the identification discussion more transparent, we focus on a special case of the model in which the loyalty advantage parameters are constant across consumers (i.e. $\lambda\left(z_{i}^{2}\right)=\lambda$ and $\Delta\left(z_{i}^{2}\right)=\Delta$ for all $i$, and the search-cost distribution is independent of consumer observable characteristics (i.e. $H\left(\kappa \mid z_{i}^{1}\right)=H(\kappa)$ ). We also abstract from observable differences between lenders (i.e. $G_{j}(\omega)=G(\omega)$ for all $j$ ). Both assumptions can be relaxed without affecting the key results.

## C. 1 Identification problem and observed outcomes

The identification problem can be summarized as follows. First, we need to separately identify two sources of unobserved lending cost heterogeneity, the common component $c_{i}$ and the lender-specific idiosyncratic component $\omega_{i j}$, using only data on transaction prices. Second, we need to distinguish between two sources of brand loyalty - search costs and loyalty premia-using data on the conditional probability of remaining loyal to the home bank. Finally, we need to demonstrate that the two sources of the loyalty advantage - cost and WTP advantages - are separately identified.

Consider an ideal data set (i.e. one that satisfies Assumption 2), which allows us to measure three empirical distributions: (i) the conditional distribution of prices for switching consumers, (ii) the conditional distribution of prices for loyal consumers, and (iii) the conditional switching probability. These three outcomes are observed conditional on the vector of observed borrower characteristics, and over the full support of the distribution of the posted price and the number of lenders. The following three equations summarize the link between the model and the data:

$$
\begin{align*}
\Phi^{S}\left(p \mid x_{i}, \bar{p}, n\right) & =\frac{\operatorname{Pr}\left(p^{*}(\omega, s) \leq p, \omega_{(1)}<-\gamma, \kappa_{i}<\bar{\kappa}(s) \mid x_{i}, \bar{p}, n\right)}{\operatorname{Pr}\left(\omega_{(1)}<-\gamma, \kappa_{i}<\bar{\kappa}(s) \mid x_{i}, \bar{p}, n\right)}  \tag{18}\\
\Phi^{L}\left(p \mid x_{i}, \bar{p}, n\right) & =\frac{\operatorname{Pr}\left(p^{0}(s) \leq p, \bar{\kappa}_{i}>\bar{\kappa}(s) \mid x_{i}, \bar{p}, n\right)+\operatorname{Pr}\left(p^{*}(\omega, s) \leq p, \omega_{(1)}>-\gamma, \bar{\kappa}_{i}>\bar{\kappa}(s) \mid x_{i}, \bar{p}, n\right)}{1-\operatorname{Pr}\left(\omega_{(1)}<-\gamma, \kappa_{i}<\bar{\kappa}(s) \mid x_{i}, \bar{p}, n\right)}  \tag{19}\\
S\left(x_{i}, \bar{p}, n\right) & =\operatorname{Pr}\left(\omega_{(1)}<-\gamma, \kappa_{i}<\bar{\kappa}(s) \mid x_{i}, \bar{p}, n\right) \tag{20}
\end{align*}
$$

where $\Phi^{S}\left(p \mid x_{i}, \bar{p}, n\right)$ and $\Phi^{L}\left(p \mid x_{i}, \bar{p}, n\right)$ are the empirical distributions of prices for switching and loyal consumers, respectively, and $S\left(x_{i}, \bar{p}, n\right)$ is the empirical switching probability. The first two equations highlight the fact that the distribution of prices for loyal consumers is a mixture of prices coming from searchers and non-searchers, while the distribution of prices for switching consumers is solely a function of the outcome of the auction. Similarly, the switching probability is a combination
of two factors: consumers rejecting the initial offer, and the home bank losing at the competition stage.

## C. 2 Identification of the lending cost and loyalty advantage

We first discuss identification of the distributions of $c_{i}$ and $\omega_{i j}$, as well as the loyalty advantage $(\gamma=\lambda+\Delta)$. To do so, we focus solely on the distribution of prices for switching consumers described in equation (18). This sub-sample is appealing, since prices are generated directly from the auction, and therefore are not directly a function of the search-cost distribution.

The challenge is to address a potential selection bias: consumers reaching the second stage of the game must first reject $p^{0}(s)$, which is a function of the unobserved common lending cost component. This endogenous selection implies that the lending cost distribution among switchers is different from the unconditional distribution $F\left(c \mid x_{i}\right)$.

To get around this problem we use the full-support assumption of $\bar{p}$ to eliminate the dependence of the selection probability on $c$. In particular, note that as $\bar{p} \rightarrow \infty$ the switching probability is independent of the common lending cost distribution:

$$
\begin{align*}
\lim _{\bar{p} \rightarrow \infty} \operatorname{Pr}\left(\omega_{(1)}<-\gamma, \kappa_{i}<\bar{\kappa}(s) \mid x_{i}, \bar{p}, n\right) & =\lim _{\bar{p} \rightarrow \infty} \int G_{(1)}(-\gamma) H(\bar{\kappa}(s)) d F\left(c \mid x_{i}\right) \\
& =H(\bar{\kappa}(\lambda, \Delta, n)) G_{(1)}(-\gamma) \tag{21}
\end{align*}
$$

where the last line follows from the fact that the search probability is independent of $c$ when the initial quote is unconstrained (Corollary $1(i)$ ).

This result implies that as $\bar{p} \rightarrow \infty$ the conditional distribution of prices for switching consumers is independent of the search-cost distribution:

$$
\begin{align*}
\lim _{\bar{p} \rightarrow \infty} \Phi^{S}\left(p \mid x_{i}, \bar{p}, n\right) & =\lim _{\bar{p} \rightarrow \infty} \frac{\operatorname{Pr}\left(p^{*}(\omega, s) \leq p, \omega_{(1)}<-\gamma, \kappa_{i}<\bar{\kappa}(s) \mid x_{i}, \bar{p}, n\right)}{\operatorname{Pr}\left(\omega_{(1)}<-\gamma, \kappa_{i}<\bar{\kappa}(s) \mid x_{i}, \bar{p}, n\right)} \\
& =\frac{H\left(\bar{\kappa}(\lambda, \Delta, n) \operatorname{Pr}\left(c_{i}+\min \left\{-\gamma, \omega_{(2)}\right\}<p, \omega_{(1)}<-\gamma \mid x_{i}, n\right)\right.}{H(\bar{\kappa}(\lambda, \Delta, n)) G_{(1)}(-\gamma)} \\
& =\operatorname{Pr}\left(c_{i}+\min \left\{-\gamma, \omega_{(2)}\right\}<p \mid \omega_{(1)}<-\gamma, x_{i}, n\right) . \tag{22}
\end{align*}
$$

The second equality follows from equation (21). Therefore, the distribution of $c_{i}$ within the sample of unconstrained switching consumers is equal to the unconditional distribution of the common lending cost $F\left(c \mid x_{i}\right) .{ }^{32}$

Using this sub-sample, it is easy to show that the distributions of $c_{i}$ and $\omega_{i j}$ and the loyalty advantage $\gamma$ are separately identified. To see this, the distribution of prices in markets with two lenders is truncated from below by $\underline{c}-\gamma$. This is because the home bank faces only one rival ( $n=1$ ), and the price paid by switchers is equal to $p^{*}=c+\gamma$. Therefore, the minimum price paid by unconstrained switchers can be used to identify $\gamma$, while the remaining distribution of prices directly identifies the common lending cost:

$$
F\left(c \mid x_{i}\right)=\Phi^{S}\left(p+\gamma \mid x_{i}, \bar{p}=\infty, n=1\right) .
$$

[^23]The distribution of idiosyncratic cost differences $\omega_{j}$ can be identified using minimal variation in the number of bidders. In particular, the distribution of prices for each $n>1$ is given by:

$$
\begin{align*}
& \Phi^{S}\left(p+\gamma \mid x_{i}, \bar{p}=\infty, n\right) \\
& \quad=F\left(p-\gamma \mid x_{i}\right) \frac{\left[G_{(1)}(-\gamma \mid n)-G_{(2)}(-\gamma \mid n)\right]}{G_{(1)}(-\gamma \mid n)}+\int_{\omega_{(2)}<-\gamma} F\left(p-\omega_{(2)}\right) \frac{d G_{(2)}\left(\omega_{(2)} \mid n\right)}{G_{(1)}(-\gamma \mid n)} . \tag{23}
\end{align*}
$$

Since $\gamma$ and $F\left(c \mid x_{i}\right)$ are known, equation (23) can be inverted under standard conditions to recover the distribution of idiosyncratic cost differences $G(\omega)$.

The previous argument depends on observing at least two market structures, including $n=1$. However, observing duopoly markets is not necessary. For instance, Quint (2015) shows that observations of transaction prices and at least two different market structures of any size are sufficient to non-parametrically identify ascending auction models with additively-separable unobserved heterogeneity. Quint's identification argument relies on a condition that valuations be bounded below by zero. Quint's proof can be adapted to our procurement context with observations only for transaction prices from switchers. In this setting, identification requires that the common lending cost be bounded above as in Assumption 2(i). ${ }^{33}$

If more than two sizes of auctions are observed, the model is over-identified, which allows us to identify $\gamma$ without using the distribution of prices in $n=1$. Intuitively, the loyalty advantage is identified by observing how the distribution of prices for switchers changes with the number of lenders. In markets with a small number of lenders, the presence of a positive loyalty advantage implies that the distribution of prices for switchers mostly reflects the common cost component, since the home bank is likely the next-best alternative for switching consumers. This is because $p^{*}=c_{i}-\gamma$ if $\omega_{(2)}>-\gamma$, and $\operatorname{Pr}\left(\omega_{(2)}>-\gamma \mid n\right)$ is close to one when $n$ is small and $\gamma$ is large. In contrast, as the number of rival lenders increases, the probability that $\omega_{(2)}<-\gamma$ converges to one, which implies a stronger correlation between $n$ and the price paid by switchers. Therefore, the loyalty advantage is identified from the strength of the correlation between $n$ and $p^{*}$, as the number of competitors becomes large.

## C. 3 Identification of the search-cost distribution and willingness-to-pay

In the previous subsection, we explained how to use variation in $n$ and the full-support assumption to identify $F\left(c \mid x_{i}\right), G(\omega)$ and $\gamma$. To see how the home-bank WTP advantage and the search-cost distribution are identified, consider first the switching probability conditional on a guess of $\lambda$ :

$$
\begin{align*}
S\left(x_{i}, \bar{p}, n\right) & =\operatorname{Pr}\left(\omega_{(1)}<-\gamma, \kappa_{i}<\bar{\kappa}(s) \mid x_{i}, \bar{p}, n\right) \\
& =\int G_{(1)}(-\gamma) H(\bar{\kappa}(s)) d F\left(c \mid x_{i}\right)=\int m(s) d F\left(c \mid x_{i}\right), \tag{24}
\end{align*}
$$

where $\bar{\kappa}(s)=p^{0}(s)-E\left(p^{*} \mid p^{0}(s)\right)-\lambda G_{(1)}(-\gamma)$ is the search-cost threshold.
Contrary to the argument used in the previous section, the search-cost distribution cannot be non-parametrically identified if all consumers are unconstrained by the posted price. To see this, note that when $\bar{p} \rightarrow \infty$ the predicted search probability is constant for all consumers facing the same market structure. At most, equation (24) would allow us to estimate the search probability

[^24]for each $n=1,2, \ldots, \bar{n}$. Since the threshold function $\bar{\kappa}(s)$ is an implicit function of the entire search-cost distribution, this is clearly not enough to identify the entire distribution.

As a solution to this problem, we need to exploit variation in the probability of being constrained by the posted price. This can be done for instance by varying $\bar{p}$, or by varying elements of $x_{i}$ that are positively correlated with the lending cost. For consumers receiving an initial quote of $\bar{p}$, the search threshold is a known function of $\{F(c \mid x), G(\omega), \Delta, \lambda\}$ :

$$
\bar{\kappa}(s)=\bar{p}-E\left(p^{*} \mid \bar{p}, s\right)-\lambda G_{(1)}(-\gamma), \quad \text { if } p^{0}(s)=\bar{p}
$$

It is easy to show that for these consumers, the search probability is an increasing function of $\bar{p}$, and a decreasing function of $c_{i}$. Therefore, exogenous variation in the posted-price and in the observable risk of consumers can be used to trace out the support of $\kappa_{i}$ in equation (24) by varying continuously the search threshold of consumers.

More formally, conditional on knowing $\left\{F\left(c \mid x_{i}\right), G(\omega), \Delta, \lambda\right\}$, the identification of the searchcost distribution is analogous to the identification of non-parametric instrumental regression models (e.g. Newey and Powell (2003)). Under standard completeness conditions, one can show that there is a unique solution $m(s) \equiv G_{(1)}(-\gamma) H(\bar{\kappa}(s))$ to equation (24). With this non-parametric function in hand, we can use the solution of the model to compute the equilibrium search-cost thresholds and characterize the entire search-cost distribution.

Finally, to distinguish between the home-bank WTP and cost advantages, we must rely on the distribution of prices among loyal consumers. Recall from equation (19) that this distribution is a mixture of initial quote offers and auction prices. We know from Corollary 1 (iv) that $\lambda$ and $\Delta$ have different impacts on the average transaction price of loyal consumers. In contrast, $\lambda$ and $\Delta$ affect symmetrically the equilibrium search probability (Corollary $1(i i)$ ), and the distribution of prices for switchers is only a function of the sum $\gamma$ (Corollary $1(i i i)$ ). Therefore, while both parameters influence in the same way the observed retention probability, they have different effects on the average price difference between loyal and switching consumers. This moment can thus be used to identify $\lambda$ separately from $\Delta$.

## D Full derivation of the likelihood function

The model is described by the following random variables:

- Lending cost: $c_{i j}=c_{i}+\omega_{i j}$.
- Home bank cost differential: $\operatorname{Pr}\left(\omega_{i h}<z\right)=G_{h}\left(z \mid \sigma_{\omega}\right)$.
- Home bank quality: $\lambda$.
- Rival banks cost differential: $\operatorname{Pr}\left(\omega_{i j}<\omega\right)=G_{j}\left(\omega \mid \sigma_{\omega}\right)$.
- Common cost component: $\operatorname{Pr}\left(c_{i}<c \mid x_{i}\right)=F\left(c \mid x_{i}, \beta, \sigma_{\epsilon}\right)$.
- Search cost: $\operatorname{Pr}\left(\kappa_{i}<\kappa \mid z_{i}^{1}\right)=H\left(\kappa \mid z_{i}^{1}\right)$.
- Set of rival banks: $n(i)$.
- Posted rate at negotiated date: $\bar{p}_{t(i)}$.

To simplify the notation, we abstract completely from consumer $i$ 's information. This gives us three distributions: $G_{j}(\cdot), F(\cdot)$ and $H(\cdot)$. Also, we use the subscript $b$ to index the chosen lender, and $-b$ to index the most efficient lender other than $b$ in the choice-set of consumers. Moreover, since $\Delta$ is realized and common knowledge at the time of negotiation, we will treat it as a constant.

Let $p^{0}$ denote the realized initial quote. If the home bank qualifies for a loan (i.e. $c-\Delta<\bar{p}$ ), the auction outcome is:

$$
p^{*}= \begin{cases}p^{0} & \text { If } \omega_{(1)}>p^{0}-c-\lambda, \\ c+\omega_{(1)}+\lambda & \text { If }-\Delta-\lambda<\omega_{(1)}<p^{0}-c-\lambda, \\ c+\min \left\{\omega_{(2)},-\Delta-\lambda\right\} & \text { If }-\Delta-\lambda>\omega_{(1)} .\end{cases}
$$

If the home bank fails to qualify $(c-\Delta>\bar{p})$, the reserve price is $p^{0}=\bar{p}$ and the auction outcome is:

$$
p^{*}= \begin{cases}\bar{p} & \text { If } \omega_{(2)}>p-c>\omega_{(1)}, \\ c+\omega_{(2)} & \text { If } \omega_{(2)}<\bar{p}-c .\end{cases}
$$

The initial quote is the following step function:

$$
p^{0}= \begin{cases}+\infty & \text { If } c>\bar{p}+\Delta \\ \bar{p} & \text { If } \bar{p}>c>\bar{p}-\mu_{i} \\ c+\mu_{i} & \text { If } c<\bar{p}-\mu_{i}\end{cases}
$$

where $\mu_{i} \equiv \mu(n, \Delta, \lambda)$ is the initial quote markup (interior solution). Consumers decide to search based on a cut-off rule: $\kappa_{i}<\bar{\kappa}(s)$ where $s=(c, \bar{p}, \lambda, \Delta, n)$. To highlight the dependence of the search threshold with $c$ and simplify the notation, we use $H(\bar{\kappa}(s)) \equiv H(\bar{\kappa}(c))$ to denote equilibrium search probability. The other state variables are implicit in the consumer index $i$.

To construct the likelihood we consider three separate cases: (i) Switching consumers, (ii) Loyal consumers going to the auction, and (iii) Loyal consumers accepting the initial quote. Within each of these cases, it is useful to consider separately consumers paying the posted rate and consumers obtaining a discount.

## Case 1: Switching consumers

The population of switchers receiving a discount and dealing with lender $b$ is composed of searchers who rejected $p^{0}$ (or didn't receive an initial quote) and whose home bank was not competitive at the auction stage. Recall that the price that switchers pay is given by:

$$
p_{i}= \begin{cases}c-\Delta-\lambda & \text { If } \omega_{i, b}<-\Delta-\lambda<\omega_{i,-b} \text { and } c<\bar{p}+\Delta \\ c+\omega_{i,-b} & \text { Else. }\end{cases}
$$

The joint probability of observing an unconstrained price lower than $p$ from a switching con-
sumer is given by:

$$
\begin{aligned}
\operatorname{Pr} & \left(P_{i}<p, B_{i}=b, B_{i} \neq h\right) \\
= & \int_{-\infty}^{\bar{p}+\Delta} \int_{\omega_{i,-b}} \int_{\omega_{b}} 1\left(c+\min \left\{-\Delta-\lambda, \omega_{i,-b}\right\}<p\right) 1\left(\omega_{i, b}<\min \left\{-\Delta-\lambda, \omega_{i,-b}\right\}\right) H(\bar{\kappa}(c)) d G_{-b} d G_{b} d F \\
& +\int_{\bar{p}+\Delta}^{\infty} \int_{\omega_{i,-b}} \int_{\omega_{b}} 1\left(c+\omega_{i,-b}<p\right) 1\left(\omega_{i, b}<\omega_{i,-b}\right) d G_{-b} d G_{b} d F \\
= & \int_{-\infty}^{\bar{p}+\Delta} \int_{-\Delta-\lambda}^{+\infty} 1(c<p+\Delta+\lambda) G_{b}(-\Delta-\lambda) H(\bar{\kappa}(c)) d G_{-b} d F \\
& +\int_{-\infty}^{\bar{p}+\Delta} \int_{-\infty}^{-\Delta-\lambda} 1\left(\omega_{i,-b}<p-c\right) G_{b}\left(\omega_{i,-b}\right) H(\bar{\kappa}(c)) d G_{-b} d F+\int_{\bar{p}+\Delta}^{\infty} \int_{-\infty}^{p-c} G_{b}\left(\omega_{i,-b}\right) d G_{-b} d F \\
= & \int_{-\infty}^{\min \{p+\Delta+\lambda, \bar{p}+\Delta\}}\left(1-G_{-b}(-\Delta-\lambda)\right) G_{b}\left(\omega_{h}-\lambda\right) H(\bar{\kappa}(c)) d F \\
& +\int_{-\infty}^{\bar{p}+\Delta} \int_{-\infty}^{\min \{-\Delta-\lambda, p-c\}} G_{b}\left(\omega_{i,-b}\right) H(\bar{\kappa}(c)) d G_{-b} d F+\int_{\bar{p}+\Delta}^{\infty} \int_{-\infty}^{p-c} G_{b}\left(\omega_{i,-b}\right) d G_{-b} d F
\end{aligned}
$$

The derivative of the previous probability with respect to $p$ yields the likelihood contribution of unconstrained switching consumers:

$$
\begin{aligned}
l_{i}(p, b)= & 1(\bar{p}>p+\lambda)\left(1-G_{-b}(-\Delta-\lambda)\right) G_{b}(-\Delta-\lambda) H(\bar{\kappa}(p+\Delta+\lambda)) f(p+\Delta+\lambda) \\
& \quad+\int_{-\infty}^{\bar{p}+\Delta} 1(p-c<-\Delta-\lambda) g_{-b}(p-c) G_{b}(p-c) H(\bar{\kappa}(c)) d F+\int_{\bar{p}+\Delta}^{\infty} g_{-b}(p-c) G_{b}(p-c) d F \\
= & 1(\bar{p}>p+\lambda)\left[\begin{array}{l}
\left(1-G_{-b}(-\Delta-\lambda)\right) G_{b}(-\Delta-\lambda) H(\bar{\kappa}(p+\Delta+\lambda)) f(p+\Delta+\lambda) \\
+\int_{p+\Delta+\lambda}^{\bar{p}+\Delta} g_{-b}(p-c) G_{b}(p-c) H(\bar{\kappa}(c)) d F
\end{array}\right] \\
& \quad+\int_{\bar{p}+\Delta}^{\infty} g_{-b}(p-c) G_{b}(p-c) d F \\
= & 1(\bar{p}>p+\lambda)\left(1-G_{-b}(-\Delta-\lambda)\right) G_{b}(-\Delta-\lambda) H(\bar{\kappa}(p+\Delta+\lambda)) f(p+\Delta+\lambda) \\
& \quad+\int_{p+\Delta+\lambda}^{\infty} g_{-b}(p-c) G_{b}(p-c) H(\bar{\kappa}(c)) d F
\end{aligned}
$$

The likelihood is more straightforward for switching consumers paying the posted rate. This event occurs only if: (i) the home bank fails to quality, (ii) the lowest cost lender is the only qualifying firm. The probability of observing this event is the likelihood contribution of constrained switching consumers:

$$
\begin{align*}
l_{i}(p=\bar{p}, b) & =\int_{c>\bar{p}+\Delta} \int_{\omega_{i,-b}} \int_{\omega_{i, b}} 1\left(\omega_{i,-b}>\bar{p}-c\right) 1\left(\omega_{i, b}<\bar{p}-c\right) d G_{b} d G_{-b} d F \\
& =\int_{\bar{p}+\Delta}^{\infty} G_{b}(\bar{p}-c)\left(1-G_{-b}(\bar{p}-c)\right) d F \tag{25}
\end{align*}
$$

## Case 2: Loyal consumers going to the auction

We consider first the case of loyal consumers going to the auction and receiving a discount. The population of consumers receiving a discount and staying with their home bank despite rejecting the initial offer is composed of consumers: (i) who qualify at their home bank, (ii) for whom the home bank cost advantage is large enough, and (iii) who chose to search. Two events can lead to this case:

$$
p_{i}= \begin{cases}c+\mu & \text { If } c+\mu<\bar{p} \text { and } \omega_{(1)}>\mu-\lambda, \\ c+\omega_{(1)}+\lambda & \text { If } c<\bar{p}+\Delta \text { and }-\Delta-\lambda<\omega_{(1)}<p^{0}(c)-c-\lambda,\end{cases}
$$

where $p^{0}(c)=\min \{\bar{p}, c+\mu(c, \Delta, \lambda)\}$ is the initial quote function.
The joint probability of observing a price lower than $p$ from a loyal consumer is given by:

$$
\begin{aligned}
\operatorname{Pr}\left(P_{i}<p, B_{i}=h\right)= & \int_{-\infty}^{\bar{p}-\mu} \int_{\mu-\lambda}^{\infty} 1(c<p-\mu) H(\bar{\kappa}(c)) d G_{(1)} d F \\
& +\int_{-\infty}^{\bar{p}+\Delta} \int_{-\Delta-\lambda}^{p^{0}(c)-c-\lambda} 1\left(\omega_{1}<p-c-\lambda\right) H(\bar{\kappa}(c)) d G_{(1)} d F \\
= & \int_{-\infty}^{p-\mu}\left[1-G_{(1)}(\mu-\lambda)\right] H(\bar{\kappa}(c)) d F \\
& +\int_{-\infty}^{\bar{p}+\Delta}\left[G_{(1)}\left(\min \left\{p-c-\lambda, p^{0}(c)-c-\lambda\right\}\right)-G_{(1)}(-\Delta-\lambda)\right] H(\bar{\kappa}(c)) d F \\
= & \int_{-\infty}^{p-\mu}\left[1-G_{(1)}(\mu-\lambda)\right] H(\bar{\kappa}(c)) d F \\
& +\int_{\bar{p}-\mu}^{\bar{p}+\Delta} 1(c<p+\Delta)\left[G_{(1)}(p-c-\lambda)-G_{(1)}(-\Delta-\lambda)\right] H(\bar{\kappa}(c)) d F \quad \quad\left[\text { Note: } p^{0}=\bar{p}\right] \\
& +\int_{-\infty}^{\bar{p}-\mu}\left[G_{(1)}(\min \{p-c-\lambda, \mu-\lambda\})-G_{(1)}(-\Delta-\lambda)\right] H(\bar{\kappa}(c)) d F \quad\left[\text { Note: } p^{0}<\bar{p}\right]
\end{aligned}
$$

The derivative of this probability with respect to $p$ corresponds to the likelihood of unconstrained
loyal consumers at the auction:

$$
\begin{aligned}
& l_{i}(p, b=h)=\left(1-G_{(1)}(\mu-\lambda)\right) H(\bar{\kappa}(p-\mu)) f(p-\mu) \\
& +1(p+\Delta>\bar{p}-\mu) \int_{\bar{p}-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda) H(\bar{\kappa}(c)) d F(c) \\
& +\int_{-\infty}^{\bar{p}-\mu} 1(p-c-\lambda<\mu-\lambda) 1(p-c-\lambda>-\Delta-\lambda) g_{(1)}(p-c-\lambda) H(\bar{\kappa}(c)) d F(c) \\
& =\left(1-G_{(1)}(\mu-\lambda)\right) H(\bar{\kappa}(p-\mu)) f(p-\mu) \\
& +1(p+\Delta>\bar{p}-\mu) \int_{\bar{p}-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda) H(\bar{\kappa}(c)) d F(c) \\
& +\int_{p-\mu}^{\min \{\bar{p}-\mu, p+\Delta\}} g_{(1)}(p-c-\lambda) H(\bar{\kappa}(c)) d F(c) \\
& = \begin{cases}\left(1-G_{(1)}(\mu-\lambda)\right) H(\bar{\kappa}(p-\mu)) f(p-\mu) & \\
+\int_{\bar{p}-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda) H(\bar{\kappa}(c)) d F(c) & \text { If } p+\Delta>\bar{p}-\mu \\
+\int_{p-\mu}^{\bar{p}-\mu} g_{(1)}(p-c-\lambda) H(\bar{\kappa}(c)) d F(c) & \\
& \\
\left(1-G_{(1)}(\mu-\lambda)\right) H(\bar{\kappa}(p-\mu)) f(p-\mu) & \\
+0+\int_{p-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda) H(\bar{\kappa}(c)) d F(c) & \text { If } p+\Delta<\bar{p}-\mu\end{cases} \\
& =\left(1-G_{(1)}(\mu-\lambda)\right) H(\bar{\kappa}(p-\mu)) f(p-\mu)+\int_{p-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda) H(\bar{\kappa}(c)) d F(c)
\end{aligned}
$$

The likelihood contribution for constrained consumers is given by the probability of observing a loyal consumer going to the auction and not receiving a discount:

$$
\begin{equation*}
l_{i}(p=\bar{p}, b=h)=\int_{\bar{p}-\mu}^{\bar{p}+\Delta}\left(1-G_{(1)}(\bar{p}-c-\lambda)\right) H(\bar{\kappa}(c)) d F(c) . \tag{26}
\end{equation*}
$$

## Case 3: Loyal consumers accepting the initial quote

In this case, the transaction price is equal to $p^{0}(c)=\min \{\bar{p}, c+\mu(c, \Delta, \lambda)\}$. The likelihood contribution is therefore given by a truncated distribution:

$$
l_{i}(p, b=h)= \begin{cases}\int_{\bar{p}-\mu}^{\bar{p}+\Delta}[1-H(\bar{\kappa}(c))] d F(c) & \text { If } p=\bar{p} \\ f(p-\mu)[1-H(\bar{\kappa}(p-\mu))] & \text { If } p<\bar{p}\end{cases}
$$

## Combined likelihood function

Conditional on $h$, the likelihood contribution function is given by:

## E Monte Carlo Exercise: Quasi-Likelihood versus GMM

We compare the performance of the quasi-likelihood approach described in section 5.2 to the GMM approach based on Imbens and Lancaster (1994). The goal is to present a transparent case where our MLE-based approach that incorporates auxiliary data can be compared to a GMM-based approach that incorporates auxiliary data. We find that the MLE approach is more accurate than the one based on GMM (or as accurate). We think the reason for this is that in MLE it is straightforward to add the analytical gradient (scores), but since the likelihood scores are the moment conditions in GMM, it is much more sensitive to numerical solutions. Both methods are superior to an approach that ignores the moment conditions from the auxiliary data.

We present an example that mimics the structure of the model that we estimate in the paper. There are two choices, $y_{1}$ and $y_{2}$ : one can think of $y_{1}$ as the decision to switch and $y_{2}$ the decision to search. A borrower's decision to search is a function of X's and the decision to switch is a function of the search decision. The econometrics problem is that only one outcome $\left(y_{i 1}\right)$ is observed. In addition, the econometrician observes aggregate moments on the conditional distribution of the second outcome variable $\left(y_{i 2}\right)$. We first present the likelihood for our discrete choice problem and then the GMM problem. Lastly, we present the Monte Carlo results.

Consider the following discrete choice model:

$$
\begin{aligned}
y_{i 1} & =1\left(\alpha_{0}+\alpha_{1} y_{i 2}+\epsilon_{i 1}>0\right) \\
y_{i 2} & =1\left(x_{i} \beta+\epsilon_{i 2}>0\right)
\end{aligned}
$$

where $\left(\epsilon_{i 1}, \epsilon_{i 2}\right)^{\prime} \sim N(0, I)$. The main data-set contains only information on $\left(Y_{1}, X\right)=\left\{y_{i 1}, x_{i 1}, x_{i 2}\right\}_{i=1, \ldots, N}$. The auxiliary data-set contains the sample frequency of $y_{i 2}$ conditional on a subset of $x_{i}$.

## E. 1 Likelihood Approach

The likelihood of observing $\left(y_{i 1}, x_{i}\right)$ is:

$$
L\left(y_{i 1}, x_{i} ; \theta\right)=\left\{\begin{array}{cl}
\left(1-\Phi\left(-\alpha_{0}-\alpha_{1}\right)\right) \times\left(1-\Phi\left(-x_{i} \beta\right)\right) & \text { If } y_{i 1}=1  \tag{27}\\
+\left(1-\Phi\left(-\alpha_{0}\right)\right) \times \Phi\left(-x_{i} \beta\right) & \\
\Phi\left(-\alpha_{0}-\alpha_{1}\right) \times\left(1-\Phi\left(-x_{i} \beta\right)\right) & \text { If } y_{i 1}=0 \\
+\Phi\left(-\alpha_{0}\right) \times \Phi\left(-x_{i} \beta\right) &
\end{array}\right.
$$

The log-likelihood contribution of observation $i$ is thus given by:

$$
\begin{aligned}
& \log L\left(y_{i 1}, x_{i} ; \theta\right) \\
& =y_{i 1} \log \binom{\left(1-\Phi\left(-\alpha_{0}-\alpha_{1}\right)\right) \times\left(1-\Phi\left(-x_{i} \beta\right)\right)}{+\left(1-\Phi\left(-\alpha_{0}\right)\right) \times \Phi\left(-x_{i} \beta\right)}+\left(1-y_{i 1}\right) \log \binom{\Phi\left(-\alpha_{0}-\alpha_{1}\right) \times\left(1-\Phi\left(-x_{i} \beta\right)\right)}{+\Phi\left(-\alpha_{0}\right) \times \Phi\left(-x_{i} \beta\right)} \\
& =y_{i 1} \log P\left(x_{i} ; \theta\right)+\left(1-y_{i 1}\right) \log \left(1-P\left(x_{i} ; \theta\right)\right)
\end{aligned}
$$

The derivative of the log-likelihood contribution is given by:

$$
\begin{equation*}
s_{i, j}\left(y_{i 1}, x_{i} ; \theta\right)=\frac{\partial \log L\left(y_{i 1}, x_{i} ; \theta\right)}{\partial \theta_{j}}=\left(\frac{y_{i 1}}{P\left(x_{i} ; \theta\right)}-\frac{1-y_{i 1}}{1-P\left(x_{i} ; \theta\right)}\right) \frac{\partial P\left(x_{i} ; \theta\right)}{\partial \theta_{j}} \tag{28}
\end{equation*}
$$

Let $s_{i}\left(y_{i}, x_{i} ; \theta\right)$ denote a $1 \times|\theta|$ vector of partial derivatives.
In addition, using an auxiliary data-set we observe the average of the second outcome variable conditional on one of the $X s$. In particular, let $x_{i}=\left\{x_{i}^{(1)}, x_{i}^{(2)}\right\}$, where $x_{i}^{(1)}$ takes $K$ discrete values. Let $M_{k}$ denote the number of observations in the second survey for which $x_{i}^{(1)}=k$. Assume that the $x_{i}$ in both surveys are drawn from the same distribution, and that $\max _{k}\left\{M_{k}\right\} \ll N$.

Let $h_{k}^{*}(\theta)$ denote the "true" conditional average of $y_{i 2}$ in the population, given parameter values $\theta$. Since $N$ is large, we estimate this moment using the conditional distribution of $x_{i}^{(2)}$ in the large sample:

$$
\begin{equation*}
h_{k}^{*}(\theta)=\int\left(1-\Phi\left(-x_{i} \beta\right)\right) d G\left(x_{i}^{(2)} \mid x_{i}^{(1)}=k\right) \approx \frac{1}{N_{k}} \sum_{i} 1\left(x_{i}^{(1)}=k\right)\left(1-\Phi\left(-x_{i} \beta\right)\right), \tag{29}
\end{equation*}
$$

where $N_{k}$ is the number of individuals in the large survey with $x_{i}^{(1)}=k$, and $G(\cdot)$ is the true conditional distribution of $x_{i}^{(2)}$.

In addition, let $\operatorname{var}_{k}(\theta)$ denote an estimate of the conditional variance of $\operatorname{Pr}\left(y_{i 2}=1 \mid x_{i}^{(1)}=k\right)$. As in the paper, we calculate this using the sample variance of $\operatorname{Pr}\left(y_{i 2}=1 \mid x_{i}\right)$ calculated in the large sample. That is, using the $N_{k}$ observations with $x_{i}^{(1)}=k$. Note that this variance depends on the parameter $\theta$. In the paper, we calculate this variance at a preliminary estimate of the $\theta$. In the Monte-Carlo simulations below, we calculate the variance at the true value of the parameters.

The sample analogue of the conditional expectation in the auxiliary data-set is given by:

$$
\begin{equation*}
\bar{h}_{k}=\frac{1}{M_{k}} \sum_{i} 1\left(x_{i 1}=k\right) y_{i 2} . \tag{30}
\end{equation*}
$$

From the central-limit theorem, we know that the asymptotic distribution of $\bar{h}_{k}$ can be described as:

$$
\begin{equation*}
\bar{h}_{k} \sim N\left(h_{k}^{*}(\theta), \frac{\operatorname{var}_{k}(\theta)}{M_{k}}\right) . \tag{31}
\end{equation*}
$$

Let $\sigma_{k}\left(\theta, M_{k}\right)=\sqrt{\frac{\operatorname{var}_{k}(\theta)}{M_{k}}}$, and define $W_{2}$ to be a $K \times K$ diagonal matrix with element $(k, k)$ equal to $\sigma_{k}\left(\theta, M_{k}\right)^{2}$. Also, let $g(\theta)=h-h^{*}(\theta)$ denote the $K \times 1$ vector of moments coming from the auxiliary data-set, with element $k$ given by: $g_{k}(\theta)=\bar{h}_{k}-h_{k}^{*}(\theta)$.

The density function for the external moments is given by:

$$
\begin{equation*}
f(\theta)=\frac{1}{\sqrt{\left|2 \pi W_{2}\right|}} \exp \left(-\frac{1}{2} g(\theta)^{T} W_{2}^{-1} g(\theta)\right) . \tag{32}
\end{equation*}
$$

We maximize the following quasi likelihood function:

$$
\begin{align*}
L(Y, X ; \theta) & =\sum_{i} y_{i 1} \log P\left(x_{i} ; \theta\right)+\left(1-y_{i 1}\right) \log \left(1-P\left(x_{i} ; \theta\right)\right)+\log f(\theta)  \tag{33}\\
& =\underbrace{\sum_{i} y_{i 1} \log P\left(x_{i} ; \theta\right)+\left(1-y_{i 1}\right) \log \left(1-P\left(x_{i} ; \theta\right)\right)}_{\text {Unconstrained likelihood }}-\underbrace{\frac{1}{2} g(\theta)^{T} W_{2}^{-1} g(\theta)}_{\text {External moments }}+\text { Constant }
\end{align*}
$$

Note that this likelihood is very similar to a constrained likelihood function. A constrained likelihood function would maximize the first component (i.e. unconstrained likelihood), subject to the constraint that the moments from the auxiliary data-set are satisfied. Under this interpretation the weighting matrix $W_{2}$ would be replaced by a vector of Lagrangian multipliers. Our formulation instead assumes that the weights are inversely proportional to the precision of the moments from the auxiliary data-set.

## E. 2 GMM Approach

To construct the analogue GMM problem we combine two sets of moment conditions: (i) the likelihood score (i.e. $|\theta|$ moments), and (ii) the auxiliary survey moments (i.e. $K$ ). Importantly, the likelihood score corresponds to the FOC of the unconstrained likelihood function defined above.

The element $j=1, \ldots,|\theta|$ in the first set of moments is given by:

$$
\begin{equation*}
s_{j}(\theta)=\frac{1}{N} \sum_{i} s_{i, j}\left(y_{i 1}, x_{i} ; \theta\right) \tag{34}
\end{equation*}
$$

where $s_{i, j}\left(y_{i 1}, x_{i} ; \theta\right)$ is defined above. Let $s(\theta)$ denote the vector $|\theta| \times 1$ of stacked moments. Further, let $W_{1}$ denote the $|\theta| \times|\theta|$ weighting matrix. This matrix can be estimated in a two-step procedure using the identity weighting matrix first, and a consistent estimate in the second-stage:

$$
\widehat{W}_{1}=\sum_{i} s_{i}\left(y_{i 1}, x_{i} ; \hat{\theta}\right) s_{i}\left(y_{i 1}, x_{i} ; \hat{\theta}\right)^{T}
$$

The moments from the auxiliary data-set take the same form as in equation 31 , with $W_{2}$ denoting the $K \times K$ weighting matrix. As we discussed above, in the Monte Carlo simulation we evaluate the two weighting matrices at the true parameter values, instead of performing a two-step procedure. The two sets of moments are combined to form the non-linear GMM problem:

$$
\begin{equation*}
\min _{\theta} m(\theta)^{T} W^{-1} m(\theta) \tag{35}
\end{equation*}
$$

where $m(\theta)=[s(\theta) ; g(\theta)]$ and $W$ is a block-diagonal matrix with dimension $(|\theta|+K) \times(|\theta|+K)$.

## E. 3 Monte Carlo

For the Monte Carlo exercise we use the discrete choice problem stated initially with the following parameter values: $\theta=\{0,0.5,-1.5,0.5,0.5\}$. We chose these parameter values so that the conditional means of $y_{1}$ and $y_{2}$ were close to 0.5 . In addition, $\left(\epsilon_{i 1}, \epsilon_{i 2}\right)^{\prime} \sim N(0, I)$ and we have the following regressors: $X_{1}$ has a discrete uniform distribution taking on $\mathrm{K}=5$ values, and $X_{2}$ is standard normal.

We report the average bias and root-mean-square-error (RMSE) for MLE and GMM, as well as results for MLE without using the auxiliary moment conditions for each of these cases. Note that GMM and MLE are equivalent without the additional moment conditions, but the ML problem is faster to solve. We have 10 moment conditions and 5 parameters. The 10 moment conditions are the 5 score functions and the 5 moment conditions coming from the small data-set: $\bar{h}-h^{*}(\theta)$. We set the sample size of the main data-set equal to 15,000 , and vary the size of the auxiliary data-set from 500 to 2000 . The survey data set consists of 841 households.

The results from the Monte Carlo show that both in terms of bias and RMSE the MLE estimator performs somewhat better or equivalently to the GMM estimator. The biggest difference between the two approaches is in the estimation of the intercept $\left(\beta_{0}\right)$ in the 'search' equation, with MLE outperforming GMM. Otherwise the approaches are similar. Note also that because we use the likelihood scores as the moment conditions in GMM, it is much more sensitive to numerical problems. The RMSE and bias reported exclude outliers, which happens in the case of GMM but not for MLE. Finally, MLE without the auxiliary data performs much worse than the other two approaches. The same is obviously true for GMM without the auxiliary data since the optimization problem is identical. It is important to highlight that the model is only weakly identified without the auxiliary information. This is especially true for the intercept in the "search" equation $\left(\beta_{0}\right)$.

In conclusion, the results from Monte Carlo simulations demonstrate two points: (i) incorporating moments from the auxiliary sample substantially reduces the bias and improves the precision of the estimates (i.e. relative to the unconstrained MLE or GMM), and (ii) the improvements obtained using the MLE and GMM approaches are similar in magnitude. Importantly, the efficiency gains are sizeable despite the fact that the moments obtained from the auxiliary data-set are much less precise than those obtained from the main data-set. The MLE approach incorporates this feature by adjusting the asymptotic variance of the "moment errors", while the GMM approach calculates the efficient weighting matrix of the two sets of moment conditions accounting for the differences in the sample sizes.

Table 8: Monte Carlo

| M |  | True | Bias |  |  |  |  | RMSE |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | MLE | GMM | MLE (unc) | MLE | GMM | MLE (unc) |  |  |
| 500 | $\alpha_{0}$ | 0 | 0.000 | -0.001 | -0.079 | 0.012 | 0.016 | 0.336 |  |  |
|  | $\alpha_{1}$ | 0.5 | 0.000 | -0.014 | 0.173 | 0.021 | 0.026 | 0.625 |  |  |
|  | $\beta_{0}$ | -1.5 | 0.000 | -0.013 | 0.050 | 0.010 | 0.049 | 0.725 |  |  |
|  | $\beta_{1}$ | 0.5 | 0.000 | 0.057 | 0.019 | 0.017 | 0.082 | 0.222 |  |  |
|  | $\beta_{2}$ | 0.5 | -0.003 | 0.022 | 0.080 | 0.080 | 0.078 | 0.246 |  |  |
| 1000 |  |  |  |  |  |  |  |  |  |  |
|  | $\alpha_{0}$ | 0 | 0.000 | -0.008 | -0.079 | 0.014 | 0.014 | 0.336 |  |  |
|  | $\alpha_{1}$ | 0.5 | -0.000 | -0.008 | 0.173 | 0.024 | 0.016 | 0.625 |  |  |
|  | $\beta_{0}$ | -1.5 | -0.000 | -0.002 | 0.050 | 0.009 | 0.030 | 0.725 |  |  |
|  | $\beta_{1}$ | 0.5 | 0.000 | 0.040 | 0.019 | 0.014 | 0.067 | 0.222 |  |  |
|  | $\beta_{2}$ | 0.5 | -0.003 | 0.006 | 0.080 | 0.085 | 0.045 | 0.246 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| 2000 | $\alpha_{0}$ | 0 | 0.000 | -0.004 | -0.079 | 0.015 | 0.011 | 0.336 |  |  |
|  | $\alpha_{1}$ | 0.5 | -0.000 | -0.004 | 0.173 | 0.028 | 0.014 | 0.625 |  |  |
|  | $\beta_{0}$ | -1.5 | 0.000 | 0.003 | 0.050 | 0.008 | 0.011 | 0.725 |  |  |
|  | $\beta_{1}$ | 0.5 | 0.000 | 0.030 | 0.019 | 0.010 | 0.049 | 0.222 |  |  |
|  | $\beta_{2}$ | 0.5 | 0.001 | 0.000 | 0.080 | 0.082 | 0.021 | 0.246 |  |  |

[^25]
## F Goodness of Fit

In this appendix we provide further details on the goodness of fit.
Table 4 in the text showed that the predicted and observed discount distributions are also very similar, but the model tends to under-predict the median discount ( 86.7 vs 95 bps ), as well as the fraction of borrowers paying the posted rate (i.e. $9.2 \%$ vs $12.7 \%$ ). Figure 2 shows that the shortcomings can largely be explained by the fact that the predicted distribution of discounts is smoother than the empirical distribution. For instance, when we group discounts into 25 bps bins, the model accurately predicts the fraction of consumers receiving zero or very small discounts, suggesting that few consumers in the observed sample receive discounts between 0 and 12 bps . Similarly, the empirical distribution of discounts exhibits a large mass around 100 bps , and as a result the density is sharply decreasing between 0 and 50 . This is consistent with some lenders using 100 bps as a focal point discount. The model does not have any such prediction. Instead, the model predicts a smoother decrease in the density between 0 and 50 , and a less pronounced peak at 100 pbs.

Figure 2: Predicted and observed distribution of negotiated discounts


Figure 3 shows that the model reproduces very well the lenders' aggregate market shares.
In Table 9, we contrast the predicted search probabilities from the model, with the average frequencies reported in the national survey of new home buyers. We report the probabilities by income, city-size, and region. The last two columns correspond to auxiliary moments in the likelihood function. The first column reports the predictions from the baseline specification. On average, the model predicts that $65.7 \%$ of consumers reject the initial offer and search, compared to $62.5 \%$ in the survey. This difference is significantly different from zero at a $10 \%$ significance level.

The model reproduces the general patterns of the survey across regions and city sizes, but

Figure 3: Observed and predicted market shares

tends to under-estimate the amount of heterogeneity across demographics groups. For instance, the survey suggests that there is a 10 percentage point difference in the search probabilities for small and large cities, while the model implies that the difference is $5 \%$. Similarly, both the model and the survey predict that high income borrowers search more. The baseline specification predicts that low income borrowers search with probability $63.9 \%$ compared to $59.6 \%$ in the survey. Most of the differences between the model-predicted probabilities and the survey results are not statistically significant.

In columns (2) and (3) we report the predicted search probabilities from the two alternative specifications in Table 11, which vary the weight placed on the search moments. As discussed above, when the search moments are not used in the estimation (middle column), the model tends to predict a larger search probability (69.4), but reproduces the same qualitative patterns from the survey. The differences are now statistically significant from zero in 6 out of 10 cases. In contrast, by assigning larger weights to the search moments (third column), the model is able to reproduce almost perfectly the survey predictions.

In Table 10 we evaluate the ability of the model to reproduce the reduced-form relationships observed in the data between rates, loyalty, and transaction characteristics. Regressions in columns (1)-(3) are estimated using the observed sample, and regressions in columns (4)-(6) are estimated using the simulated sample from the baseline specification. In columns (1) and (4), we regress discounts on characteristics, using the sample of consumers paying less than the posted-rate. In the remaining columns we estimate linear probability models describing the probability of paying the posted-rate ((2) and (5)), and the probability of remaining loyal to the home bank ((3)and (6)).

In general, the model does a good job at predicting the relationship between discounts and financial attributes. The $R^{2} \mathrm{~s}$ from the different specifications are nearly identical, suggesting that

Table 9: Observed and predicted search probability by demographic groups

|  | Model predictions |  |  |  | Survey data |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Baseline <br> $(\rho=1)$ | Zero moment <br> weight $(\rho=0)$ | Large moment <br> weight $(\rho=100)$ | Freq. | $M_{g}$ |  |
| City size |  |  |  |  |  |  |
| Pop. $>1 \mathrm{M}$ | 0.673 | $0.717^{b}$ | 0.661 | 0.660 | 338 |  |
| $1 \mathrm{M}>$ Pop. $>100 \mathrm{~K}$ | 0.657 | 0.695 | 0.639 | 0.654 | 268 |  |
| Pop. $\leq 100 \mathrm{~K}$ | $0.628^{b}$ | $0.655^{a}$ | 0.584 | 0.560 | 275 |  |
| Regions |  |  |  |  |  |  |
| East | $0.626^{a}$ | $0.656^{a}$ | 0.582 | 0.557 | 289 |  |
| West | 0.651 | $0.688^{c}$ | 0.628 | 0.643 | 327 |  |
| Ontario | 0.673 | 0.713 | 0.659 | 0.668 | 265 |  |
| Income |  |  |  |  |  |  |
| $>\$ 60 \mathrm{~K}$ | $0.639^{a}$ | $0.670^{a}$ | 0.586 | 0.579 | 400 |  |
| $\leq \$ 60 \mathrm{~K}$ | 0.669 | $0.712^{b}$ | 0.670 | 0.666 | 441 |  |
| Total | $0.657^{c}$ | $0.694^{a}$ | 0.635 | 0.625 | 841 |  |

The simulated sample is obtained by simulating 100,000 contracts from the model, and dropping consumers who fail to qualify for a loan (5.5\%). Source: FIRM survey by Ipso Reid. Null hypothesis: Survey average $=$ Model average. Significance levels: $a=1 \%, b=5 \%, c=10 \%$. P-values are calculated using the asymptotic standard-errors of the survey.
the model predicts more or less the same magnitude of residual rate dispersion observed in the data. The regression coefficients for loan-to-value and FICO scores are also similar in the simulated and observed samples, and the model captures well the non-linear relationship between income/loan size, and discounts (see marginal effects at the bottom). Similarly, the effect of competition on the probability of obtaining a discount and the magnitude of discounts has the same sign and similar magnitudes in both samples.

The fit of the model is not as good when it comes to the rate difference between loyal and switching consumers. In the data we estimate that loyal consumers obtain on average 9.1 bps lower discounts than do switching consumers, and are $2.3 \%$ more likely to pay the posted rate. In contrast, the model predicts that loyal consumers have 16 bps lower discounts, and a $5.9 \%$ greater probability of paying the posted-rate. This is because the model is restrictive in terms of the timing of moves, such that "switching" consumers must have rejected an initial offer and must pay a competitive price. In practice, the timing of moves probably differs across consumers, in ways that we cannot measure.

This type of measurement error likely explains why the model does a relatively poor job of matching the reduced-form loyalty probability regression. Since search cost and loyalty advantage depend on the ownership-status of borrowers, the model is able to reproduce very well the fact that previous owners are over $10 \%$ more likely to remain loyal to their home banks. The model is also able to match the sign of the relationships between loyalty and key attributes of the transaction: consumers are more likely to switch in more competitive markets, more likely to remain loyal to a large network institution, more likely to switch when financing a larger loan, and more likely to remain loyal when of high income. However, the magnitudes of these marginal effects are not always accurate.

Table 10: Reduced-form discount and loyalty rate regressions

|  | Observed Sample |  |  | Simulated Sample |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ |
|  | Discount | $1($ Disc. $=0)$ | Loyal | Discount | $1($ Disc. $=0)$ | Loyal |
| VARIABLES | $-0.091^{a}$ | $0.023^{a}$ |  | $-0.16^{a}$ | $0.059^{a}$ |  |
| Loyal dummy | $(0.0075)$ | $(0.0053)$ |  | $(0.0022)$ | $(0.0011)$ |  |
|  | 0.010 | 0.0083 | 0.011 | 0.0060 | -0.0015 | $-0.016^{a}$ |
| Total loan (X 100K) | $(0.018)$ | $(0.013)$ | $(0.017)$ | $(0.0055)$ | $(0.0034)$ | $(0.0047)$ |
|  | $0.20^{a}$ | $-0.13^{a}$ | -0.0033 | $0.23^{a}$ | $-0.098^{a}$ | $0.018^{b}$ |
| Annual income (X 100K) | $(0.034)$ | $(0.025)$ | $(0.030)$ | $(0.010)$ | $(0.0068)$ | $(0.0086)$ |
|  | $0.100^{a}$ | $-0.081^{a}$ | $-0.050^{a}$ | $0.11^{a}$ | $-0.060^{a}$ | $-0.015^{a}$ |
| Loan/Income | $(0.014)$ | $(0.0096)$ | $(0.013)$ | $(0.0042)$ | $(0.0026)$ | $(0.0036)$ |
|  | $-0.022^{a}$ | 0.0036 | $0.11^{a}$ | $-0.017^{a}$ | $0.013^{a}$ | $0.13^{a}$ |
| Previous home-owner | $(0.0080)$ | $(0.0057)$ | $(0.0072)$ | $(0.0024)$ | $(0.0014)$ | $(0.0020)$ |
|  | $0.61^{a}$ | $-0.31^{a}$ | $0.27^{a}$ | $0.73^{a}$ | $-0.23^{a}$ | -0.0056 |
| FICO (mid-point) | $(0.045)$ | $(0.034)$ | $(0.045)$ | $(0.014)$ | $(0.0085)$ | $(0.012)$ |
|  | $-0.63^{a}$ | $0.37^{a}$ | -0.14 | $-0.77^{a}$ | $0.19^{a}$ | -0.030 |
| Loan to Value Ratio | $(0.11)$ | $(0.073)$ | $(0.10)$ | $(0.034)$ | $(0.017)$ | $(0.029)$ |
|  | $-0.023^{b}$ | $0.013^{c}$ | $-0.021^{b}$ | $-0.034^{a}$ | $0.012^{a}$ | 0.0033 |
| LTV = 0.95 | $(0.0098)$ | $(0.0072)$ | $(0.0094)$ | $(0.0030)$ | $(0.0017)$ | $(0.0026)$ |
| Posted-rate spread | $0.30^{a}$ | $-0.13^{a}$ | $-0.039^{c}$ | $0.70^{a}$ | $-0.20^{a}$ | -0.0023 |
|  | $(0.023)$ | $(0.015)$ | $(0.021)$ | $(0.0069)$ | $(0.0041)$ | $(0.0060)$ |
| Bond rate | $0.27^{a}$ | $-0.13^{a}$ | $-0.039^{c}$ | $0.48^{a}$ | $-0.14^{a}$ | 0.0076 |
| Relative network size | $(0.023)$ | $(0.015)$ | $(0.021)$ | $(0.0070)$ | $(0.0039)$ | $(0.0061)$ |
|  | $-0.0094^{b}$ | $0.019^{a}$ | $0.026^{a}$ | $0.0055^{a}$ | 0.00064 | $0.048^{a}$ |
| Nb. Lenders (log) | $(0.0045)$ | $(0.0036)$ | $(0.0045)$ | $(0.0013)$ | $(0.00077)$ | $(0.0014)$ |
| Observations | $0.085^{a}$ | $-0.070^{a}$ | $-0.076^{a}$ | $0.059^{a}$ | $-0.040^{a}$ | $-0.17^{a}$ |
| R-squared | $(0.021)$ | $(0.016)$ | $(0.019)$ | $(0.0062)$ | $(0.0045)$ | $(0.0052)$ |
| Marginal effect: income | -0.16 | 0.17 | 0.18 | -0.18 | 0.12 | 0.071 |
| Marginal effect: loan | -0.34 | 0.18 | -0.19 | -0.43 | 0.054 | -0.065 |

The simulated sample is obtained by simulating 300,000 contracts from the model, and dropping consumers who fail to qualify for a loan ( $5.5 \%$ ). Sample selection: All specifications exclude mortgages originated from lender(s) with missing loyalty variable, and specifications (1) and (4) exclude transactions with zero discounts. Dependent variables: Discount $=$ Posted rate - negotiated negotiated rate (if $\left.\bar{r}>r_{i}\right), 1($ Disc. $=0)=$ Indicator variable equal to 1 Discount $>0$, Loyal $=$ indicator variable equal to 1 if lender is $h$. Control variables: Income, loan size, loan/income, 5-year bondrate, region fixed-effects, year/quarter fixed-effects. Robust standard errors in parentheses. Significance levels: ${ }^{a}=$ $1 \%,^{b}=5 \%,^{c}=10 \%$.

## G Robustness

Table 11: Maximum likelihood estimation results with or without extra weight on search moments

|  | Specification 4 |  | Specification 5 |  | Specification 6 |  | Specification 7 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | est | se | est | se | est | se | est | se |
| Heterogeneity |  |  |  |  |  |  |  |  |
| Common shock ( $\sigma_{c}$ ) | 0.356 | (0.003) | 0.357 | (0.003) | 0.365 | (0.004) | 0.365 | 0.004 |
| Idiosyncratic shock ( $\sigma_{\omega}$ ) | 0.120 | (0.003) | 0.112 | (0.003) | 0.108 | (0.002) | 0.111 | 0.002 |
| Avg. search cost ( $\log$ ) |  |  |  |  |  |  |  |  |
| $\alpha_{0}$ | -1.644 | (0.042) | -1.766 | (0.045) | -1.711 | (0.023) | -1.766 | 0.045 |
| $\alpha_{\text {inc }}$ | 0.399 | (0.068) | 0.325 | (0.066) | $-0.363$ | (0.024) | 0.325 | 0.066 |
| $\alpha_{\text {owner }}$ | 0.004 | (0.08) | 0.103 | (0.082) | 0.319 | (0.038) | 0.103 | 0.082 |
| Home-bank WTP |  |  |  |  |  |  |  |  |
| $\lambda_{0}$ |  |  | 0.011 | (0.008) |  |  | -0.051 | 0.008 |
| $\lambda_{\text {owner }}$ |  |  | -0.019 | (0.008) |  |  | -0.032 | 0.008 |
| $\lambda_{\text {inc }}$ |  |  | 0.023 | (0.011) |  |  | 0.074 | 0.01 |
| Home-bank cost-adv. |  |  |  |  |  |  |  |  |
| $\Delta_{0}$ | 0.176 | (0.009) | 0.154 | (0.01) | 0.18 | (0.006) | 0.23 | 0.01 |
| $\Delta_{\text {owner }}$ | 0.078 | (0.005) | 0.089 | (0.009) | 0.066 | (0.004) | 0.097 | 0.009 |
| $\Delta_{\text {inc }}$ | 0.019 | (0.007) | -0.001 | (0.011) | 0.008 | (0.006) | -0.061 | 0.011 |
| Cost function |  |  |  |  |  |  |  |  |
| Intercept | 5.548 | (0.253) | 5.532 | (0.228) | 5.414 | (0.223) | 5.455 | 0.243 |
| Bond rate | 0.308 | (0.02) | 0.308 | (0.025) | 0.291 | (0.028) | 0.29 | 0.027 |
| Range posted-rate | -0.144 | (0.018) | -0.145 | (0.017) | -0.153 | (0.018) | -0.153 | 0.018 |
| Total loan | -0.200 | (0.127) | -0.201 | (0.073) | -0.276 | (0.064) | -0.27 | 0.078 |
| Income | -0.207 | (0.026) | -0.218 | (0.027) | -0.199 | (0.027) | -0.261 | 0.028 |
| Loan/Income | -0.102 | (0.01) | -0.103 | (0.01) | -0.114 | (0.011) | -0.118 | 0.011 |
| Previous owner | 0.055 | (0.008) | 0.059 | (0.009) | 0.043 | (0.007) | 0.065 | 0.008 |
| House price | 0.202 | (0.116) | 0.202 | (0.066) | 0.279 | (0.057) | 0.281 | 0.07 |
| FICO | -0.652 | (0.037) | -0.656 | (0.038) | -0.68 | (0.039) | -0.681 | 0.04 |
| LTV | 1.062 | (0.32) | 1.063 | (0.156) | 1.303 | (0.141) | 1.314 | 0.172 |
| $1(L T V=95 \%)$ | 0.029 | (0.01) | 0.029 | (0.008) | 0.03 | (0.008) | 0.029 | 0.008 |
| Rel. network size | -0.046 | (0.002) | -0.043 | (0.002) | -0.04 | (0.002) | -0.041 | 0.002 |
| Range of Bank FE | [-0.103 | , 0.070 ] | [-0.096 | , 0.066 ] | [-0.084 | , 0.070 ] | [-0.086 | 0.072 ] |
| Quarter-year FE |  |  |  |  |  |  |  |  |
| Region FE |  | , |  |  |  |  |  |  |
| Sample size |  | 218 |  | 218 |  | 218 |  |  |
| LLF/N |  | 246 |  | , 045 |  | 989 |  |  |
| Search moments weight |  | , |  | ) |  | 0 |  |  |
| Likelihood ratio test ( $\chi^{2}(3)$ ) |  | 226 |  |  |  | 890 |  |  |

Units: $\$ / 100$ per month. Average search cost function: $\log \alpha\left(z_{i}^{1}\right)=\alpha_{0}+\alpha_{\text {inc }} \log \left(\operatorname{Income}_{i}\right)+\alpha_{o w n e r} \mathrm{EHB}_{i}$. Homebank willingness-to-pay: $\lambda\left(z_{i}^{2}\right)=L_{i} \times\left(\lambda_{0}+\lambda_{\left.\text {inc }_{c} \text { Income }_{i}+\lambda_{\text {owner }} \text { Previous owner }_{i}\right) \text {. Home-bank cost advantage: }}\right.$ $\Delta\left(z_{i}^{2}\right)=L_{i} \times\left(\Delta_{0}+\Delta_{\text {inc }}\right.$ Income $_{i}+\Delta_{\text {owner }}$ Previous owner $\left._{i}\right)$. Cost function: $c_{i j}=L_{i} \times\left(c_{i}+\omega_{i j}\right)$, where $c_{i} \sim N\left(x_{i} \beta, \sigma_{c}^{2}\right)$ and $\omega_{i j} \sim \operatorname{T1EV}\left(\bar{\xi}_{j}+\xi_{\text {branch }}\right.$ Rel. network $\left.\operatorname{size}_{i j}-e \sigma_{\omega}, \sigma_{\omega}\right)$. The likelihood-ratio test reported in the last row test Model 1 and 2 against Model 3 (alternative hypothesis). The $1 \%$ significance level critical value is 16.266. Specification 2 is our baseline model.


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[^1]:    ${ }^{1}$ As a result we cannot use recent approaches based on the simultaneous complete-information multi-lateral negotiation game proposed by Horn and Wolinsky (1988) that have modeled the outside option as observed prices paid by a given buyer to alternative suppliers (Crawford and Yurukoglu (2012), Grennan (2013), Lewis and Pflum (2015), Gowrisankaran et al. (2015) and Ho and Lee (2017)).
    ${ }^{2}$ For instance, the data-set used by Goldberg (1996) contains information on the price the consumer paid, the brand of the purchased vehicle and whether the consumer previously bought the same brand. Cicala (2015) has data on coal deliveries to power plants and transaction prices, while Salz (2015) has information on the contract terms between businesses and waste carters. Their data-sets allow them to measure the duration of contractual relationships with incumbent suppliers.

[^2]:    ${ }^{3}$ Appendix A describes the insurance rules, and defines all of the variables included in the data-set.

[^3]:    ${ }^{4}$ In Canada pricing over the posted rate is illegal, and therefore this is a natural assumption. A similar setup is implied in other retail markets featuring negotiation in the presence of manufacturer's suggested retail prices.
    ${ }^{5}$ Local branch managers compete against rival banks, but not against other branches of the same bank. Brokers are "hired" by borrowers to gather the best quotes from multiple lenders but compensated by lenders.
    ${ }^{6}$ The FSA is the first half of a postal code. We observe nearly $1,300 \mathrm{FSA}$ in the sample. While the average FSA has a radius of 7.6 kilometers, the median is 2.6 kilometers.
    ${ }^{7}$ Based on the closing date we construct a posted price associated with each contract as the posted rate at closing if this yields a non-negative discount. If the generated discount is negative, the posted rate is taken to be the nearest-one that yields a positive discount.
    ${ }^{8}$ The "Other Bank" category includes mostly two institutions: Laurentian Bank and HSBC. The former is only present in Québec and Eastern Ontario, while the latter is present mostly in British Colombia and Ontario. We exploit this geographic segmentation and assign the "Other banks" customers to HSBC or Laurentian based on their relative presence in the local market around each home location.

[^4]:    ${ }^{9}$ Source: Consumer finance monitor (CFM), Ipsos-Reid, 1999-2002.
    ${ }^{10}$ Our results are robust to alternative neighborhood size definitions. We also considered a 5 KM neighborhood, since this captures the fact that the average distance to chosen lenders is about 2 KM , compared to slightly less than 4KM for the average distance to other financial institutions. Results using this definition are available upon request.

[^5]:    Sample size $=26,218$. Number of missing loyal observations $=5,599$. The sample covers the period from Jan 1999 to Oct 2002. We trim the top and bottom $0.5 \%$ of observations in terms of income and loan size. Interest rates and discounts are expressed in percentage basis points (bps). The number of lenders is within 10 KM of the borrowers new home (neighborhood). Relative branch is defined as the average network size of the chosen institution relative to the average size of others present in the same neighborhood. Each regression also includes market and quarter/year fixed-effects, and other financial characteristics (i.e. posted-rate, bond-rate, FICO score, LTV, 1(LTV Max), loan size, income, loan/income.). Robust standard errors in parenthesis. Significance levels: ${ }^{a} \mathrm{p}<0.01,{ }^{b} \mathrm{p}<0.05,{ }^{c} \mathrm{p}<0.1$.

[^6]:    ${ }^{11}$ While lenders are fully insured against default risk, the event of default implies additional transaction costs to lenders that lower the value of lending to risky borrowers. Pre-payment risk is perhaps more relevant in our context, since consumers are allowed to reimburse up to $20 \%$ of their mortgage every year without penalty.
    ${ }^{12}$ For instance, credit cards have a $50 \%-60 \%$ return on equity ( ROE ), compared to Canadian banks' overall ROE of $16 \%$.
    ${ }^{13}$ Note that we rule out the possibility that the incumbent bank has more information than other lenders, since otherwise, the problem would involve adverse selection, and the initial quote would be much more complicated. For a discussion about competition when one firm has more information about a consumer learned from their past purchases see Fudenberg and Villas-Boas (2007). A subset of this literature has focused on credit markets and the extent to which lenders can learn about the ability of their borrowers to repay loans and use this information in their future credit-decisions and pricing. See for instance Dell'Ariccia et al. (1999).
    ${ }^{14}$ As mentioned above there is almost no dispersion in posted price and so we assume that every lender has the same posted rate.

[^7]:    ${ }^{15}$ Beckert et al. (2016) take a different approach, by assuming that consumers and firms split the known surplus from the auction using a Nash-Bargaining protocol. In this context, the relative bargaining power of consumers, instead of the search cost distribution, determines the split of the surplus.
    ${ }^{16}$ Our approach is close in spirit to the literature in both labor economics and finance studying search and matching frictions in markets with bargaining (see for instance Postel-Vinay and Robin (2002) for on-the-job search and Duffie et al. (2005) for over-the-counter markets). More recently, the application of auction-like models to price negotiation settings has been used in the context of business-to-business transactions (e.g. Beckert et al. (2016), Salz (2015)), and consumer markets (e.g. Hall and Woodward (2012) and Allen et al. (2014a)).

[^8]:    ${ }^{17}$ Equations (3) and (4) also highlight the fact that the transaction price is determined by three lenders: the home bank, and the two most cost-efficient lenders. Therefore, while we assume that consumers search the entire choice set, an implication of the model is that consumers need to obtain formal quotes from at most three lenders. This is in line with a Bertrand-Nash interpretation of the game, in which consumers learn lenders' cost ranking after paying the search cost, for instance through advertising, by calling banks directly, or indirectly through a real-estate agent.

[^9]:    ${ }^{18}$ This is similar to the identification at infinity arguments used in labor economics to study "Roy-type" models (e.g. French and Taber (2011)).

[^10]:    ${ }^{19}$ Under the null hypothesis that search costs are zero, the observed probability of remaining loyal to the home bank is equal to the retention probability, $G_{(n)}(-\gamma)$. This probability can be calculated from data on the price paid by switching consumers.

[^11]:    ${ }^{20}$ The location parameter of the type-1 extreme-value distribution is adjusted by a factor $e \sigma_{\omega}$ to guarantee that the error is mean zero (i.e $e$ is the euler constant).

[^12]:    ${ }^{21}$ We use $\mathcal{N}_{i}$ rather than $n_{i}$ to characterize the choice set of consumers, since the identities of banks present in each neighborhood (not just the number) enter the distribution of lending costs.

[^13]:    ${ }^{22}$ The density $g_{-b}\left(\omega \mid \mathcal{N}_{i}\right)$ is $g_{(1)}(\omega \mid \mathcal{N} \backslash b)$.
    ${ }^{23}$ This reduces the smoothness of the likelihood, affecting primarily the parameters determining $\lambda_{i}$. To remedy this problem we smooth the likelihood by multiplying the second term in equation (11) by $\left(1+\exp \left(\left(\lambda_{i}-\bar{p}_{t(i)}+p_{i}\right) / s\right)\right)^{-1}$, where $s$ is a smoothing parameter set to 0.01 .

[^14]:    ${ }^{24}$ The parameters are estimated by maximizing the aggregate log-likelihood function using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) numerical optimization algorithm within the Ox programming language (Doornik 2007).
    ${ }^{25}$ We estimate $\sigma_{g}$ by calculating the within group variance in search probability using the sample of individual contracts. Since this variance depends on the model parameter values, we follow a two-step approach: (i) calculate $\sigma_{g}$ using an initial estimate of $\theta$ (e.g. starting with $\sigma_{g}=1$ ), and (ii) hold $\sigma_{g}$ fixed to estimate $\hat{\theta} . \hat{W}_{2}$ is a diagonal matrix with element $(g, g)$ given by $2 \hat{\sigma}_{g}^{2} / M_{g}$. The multiple 2 is coming from the fact that the log of the normal density is proportional to: $-0.5(x-\mu)^{\prime} \Sigma^{-1}(x-\mu)$.

[^15]:    ${ }^{26}$ In the empirical application, we compare the results for two values of the Lagrangian multiplier: $\rho=1, \rho=100$.

[^16]:    ${ }^{27}$ The standard deviation of an extreme-value random variable is equal to $\sigma_{\omega} \pi / \sqrt{6}$, or 0.102 in our case.

[^17]:    Units: $\$ / 100$ per month. Average search cost function: $\log \alpha\left(z_{i}^{1}\right)=\alpha_{0}+\alpha_{\text {inc }} \log \left(\right.$ Income $\left._{i}\right)+$ $\alpha_{\text {owner }}$ Previous owner $_{i}$. Home-bank willingness-to-pay: $\lambda\left(z_{i}^{2}\right)=L_{i} \times\left(\lambda_{0}+\lambda_{\text {inc }}\right.$ Income $_{i}+\lambda_{\text {owner }}$ Previous owner $\left._{i}\right)$. Home-bank cost advantage: $\Delta\left(z_{i}^{2}\right)=L_{i} \times\left(\Delta_{0}+\Delta_{\text {inc }}\right.$ Income $_{i}+\Delta_{\text {owner }}$ Previous owner $\left._{i}\right)$. Cost function: $c_{i j}=$ $L_{i} \times\left(c_{i}+\omega_{i j}\right)$, where $c_{i} \sim N\left(x_{i} \beta, \sigma_{c}^{2}\right)$ and $\omega_{i j} \sim \operatorname{T1EV}\left(\bar{\xi}_{j}+\xi_{\text {branch }}\right.$ Rel. network size $\left.{ }_{i j}-e \sigma_{\omega}, \sigma_{\omega}\right)$. The likelihoodratio test reported in the last row test Model 1 and 2 against Model 3 (alternative hypothesis). The $1 \%$ significance level critical value is 16.266 . Specification 2 is our baseline model. Robust standard errors are placed in parenthesis (White 1982).

[^18]:    ${ }^{28}$ Most of mortgage contracts in Canada involve substantial financial penalties for borrowers who decide to pre-pay their mortgage before the end of the 5 -year term period. Borrowers are free to switch lenders after this period. It is therefore reasonable to use the term period length as the planning horizon.

[^19]:    The simulated sample is obtained by simulating 300,000 contracts from the baseline model, and dropping consumers who fail to qualify for a loan (5.5\%). Spread is defined as transaction rate minus bond rate.

[^20]:    ${ }^{29}$ An alternative approach for eliminating the first-mover advantage is to set consumer search costs to zero. We chose instead to modify the order of moves by setting $\phi_{i j}=1 / N$, since doing so does not fundamentally change the degree of competition in the market. The zero search-cost counterfactual yields very similar conclusions. Results are available from the authors upon request.
    ${ }^{30}$ All ratios would be equal to one if the difference between lenders were caused only by idiosyncratic cost differences.

[^21]:    Some variables were only included by one of the mortgage insurers.

[^22]:    ${ }^{31}$ This is, in fact, not a guideline, but a legal requirement for regulated lenders.

[^23]:    ${ }^{32}$ This result is analogous to the identification at infinity arguments used in labor economics to identify Roy models (e.g. French and Taber (2011)).

[^24]:    ${ }^{33}$ The proof is available from the authors upon request.

[^25]:    Each entry is an average over 1,000 Monte Carlo simulated samples. The unconstrained MLE problem is equivalent to solving the GMM problem with only the scores, i.e. it does not use information form the auxiliary data set. The MLE problem is faster to solve.

